Comparing Customer Taste Distributions in Vertically Differentiated Mobile Service Markets

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Abstract. In this paper, we study a vertically differentiated duopoly market, where competitors (mobile service providers) offer mobile subscriptions to customers, who diversify in their preferences regarding price and quality. We consider a two-stage game where the players first select the quality and then begin a competitive process for the price or quantity, which is widely known as Bertrand or Cournot game, respectively. To capture the service provider strategy, we first introduce variable costs to improve the quality, which are linear in quality per a subscription, and then derive the market-related metrics of interest for the tractable uniform distribution of the customer's taste parameter. Further relaxing this strong assumption, we provide with a numerical procedure that helps characterize an arbitrary taste distribution as well as an arbitrary cost function. Finally, selected numerical examples report on the comparison between the uniform and the truncated exponential distribution, thus accentuating the importance of choosing an appropriate customer taste model.

1 Introduction

The telecommunications industry has already entered a new phase of its evolution, where the focus has shifted from the conventional multimedia transmission to the ubiquitous connectivity and massive traffic volumes driven by growing human demand for data as well as supported by the emerging innovations, such as the Internet of Things, wearables, and more far-fetched autonomous vehicles [1]. On this market that crossed the 100% penetration mark, competition of mobile service providers for increased market share and retention of customers becomes a vital part of their strategy.

One of the key marketing strategies for competitors to seek profitable niches is product differentiation and pricing [2]. In particular, *horizontal* differentiation refers to immeasurable distinctions in virtually identical products, such as in design or color, which are not sufficient for the mobile service provider (SP) to attract new customers, who are willing to acquire a better level of service. In contrast to that, *vertical* market differentiation is objectively measurable and based on diverse quality levels of the products [3]. Here, customers are sensitive with respect to the relation between the quality and the price levels, and may have diverse preferences regarding it [4].

Generally, the market and pricing models have already attracted significant attention of the wireless community across a wide range of various problems, from market entrance decisions for mobile SPs [5] and competition over spectrum [6] to specific studies of social welfare in case when SPs exploit unlicensed spectrum [7]. However, to the best of our knowledge no prior work on vertical differentiation of mobile service markets has been contributed so far. In this paper, we study a duopoly model where mobile SPs first determine the specification of their offered services and then decide on the prices or the quantities of services they offer according to the Bertrand or

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Cournot competition models [8] (the initial market entry [9] is assumed to have been completed).

We consider both the price and the quantity competition as they lead to dissimilar equilibrium points, while there is still no consensus in past literature as to which type of competition should be preferred. We thus analyze both game models in order to reveal the dependence of the corresponding results on the optimal choice of the SP strategies, namely, whether SPs eventually offer a homogeneous product (as shown by the Cournot game) or two differentiated products (as illustrated by the Bertrand game). Since both situations may occur in the real market, one model cannot be preferred over another upfront.

Further, in modeling the mobile service markets an important role belongs to characterizing the costs of offering improved service quality. The majority of existing studies as in [9], [10], and [11] assume zero or fixed quality improvement cost, as well as adopt diminishing [12] or quadratic [13] formulations. This work assumes linear costs of quality improvement per unit of product as this can be tackled easily while being close to what the SPs may experience in practice.

As an indicator of customer preferences, we adopt the standard utility function [14], where the willingness of a customer to pay for a better quality service is represented by a random *taste parameter* [14]. While most of the game-theoretical references study the formulations by example of analytically tractable but arguably unrealistic uniform distribution of the taste parameter from "poor" to "rich", we in this work offer guidance on how to handle an *arbitrary taste distribution* and an *arbitrary cost function*.

The remainder of this paper is organized as follows. In Section 2, we outline our system model as well as describe the two-stage game played to divide the market and set the optimal price or quantity (in Bertrand or Cournot game, respectively). **Our contributions** appear in Sections 3 and 4, where, correspondingly, we provide analytical calculations for the conventional tractable example under the linear cost assumption and then detail our flexible numerical procedure to cope with an arbitrary formulation. Finally, we provide numerical comparison of the two considered options based on representative examples.

2 System model

In this work, we study a *vertically differentiated* mobile service market under the simplifying assumption of two operating SPs (*service providers*). In our formulation, the SP i may be characterized by a pair "price-quality" (p_i, s_i) and offer an unconstrained number of mobile subscriptions, each of which guaranteeing the announced mobile service quality s_i for the price p_i. Thus offered subscriptions (e.g., SIM-cards) may be purchased by a potentially large number of consumers, hereinafter named *customers*. Based on their preferences, customers may select only one SP or else refrain from buying anything.

We emphasize here that the products on a vertically differentiated market (in our case, subscriptions) may differ in both their quality and price. Moreover, the customers are *not identical* in their preferences due to diverse taste or budget restrictions, which results in varying willingness to pay for the offer [15], [4].

2.1 Characterization of the customers

Utility function of customers For differentiated markets, it is typically assumed that all of the customers agree on ranking the mobile service offers (subscriptions) in the order of quality preference according to some utility function based on a *taste parameter* [13]. The taste parameter θ reflects the customer's preference i.e., the more a customer agrees to pay for a better quality service – the higher the parameter θ becomes. In our study, we adopt the following utility function of θ [15], given the price p_i and the quality s_i offered by the SP i:

$$U(\theta, s_i, p_i) = \theta \cdot s_i - p_i, \tag{1}$$

where, $s_i = s(T_i)$ is an increasing quality function of data volume or rate T_i guaranteed by the SP. The function $s(T_i)$ is typically non-linear and often represented in literature by a logarithmic dependence, but may also be replaced by another, more appropriate choice.

Strategy of customers All of the customers are assumed to be *rational* i.e., the strategy of any customer is to maximize its utility $U(\theta, s, p)$ by choosing *exactly one* subscription of the SP i characterized by a pair (p_i, s_i) or, alternatively, refraining from buying anything at all. We note that zero utility value $U(\theta, s_i, p_i) \leq 0$ is equivalent to not purchasing the product i, and the case $U(\theta, s_i, p_i) = U(\theta, s_j, p_j)$ yields customer's indifference to buying product i or j.

Distribution of customers In order to be able to apply the Cournot competition model, we further assume that the considered market is not covered i.e., there always are customers who never participate [11, 15]. Therefore, θ should be distributed over the interval $[0, \theta_{\max}]$, where $\theta_{\max}$ corresponds to customers able to pay the most for a better quality. We assume that within this interval θ is distributed according to a certain probability density $h_\theta(\theta)$. Below, we compare two distinct distributions $h_\theta(\theta)$: the conventional and analytically tractable uniform distribution as well as the more realistic truncated exponential distribution, for which a numerical solution may be produced.

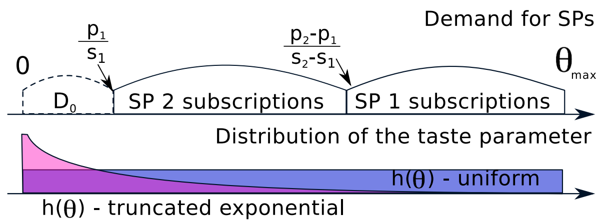

Fig. 1. Illustration of the target market structure.

2.2 Characterization of the SPs

Demand of the SPs Without loss of generality, we reorder our SPs such that $s_1 \geq s_2$. Due to the assumption on the rationality of customers, prices should also be rearranged in the non-decreasing order $p_1 \geq p_2$. For the fixed price and quality levels, we may obtain the following points of indifference for a tagged customer [13]:

- point of indifference to buying or not buying the service of the SP 2 is denoted by the parameter $\theta_{\varnothing,2} = \frac{p_2}{s_2}$ (follows from $U(\theta, s_2, p_2) = 0$),
- point of indifference to buying the service of the SP 2 or of the SP 1 corresponds to the parameter $\theta_{2,1} = \frac{p_1 - p_2}{s_1 - s_2}$ (follows from $U(\theta, s_1, p_1) = U(\theta, s_2, p_2)$).

The demand of the SPs may then be established as $D_1(\mathbf{s}; \mathbf{p}) = \int_{\theta_{2,1}}^{\theta_{\max}} h(\theta) d\theta$ and $D_2(\mathbf{s}; \mathbf{p}) = \int_{\theta_{\varnothing,2}}^{\theta_{2,1}} h(\theta) d\theta$, where $h(\theta)$ is the probability density function (PDF) of the taste parameter θ.

Profit of the SPs When making their decisions, the SPs abide by the principle of maximizing their *profit*, which is determined by the financial flow from the subscribed customers and depends on the structure of the costs. We assume that *linear costs* are incurred when improving the claimed quality s_i *per user*, so that the SP would be ready to support the respective quality of service (QoS) level for its subscribed customers. Hence, the total costs depend both on the number of served customers and on the selected quality level. These costs may reflect, for example, the initial investments into a fixed-term spectrum lease and/or the amounts of spectrum that could be resold (as e.g., in [16] or [17]).

Further, our profit function may be written as $\Pi_i(\mathbf{s}, \mathbf{p}) = D_i(\mathbf{s}, \mathbf{p})\,(p_i - \nu s_i)$, where ν is the quality cost coefficient. The latter may be roughly estimated from the value of the spectrum license costs to support the announced QoS, normalized by unit time as well as the total number of customers in the region of interest. We note that our assumption on the linear costs is relaxed in Section 4 and replaced by another suitable function.

2.3 Two-stage differentiated market game

In this work, we model *both alternatives*: the price and the quantity competition, which are known as the Bertrand and Cournot competition models, correspondingly. We focus on a differentiated market game with the following *two phases*:

1. In the first phase, both SPs select the quality level s_i (equivalent to e.g., a data rate package with the announced throughput). Importantly, at this stage the SPs are aware of each other, but make their decisions sequentially.
2. Second, given the fixed quality level s_i the SPs compete in price or, alternatively, in quantity. More specifically, in the Bertrand game the SPs decide on the prices $p_i, i = 1, 2$ that are announced to the customers purchasing their subscriptions. In contrast to that, in the Cournot game the SPs decide on the quantity, which in our modeling translates into the number of subscribed customers or, equivalently, sold subscriptions.

Further, we aim at determining the Nash equilibrium of our game and to do so we apply the principle of *backward induction*. Accordingly, we begin by finding an equilibrium for the second phase (price/quantity competition for the fixed levels of s_i) and then obtain the optimal values of s_i which are selected in the first phase.

3 Conventional example: uniform taste distribution

In this section, we consider a tractable example of the customer taste distribution $h(\theta)$, namely, a uniform distribution $h_U(\theta) = \frac{1}{\theta_{\max}}$ over the said interval $[0, \theta_{\max}]$ and thus the expressions for the demand may be rewritten as:

$$D_1(\mathbf{s}; \mathbf{p}) = \tfrac{1}{\theta_{\max}}(\theta_{\max} - \theta_{2,1}), \quad D_2(\mathbf{s}; \mathbf{p}) = \tfrac{1}{\theta_{\max}}(\theta_{2,1} - \theta_{\varnothing,2}). \tag{2}$$

In what follows, we consider the Bertrand price competition and the Cournot quantity competition models separately for both options.

3.1 Bertrand price competition for the uniform distribution

In the Bertrand game, the SP selects its own price p_i in order to maximize the profit $\Pi_i(\mathbf{s}; \mathbf{p}) = D_i(\mathbf{s}; \mathbf{p})\,(p_i - \nu s_i)$ for the selected quality function values s_i. By differentiating Π_i over p_i, one may calculate the optimal prices (can be verified for $\nu = 0$ by [13]) for the fixed levels of quality, while the solution is readily obtained as follows:

$$p_1^*(\mathbf{s}) = s_1 \tfrac{2\theta_{\max}(s_1 - s_2) + v(2s_1 + s_2)}{4s_1 - s_2}, \quad p_2^*(\mathbf{s}) = s_2 \tfrac{\theta_{\max}(s_1 - s_2) + 3s_1 v}{4s_1 - s_2}. \tag{3}$$

It can be easily demonstrated that the latter is a unique point of maximum for $0 \le s_2 \le s_1$, which is achieved during the price competition if all of the participants maximize their profits. The solution (3) represents a result of long-term price adjustment.

At the next step of our backward induction, we consider the first phase of the game, when the SPs select the quality s_i. Each of them maximizes the function $\Pi_i(\mathbf{s})$ over the only varying argument s_i, where:

$$\Pi_1(\mathbf{s}) = 4s_1^2 \tfrac{(\theta_{\max}-\nu)^2(s_1-s_2)}{\theta_{\max}(4s_1-s_2)^2}, \quad \Pi_2(\mathbf{s}) = s_1 s_2 \tfrac{(\theta_{\max}-\nu)^2(s_1-s_2)}{\theta_{\max}(4s_1-s_2)^2}. \tag{4}$$

The first-order condition of maximum for these two independent optimization problems may be formulated as follows:

$$\frac{4\,s_1\,(\theta_{\max}-v)^2\left(4\,{s_1}^2-3\,s_1\,s_2+2\,{s_2}^2\right)}{\theta_{\max}(4\,s_1-s_2)^3}=0, \quad \frac{{s_1}^2\,(4\,s_1-7\,s_2)\,(\theta_{\max}-v)^2}{\theta_{\max}(4\,s_1-s_2)^3}=0. \tag{5}$$

Denoting $\frac{s_1}{s_2}$ as x, we may then locate the maximum points for both SPs. We note that due to the absence of roots for the first equation and the fact that $\frac{\partial \Pi_1}{\partial s_1} > 0$, the maximum is located at the border $s_1^* = s_{\max}$, while the optimal quality $s_2^* = s_{\max}\xi$, where $\xi = 4/7$ (the second-order condition of maximum $\left.\frac{\partial^2 \Pi_2}{\partial s_2^2}\right|_{s_1^*, s_2^*} < 0$ could be verified easily). The latter corresponds to the rule of 4/7 [11].

Theorem 1 *The obtained solution for the Bertrand game is unique and represents the Nash equilibrium.*

Proof. The proof is fairly straightforward and is based on demonstrating that the following holds:

$$\begin{aligned} \Pi_i(s_1^*, s_2^*) &\geq \Pi_i(s_1, s_2^*), \quad \text{for any } s_1 < s_1^*, \\ \text{and } \Pi_i(s_1^*, s_2^*) &\geq \Pi_i(s_1^*, s_2), \quad \text{for any } s_2 \neq s_2^*, \end{aligned} \tag{6}$$

which is based on the fact that the sought points are the points of maximum for the respective functions. Uniqueness of the sought point follows from uniqueness of $\mathbf{p}^*(\mathbf{s})$ and the solution (s_1^*, s_2^*).

Substituting the sought point $(s_{\max}, \xi s_{\max})$ into the price, demand, and profit functions, we obtain the key indicators at the equilibrium point. Then, we additionally calculate the *consumer surplus* by characterizing the integral benefit of all customers as a difference between the maximum price that they could have paid for the quality s_i (i.e., θs_i) and what they actually spend (p_i):

$$CS = \int\limits_{\theta_{1,2}}^{\theta_{\max}} (\theta s_1 - p_1)\tfrac{1}{\theta_{\max}} d\theta + \int\limits_{\theta_{\varnothing,2}}^{\theta_{1,2}} (\theta s_2 - p_2)\tfrac{1}{\theta_{\max}} d\theta = \tfrac{7 s_{\max}(\theta_{\max}-\nu)^2}{24\theta_{\max}}. \tag{7}$$

3.2 Cournot quantity competition for the uniform distribution

While in the Bertrand game the price p_i is controlled by the SP i and the share of connected customers is then determined through the demand function, in the Cournot game the SPs control the quantity (i.e., the number of subscriptions) and then the prices are derived through the inverted system of demand functions:

$$\begin{aligned} p_1(\mathbf{s};\mathbf{D}) &= -\theta_{\max}(D_1 s_1 - s_1 + D_2 s_2), \\ p_2(\mathbf{s};\mathbf{D}) &= -\theta_{\max} s_2 (D_1 + D_2 - 1). \end{aligned} \tag{8}$$

Substituting the above into the expression for the SP profit $\Pi_i = D_i(p_i - v s_i)$, we may establish the quantity response functions that maximize the profit for the fixed qualities s_1 and s_2:

$$D_1(\mathbf{s}) = \tfrac{(2s_1-s_2)(\theta_{\max}-\nu)}{\theta_{\max}(4s_1-s_2)}, \quad D_2(\mathbf{s}) = \tfrac{s_1(\theta_{\max}-\nu)}{\theta_{\max}(4s_1-s_2)}.$$

After substituting these functions into (8), we obtain the prices set by the SPs:

$$p_1 = s_1 \tfrac{2\theta_{\max} s_1 - \theta_{\max} s_2 + 2 s_1 \nu}{(4s_1-s_2)}, \quad p_2 = s_2 \tfrac{\theta_{\max} s_1 + 3 s_1 v - s_2 \nu}{(4s_1-s_2)},$$

and, correspondingly, characterize the resulting profit:

$$\Pi_1(\mathbf{s}) = \frac{s_1(2s_1-s_2)^2(\theta_{\max}-\nu)^2}{\theta_{\max}(4s_1-s_2)^2}, \quad \Pi_2(\mathbf{s}) = \frac{s_1^2 s_2(\theta_{\max}-\nu)^2}{\theta_{\max}(4s_1-s_2)^2}. \tag{9}$$

In the second phase of the backward induction, we derive the optimal level of qualities that maximize the profit (9) by finding the stationary points of the following equations:

$$\frac{\partial \Pi_1(\mathbf{s})}{\partial s_1} = \frac{(\theta_{\max}-\nu)^2(2s_1-s_2)(8s_1^2-2s_1s_2+s_2^2)}{\theta_{\max}(4s_1-s_2)^3}, \quad \frac{\partial \Pi_2(\mathbf{s})}{\partial s_2} = \frac{(\theta_{\max}-\nu)^2 s_1^2(4s_1+s_2)}{\theta_{\max}(4s_1-s_2)^3}. \tag{10}$$

Denoting $\frac{s_1}{s_2}$ as x, we may conclude that there exists no solution $x > 1$ for (10). Since both $\frac{\partial \Pi_1(\mathbf{s})}{\partial s_1}$ and $\frac{\partial \Pi_2(\mathbf{s})}{\partial s_2} > 0$, the point of maximum is located at the right border of the interval for s, that is, $s_1^* = s_{\max}$ and $s_2^* = s_{\max}$. Therefore, we have established a candidate solution for the Cournot game and can formulate a theorem similar to the one before.

Theorem 2 *The obtained solution for the Cournot game is unique and represents the Nash equilibrium.*

Proof. The proof is easy to produce similarly to that of the above Theorem for the Bertrand game.

Since the Cournot prices and qualities are equivalent, two SPs divide the corresponding market in equal proportions, if we assume that there is no weighted preference towards a certain brand. Hence, the consumer surplus in this case may be derived as:

$$CS = \int_{\theta_{1,2}}^{\theta_{\max}} (\theta s_1 - p_1)h(\theta)d\theta = \frac{2s_{\max}(\theta_{\max}-\nu)^2}{9\theta_{\max}}. \tag{11}$$

4 Arbitrary taste distribution and cost function

In this section, we contribute an algorithm that allows for establishing an equilibrium point for an arbitrary taste distribution and cost function. As a particular example, we refer to the truncated exponential distribution:

$$h_U(\theta) = \frac{\lambda e^{-\lambda\theta}}{1-e^{-\lambda\theta_{\max}}}, \theta \in [0, \theta_{\max}], \quad H_U(\theta) = \frac{1-e^{-\lambda\theta}}{1-e^{-\lambda\theta_{\max}}}, \theta \in [0, \theta_{\max}]. \tag{12}$$

The use of the exponential shape follows from [18], where the authors analyze a real mobile service market by polling the consumers and processing the results. Further, we truncate the exponential distribution by $\theta_{\max}$ to provide a better correspondence with the parameter of the uniform distribution. Hence, the corresponding expressions for the demand may be rewritten as:

$$D_1(\mathbf{s};\mathbf{p}) = C_0\left(e^{-\lambda\frac{p_1-p_2}{s_1-s2}} - e^{-\theta_{\max}\lambda}\right), \quad D_2(\mathbf{s};\mathbf{p}) = C_0\left(e^{-\frac{\lambda p_2}{s_2}} - e^{-\lambda\frac{p_1-p_2}{s1-s2}}\right),$$

where $C_0 = \frac{1}{1-e^{-\lambda\theta_{\max}}}$ is a constant introduced for brevity. We build our numerical comparison later on in Section 5 on the example of the truncated exponential distribution, which we believe to better represent the properties of the target market. However, below we formulate the essential steps of our proposed procedure in a general form as well as introduce an arbitrary cost function.

4.1 Bertrand price competition for an arbitrary distribution

The profit function in its general form is defined as $\Pi_i = D_i p_i - D_i f_c(s_i)$, where $f_c(s_i)$ is the cost per a subscription represented by the twice differentiable function of quality s_i. In this general case, we therefore have:

$$\begin{aligned} \Pi_1(\mathbf{s};\mathbf{p}) &= \left(1 - H\left(\frac{p_1-p_2}{s_1-s_2}\right)\right)(p_1 - f_c(s_1)), \\ \Pi_2(\mathbf{s};\mathbf{p}) &= \left(H\left(\frac{p_1-p_2}{s_1-s_2}\right) - H\left(\frac{p_2}{s_2}\right)\right)(p_2 - f_c(s_2)), \end{aligned} \tag{13}$$

where $H(x)$ is the cumulative distribution function (CDF) of the taste parameter and $H(\theta_{\max}) = 1$. After differentiating both expressions separately by the corresponding quality variable, we obtain a condition for further optimization:

$$\begin{aligned}\frac{\partial \Pi_1(\mathbf{s};\mathbf{p})}{\partial p_1} &= 1 - H\left(\frac{p_1-p_2}{s_1-s_2}\right) - h\left(\frac{p_1-p_2}{s_1-s_2}\right)\frac{p_1-f_c(s_1)}{s_1-s_2},\\ \frac{\partial \Pi_2(\mathbf{s};\mathbf{p})}{\partial p_2} &= H\left(\frac{p_1-p_2}{s_1-s_2}\right) - H\left(\frac{p_2}{s_2}\right) - h\left(\frac{p_1-p_2}{s_1-s_2}\right)\frac{p_2-f_c(s_2)}{s_1-s_2} - h\left(\frac{p_2}{s_2}\right)\frac{p_2-f_c(s_2)}{s_2}.\end{aligned} \tag{14}$$

We note that an analytical solution of the system $\left(\frac{\partial \Pi_i(\mathbf{s};\mathbf{p})}{\partial p_i} = 0\right)_{i=1,2}$ may not always be produced for complex distribution shapes of $f_c(s_i)$. In order to follow the steps described previously in Section 3, for an arbitrary distribution we may apply a numerical procedure to solve the system of non-linear equations (14) for any fixed point $\mathbf{s}$, $0 < s_2 < s_1$. If the second-order condition of the local maximum holds, the obtained solution $\mathbf{p}^*(\mathbf{s})$ is set as an output of the function *FindOptimalPrices(s_1, s_2)*, which corresponds to the second phase of our game (see Algorithm 1 below).

Continuing the search of the optimal solution, we consider again the second phase (the quality competition) and maximize the profit $\Pi_i(s_1, s_2)$ by varying s_i. Importantly, the functions $\Pi_i(s_1, s_2)$ are numerical and produced by the proposed function *FindOptimalPrices(s_1, s_2)*. The optimization can be conducted via explicit search, but the following theorem simplifies the needed calculations:

Theorem 3 *Maximum of the profit function $\Pi_1(s_1, s_2)$ by $s_1 \in (0, s_{\max}]$ for the SP that makes its decision the first is always located at the point $s_{\max}$, which means that for any new SP the maximum quality yields the highest profit.*

Proof. The proof is omitted here due to the space limitations.

Employing this result, it only remains to maximize the profit of another SP $\Pi_2(s_1, s_2)$ by $s_2 \in (0, s_1]$, which is a simple one-dimensional optimization that always has a solution. The entire procedure is briefly summarized in Algorithm 1. The sought variables $(s_1^*, s_2^*; p_1^*, p_2^*)$ correspond to the *Nash equilibrium*, where no player could change its strategy (that is, price and quality for the SPs and SP choice for the customers) without decreasing its profit. Based on the obtained equilibrium, we may easily estimate the corresponding market shares D_i^*, the equilibrium profit Π_i^*, and the consumer surplus CS as provided in Section 5.

4.2 Cournot quantity competition for an arbitrary distribution

In order to characterize the Cournot quantity competition for an arbitrary taste distribution and cost function, we follow the steps similar to those in Section 3. In particular, we write down the expression for the SPs demands:

$$D_1(\mathbf{s};\mathbf{p}) = 1 - H\left(\frac{p_1-p_2}{s_1-s_2}\right), \quad D_2(\mathbf{s};\mathbf{p}) = H\left(\frac{p_1-p_2}{s_1-s_2}\right) - H\left(\frac{p_2}{s_2}\right), \tag{15}$$

where $H(x)$ is the CDF of the taste parameter. From the first equation, we may establish $p_1(\mathbf{D}) = F(1-D_1)(s_1-s_2) + p_2$, where $F = H^{-1}$ is the function inverse to the CDF. Substituting it into the second equation and calculating p_2, we may obtain the following:

$$p_1(\mathbf{D}) = F(1-D_1)(s_1-s_2) + p_2, \quad p_2(\mathbf{D}) = F(1-D_2-D_1)\, s_2. \tag{16}$$

We substitute this produced expression for price into the profit function $\Pi_i(\mathbf{s};\mathbf{D}) = D_i(p_i(\mathbf{D}) - f_c(s_i))$. By analogy with subsection 4.1, we find the optimal prices after differentiating the profit by the demand D_i and then solving the system $\left(\frac{\partial \Pi_1(\mathbf{s};\mathbf{D})}{\partial D_1} = 0\right)_{i=1,2}$ as:

$$\frac{\partial \Pi_1(\mathbf{s};\mathbf{D})}{\partial D_1} = p_1(D_1) - f_c(s_1) - \frac{D_1(s_1-s_2)}{h(1-D_1)}, \quad \frac{\partial \Pi_1(\mathbf{s};\mathbf{D})}{\partial D_1} = p_2(D_2) - f_c(s_2) - \frac{D_2 s_2}{h(1-D_2-D_1)}, \tag{17}$$

where $h(\theta)$ is the given PDF. We note that the system (17) is equivalent to (14) for the Bertrand competition. Assuming that the function *FindOptimalQuantities(s_1, s_2)*

returns the solution of (17) and then replacing prices with qualities $\mathbf{D}$ in Algorithm 1, we arrive at the final equilibrium $(s_1^*, s_2^*; D_1^*, D_2^*)$ and may calculate all of the respective metrics.

Even though existence and uniqueness of the Nash equilibrium constitute an open question for different classes of distributions, in case of our truncated exponential example we can formulate the following Theorem.

Theorem 4 *For the truncated exponential distribution, there exists a unique Nash equilibrium for both the Bertrand and the Cournot game, so that the Bertrand competition results in product differentiation, while equilibrium quality for the Cournot competition is given by* $(s_{\max}, s_{\max})$.

Proof. We leave this proof out of scope of this paper.

Importantly, cooperative games for either price or quantity competition yield different solutions e.g., product differentiation in the Cournot game.

Algorithm 1 Algorithm based on the Bertrand price competition

```
s1* = s_max
s2* = MaximizeProfit2(s1*)
p* = FindOptimalPrices(s1, s2)
function MaximizeProfit2(s1*)                ▷ Maximize profit of the SP 2 by s2
    return s2* = arg max_{s2} Profit i(s1*, s2)
function Profit i(s1, s2)                    ▷ Profit of the SP based on the optimal prices
    p* = FindOptimalPrices(s1, s2)
    return Π_i(s1, s2; p*)
function FindOptimalPrices(s1, s2)           ▷ Prices maximizing the profit for fixed s
    return p*: solution of the system (14)
```

5 Numerical results and conclusion

In total, we analyze four scenarios: Bertrand and Cournot competition for both the conventional and the realistic distribution each. Even though our approach is suitable for any cost function, for the sake of comparison this section considers the same linear costs for all of the cases. Minding a multitude of possible choices, below we only provide several representative examples for comparison.

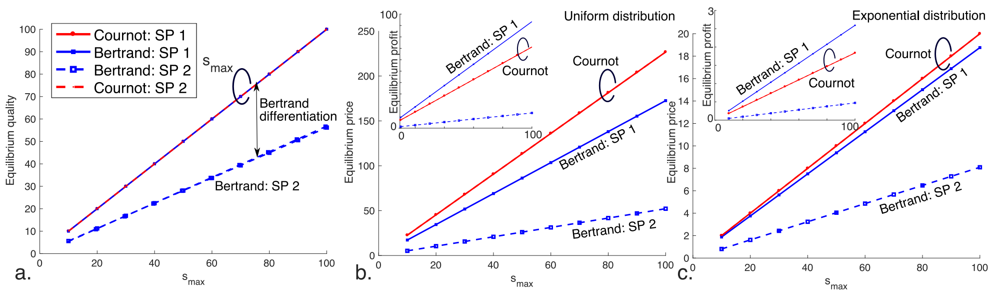

Fig. 2. Evolution of equilibrium indicators for maximum quality $s_{\max}$: (a) equilibrium quality for both distributions, (b,c) equilibrium price and quality for UD and ED.

We remind that for a particular distribution we quantify the following parameters in our model: the maximum quality $s_{\max}$ (set by default to 100), the cost coefficient v

We remind that if $s_1^* = s_2^*$ then $p_1^* = p_2^*$, and the active customers with the positive utility are indifferent to choosing either of the SPs. In this case, the demand is equally shared between the SPs and leads to equal market indicators for them.

(0.1), and the "richest" customer $\theta_{\max}$ (6.6). In Fig. 2a-c, we illustrate the evolution of our market for varying $s_{\max}$. As it is demonstrated in Fig. 2a, the equilibrium quality for both the uniform (UD) and the exponential (ED) distribution (with $\lambda = 5$) behaves similarly and confirms an identical choice for the Cournot game as well as a clear product differentiation for the Bertrand game. Importantly, we note that the latter results in the same quality for both taste distributions $h(\theta)$.

Further, Fig. 2b,c highlight the difference in prices and profits of the SPs. Intuitively, on a market where the majority of customers are "poor" (ED) the equilibrium prices as well as the profits appear to be much lower. The Cournot competition results in prices that are generally higher than those in the Bertrand competition, but for the ED market this difference diminishes together with the degree of price differentiation between the SPs.

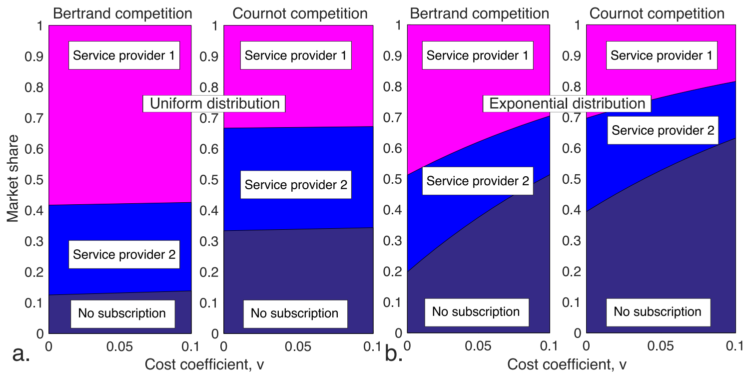

Fig. 3. Evolution of market shares vs. cost coefficient v: Bertrand and Cournot game for (a) UD and (b) ED.

Then, we investigate the impact of costs on the total demand of the SP 1, the SP 2, as well as the share of the market that is not covered. In Fig. 3a,b, we observe the volume of the market that belongs to either of these three groups. While the "wealthier" UD market is less sensitive to changes, on the ED market an increase in costs entails a rise of the equilibrium price as well as a dramatic reduction in the market shares of SPs. Customer churn eventually leads to a significant decrease in the SP profits.

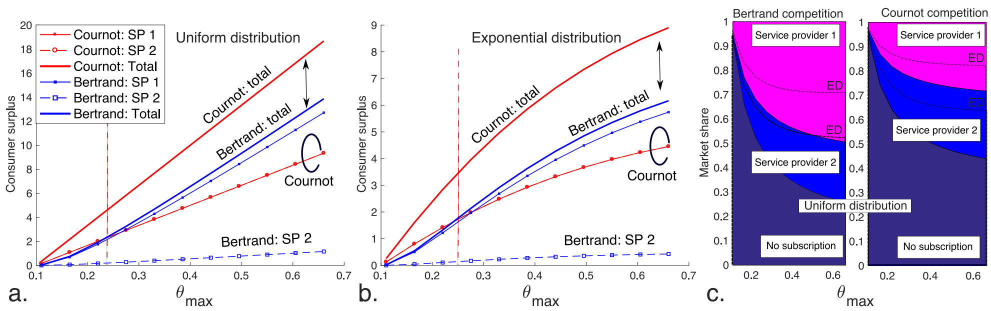

Fig. 4. Market evolution for variable restricting parameter $\theta_{\max}$: (a,b) consumer surplus for UD and ED, and (c) market shares.

Finally, we analyze all four scenarios in question by varying $\theta_{\max}$, which determines the "richest" customer on the market. As for the ED, the market shares stabilize with the growing range of taste, whereas for the UD the market broadens significantly by covering more and more customers (see Fig. 4c, where dotted lines correspond to the ED market). Further, in Fig. 4a,b for the UD and the ED, respectively, we may observe the total consumer surplus and the separate components for customers of the SP 1 and the SP 2. The relative differences are rather marginal and suggest that the Cournot game is more beneficial for the market than the Bertrand game. However, the absolute values indicate a considerable difference between the UD and the ED in terms of the resultant benefits.

In summary, this paper considered both the price and the quantity competition in a vertically differentiated market. In particular, we analyzed a tractable example with linear costs of quality improvement and proposed a numerical procedure to relax the restrictive assumptions. In contrast to most past studies, we not only evaluated the mobile service market under more realistic assumptions on the customer taste distribution, but also provided a detailed comparison of the key market indicators. While demonstrating similar general behavior, the two considered distributions – the uniform and the truncated exponential – indicate a dramatic difference in the resulting market sensitivity to the changes. The latter confirms that the choice of appropriate customer taste distribution is a crucial factor in analyzing a competitive market, while the general market trends could be understood from simpler assumptions.

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