A RCS Model of Complex Targets for Radar Performance Prediction

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Abstract—The objective of the radar performance prediction is to compute quantities of interest concerning the ability of the radar to observe its surroundings. Besides the properties of the radar system, the performance is affected by the target, whose radar cross section (RCS) is one of the predominant factors. The performance prediction is usually performed in relation to the target RCS characterized by a constant value or a particular statistical distribution. Such representations generalize real-life complex targets rendering them unsuitable for some objectives since the RCS is significantly influenced by the target aspect angle and is inherently stochastic by nature. Thus, a more dynamic description may be valuable e.g. for analyzing the radar performance on a flight path of interest. We propose representing the RCS with a histogram that includes such dynamic properties and is suitable for considering the target in different ways for performance prediction: in a more general manner or dependent on its aspect angle. We consider the case of traditional RCS with low spatial resolution and demonstrate the proposed approach through the probability of detection computed for a generic surveillance radar.

I. Introduction

The objective of radar performance prediction is to estimate and quantify the prerequisites of a radar system to sense its surroundings. Usually the factors to quantify relate to observing or identifying a target and are consequently largely affected by its scattering properties, i.e. the radar cross section (RCS). The RCS depends on the physical characteristics of the target as well as the operating parameters of the radar and other operating conditions. In practical situations concerning airborne targets, the orientation that the target is observed in is one of the predominant factors affecting its RCS, which is inherently stochastic by nature. This stochasticity is due to the unintended and uncontrollable variation of the aspect angle and the corresponding RCS, as even a minuscule change of the aspect angle may translate to an unpredictable change of the RCS. Thus, the RCS may vary significantly e.g. between radar scans or even radar pulses, especially against a maneuvering target. In statistical performance prediction, this variation can be seen in different realizations of the same flight path or flight maneuver. In [1] we proposed a method for incorporating the flight dynamics in a model of RCS, which allowed the estimation or prediction of a dynamically varying distribution for the target RCS for a particular flight path. In this paper, we extend these concepts to radar performance prediction, where we consider the target in a more generalized manner.

The traditional way of describing the ability of the radar to observe a target is through the quantities of the probability of detection (P_d) and the probability of false alarm (P_{fa}). In this paper, we focus on objects with complex shapes, such as aircraft, instead of canonical or simple objects. Due to the complicated nature of the electromagnetic scattering phenomenon, the RCS of such targets has traditionally been treated as a statistically characterized random variable [2], [3], [4]. However, the statistical characteristics of RCS are known to vary with the aspect angle and are represented inadequately with a single type of density function [5].

Similar to the statistical model based on central moments proposed in [6], our objective is to develop a non-parametric model for the RCS. The basic principle of the model is to use the RCS values produced by some other means, e.g. a scattering simulation, to generate a higher-level performance model that is able to represent the RCS of diverse complex targets in a highly realistic manner. We propose using a histogram changing with the aspect angle to allow the distribution of the target RCS to vary dynamically according to the scattering characteristics. The proposed model is intended for radar performance prediction and we demonstrate its benefits in P_d calculation. As the RCS is represented with a histogram without a predetermined distribution, the traditional methods for the P_d calculation are not directly applicable. Thus, we also propose a method for this calculation.

We incorporate the stochasticity of the aspect angle in the proposed RCS model through the probability of occurrence, which describes the uncertainty or variation of the aspect angle. Through the compilation of the RCS histogram, this description for the aspect angle is transformed into a description for the fluctuation of the RCS. In the context of radar performance prediction, the use of the probability of occurrence achieves a similar result as a Monte Carlo simulation but without the need for performing numerous simulation runs.

We compare the proposed model with a simplified version, which lacks the dependence on the aspect angle and the probability of occurrence, as well as the Swerling I model [7].

We show that both proposed models are suitable for radar performance prediction but for different objectives. A general type of analysis, considering the target overall or relating e.g. to a particular type of flight, is achievable with the histogram lacking the dependence on the aspect angle, whereas the histogram varying with the aspect angle is suitable for a more detailed analysis. In the histogram calculation, we utilize the deterministic RCS that has been precomputed with an electromagnetic scattering simulation but e.g. measured RCS could be used as well.

The structure of this paper is the following. In Section II, we describe the proposed method for representing the RCS in radar performance prediction first, and then how the probability of detection is computed using it. We demonstrate the proposed method using simulated RCS in Section III, and finally, we discuss the presented results and give the conclusions in Sections IV and V.

II. THE FORMULATION OF THE RCS FOR THE PERFORMANCE PREDICTION

In this section, we describe the proposed RCS model. First, the simplified form of the model is described, and then the model is extended to include the dependence on the aspect angle. After this, we describe how the model is utilized in P_d calculation. In this paper, we use the following notation of the aspect angle:

$$\alpha = (\phi, \theta), \tag{1}$$

where ϕ indicates the azimuth angle and θ the elevation angle. The azimuth angle $\phi = 0°$ corresponds to observing the target from the front and $\phi = 90°$ from the right side. Accordingly, the elevation angle $\theta = 90°$ is the view from the top and $\theta = -90°$ from the bottom. This definition of the aspect angle is illustrated in Fig. 1.

A. The RCS Histogram

In its simplest form, the proposed RCS model is the conventional histogram calculated from a set of RCS values. Each histogram bin

$$h_i = \frac{1}{K} \sum_{k=1}^{K} y_i(x_k), \tag{2}$$

where x_k is the RCS sample corresponding to the aspect angle α_k, K is the number of RCS samples,

$$y_i(x) = \begin{cases} 1, & \text{if } x \in \Delta_i \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

and Δ_i defines the ith histogram bin. In this formulation all the aspect angles are assumed equally probable and all the RCS samples x_k are, thus, given an equal weight in the histogram. This provides a general description of the scattering of the target in relation to the chosen set of α_k. For instance, if the α_k are selected near the elevation angle $0°$ covering the azimuth angles in a balanced manner, the resulting histogram will describe the RCS of the target during horizontal flight.
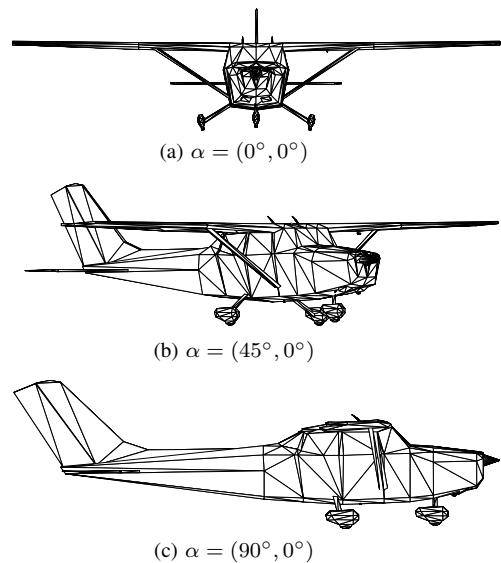

(a) $\alpha = (0°, 0°)$

(b) $\alpha = (45°, 0°)$

(c) $\alpha = (90°, 0°)$

Fig. 1. The 3D model of the Cessna 182 used in the RCS simulation viewed from three different aspect angles α, which are specified below each figure.

However, as we want to incorporate the variation of the orientation and aspect angle inherent in practical situations in the model, some modifications are needed. We will introduce the concept of the probability of occurrence $p_{j,k}$ for the aspect angle. It represents the probability of observing the target in the aspect angle α_k, when the calculatory or estimated value of the aspect angle is α_j. We compile the histogram from the RCS samples around the aspect angle of interest α_j and each sample is weighted according to $p_{j,k}$. Thus, each histogram bin i for aspect angle α_j becomes

$$h_{j,i} = \sum_{k=1}^{K} y_i(x_k) \cdot p_{j,k}. \tag{4}$$

In the context of radar performance prediction, the $p_{j,k}$ is comparable to the result of a Monte Carlo simulation of a particular flight path for one time instant, when the output variable is the target aspect angle. The individual realizations of the trajectory correspond to the probabilities assigned to different α_k with the most probable assigned as α_j. The use of $p_{j,k}$ directly, instead of performing the Monte Carlo simulation, eliminates the need for performing multiple simulation runs to produce a statistical description of the radar performance. In this paper, we assume $p_{j,k} \sim N(\alpha_j, \sigma^2)$.

B. Calculating the Probability of Detection

We demonstrate the utility of the proposed RCS model in radar performance prediction with the P_d. For a target of particular RCS, we first compute the reflected power and signal-to-noise ratio (SNR) using the traditional radar equation [8]. The SNR is then transformed to a figure of P_d related to a selected value of the P_{fa}.

The proposed RCS model represents the RCS as a histogram instead of a single value, which imposes the need for a
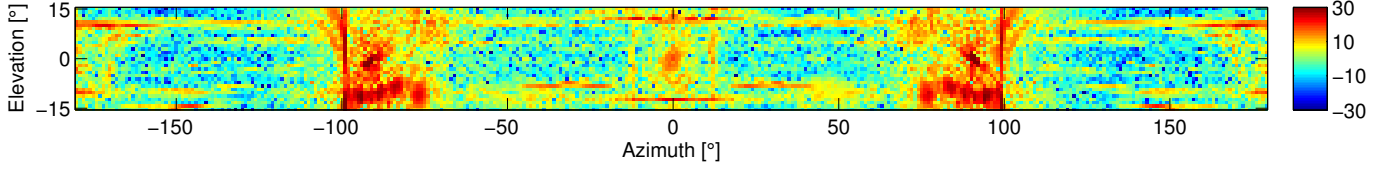
Fig. 2. The RCS in dBsm of the Cessna 182 aircraft in aspect angles $\phi \in [-180°, 180°[$ and $\theta \in [-15°, 15°]$.

modification to this formulation. The histogram can be considered as modeling the target as the superposition of multiple targets of different RCS defined by the histogram bins. As a simplification, each bin is seen as a spherical target with constant RCS that is determined by the bin center. The total P_d is calculated as the sum of the probabilities of the individual spherical targets $P_{d,i}$ [9] weighted according to the histogram values. If the histogram has been compiled independent of the aspect angle, as in (2), the probability of detection

$$P_d = \sum_{i=1}^{I} P_{d,i} \cdot h_i, \tag{5}$$

where I is the number of histogram bins. Equivalently, if the histogram varies with the aspect angle, as presented in (4), the probability of detection

$$P_d = \sum_{i=1}^{I} P_{d,i} \cdot h_{J,i}, \tag{6}$$

where J indicates the α_j that corresponds to the aspect angle of interest.

III. NUMERICAL EXAMPLES

Next, we illustrate the proposed method via examples using the simulated RCS of a Cessna 182 aircraft. The RCS values have been computed on the frequency of 3 GHz using a three-dimensional (3D) model of the Cessna aircraft. This model has been acquired from the open source FlightGear Flight Simulator and is illustrated in Fig. 1. Note that the front propeller of the aircraft has been removed from the model and is not included in the RCS simulation.

The RCS calculation has been performed with the ESPRESS method [10] that is based on physical optics. Any other RCS calculation method or software could, however, be used instead, as the means of producing the RCS values is irrelevant to the proposed approach. Information about the material of the target has not been included in the RCS calculation and its surface has been assumed a perfect electric conductor.
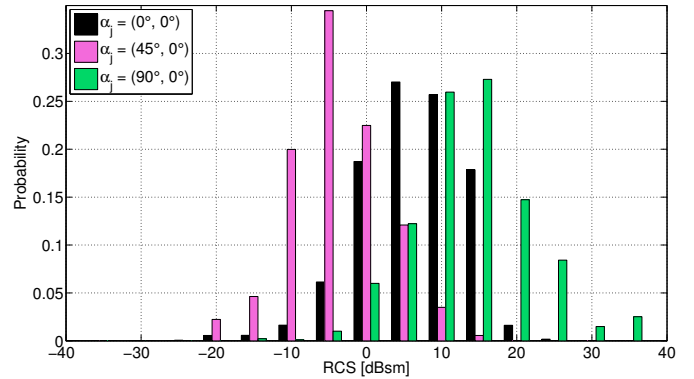
A. The RCS Histograms

The simulated RCS we use as the source data in the following examples is illustrated in Fig. 2. The RCS of the Cessna aircraft has been precomputed for $\phi \in [-180°, 180°[$ and $\theta \in [-15°, 15°]$. This is sufficient for the scope of this paper, since we limit our analysis to targets in straight and steady flight and, thus, aspect angles near $\theta = 0°$. More specifically, we choose α_j so that $\phi \in [-180°, 180°[$ and
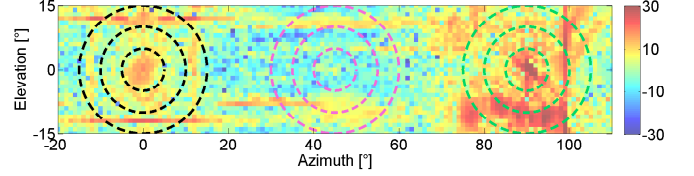
(a) Three contours ($\sigma, 2\sigma, 3\sigma$) for the probability of occurrence distribution for three α_j in different colors overlaid on the RCS data.
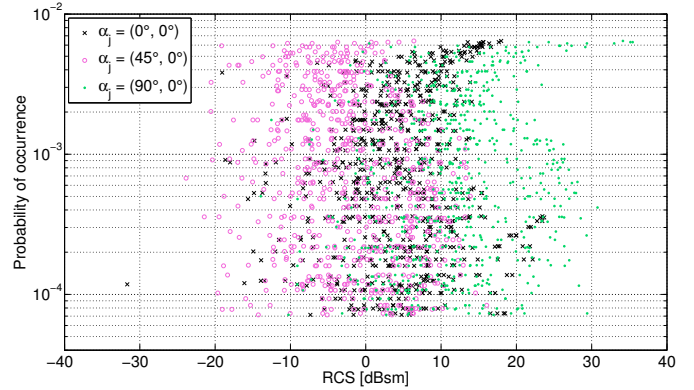
(b) The RCS samples and their probability of occurrence.

(c) The RCS histograms.

Fig. 3. Visualization of how the RCS histogram is compiled. In each figure the black color corresponds to $\alpha_j = (0°, 0°)$, the purple color to $\alpha_j = (45°, 0°)$, and the green color to $\alpha_j = (90°, 0°)$. In the top figure (a) the contours corresponding to σ, 2σ, and 3σ of the distribution for probability of occurrence for three α_j are shown overlaid on the RCS data of the Cessna 182 aircraft. These distributions are identical in shape with $\sigma = 5°$ and mean values corresponding to the α_j. The RCS samples from the top figure and their probability of occurrence is shown as a scatter plot in the figure (b). For each α_j, RCS samples have been collected within a window of 3σ from the mean value. The histogram compiled from the RCS samples weighted according to the probability of occurrence is shown in the bottom figure (c).

$\theta = 0°$. As we use $\sigma = 5°$ for the calculation of the probability

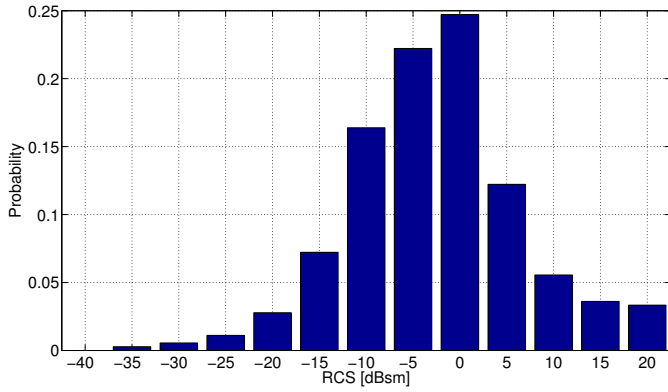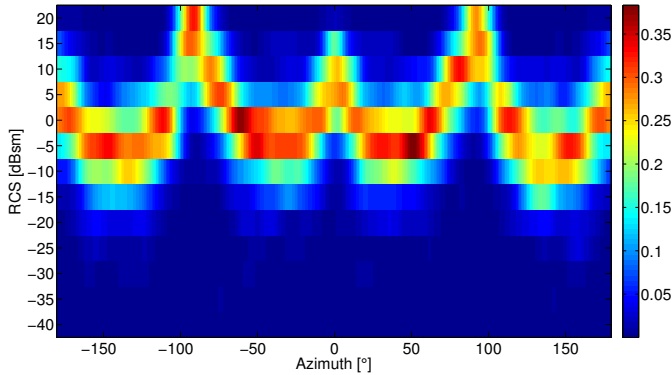Fig. 4. Histogram of the RCS of the Cessna 182 aircraft at $\theta = 0°$ and $\phi \in [-180°, 180°[$.

Fig. 5. Histograms of the Cessna 182 aircraft at α_j of $\theta = 0°$ and $\phi \in [-180°, 180°[$.

of occurrence, this $30°$ window of θ represents a range of $\pm 3\sigma$ or 99.7 percent of its distribution.

The calculation of a RCS histogram according to (4) is demonstrated in Fig. 3. The number of RCS samples used in the calculation depends on the sample spacing of the underlying RCS data. In this paper, both azimuth and elevation samples have the spacing of one degree.

The histogram calculated with the simplified version of the proposed model using the RCS values that represent horizontal flight, i.e. $\theta = 0°$ and $\phi \in [-180°, 180°[$, is shown in Fig. 4. Similarly, the histograms produced with the proposed model dependent on the aspect angle are presented in Fig. 5. Each column in Fig. 5 corresponds to an aspect angle α_j, which have been selected at $\theta = 0°$ and $\phi \in [-180°, 180°[$. The histograms have been produced with $\sigma = 5°$.

B. The P_d of a Surveillance Radar

We use the RCS histograms computed for the Cessna 182 aircraft and shown in Fig. 4 and Fig. 5 to demonstrate the P_d of a generic monostatic single-polarized surveillance radar. Fig. 6 shows the result for the simplified version of the proposed model visualized on a map. A similar result dependent on the aspect angle is presented in Fig. 7 for the target heading north. The equivalent result produced with the Swerling I model is
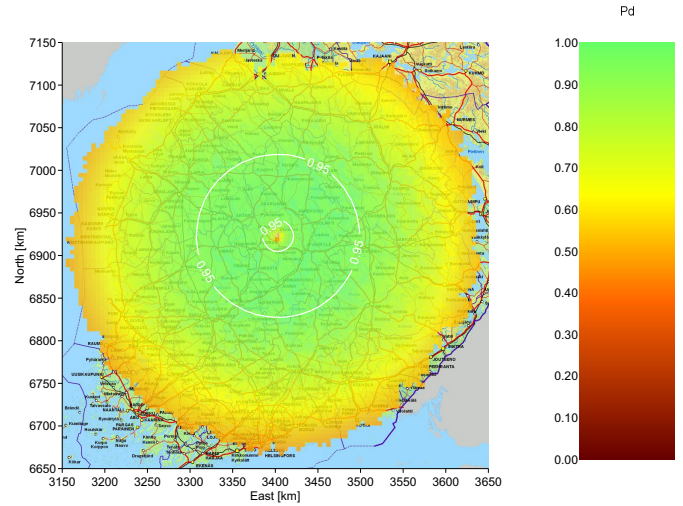

Fig. 6. The P_d of the Cessna 182 corresponding to horizontal flight and produced with the simplified version of the proposed model. The radar is located on the ground at the center of the colored blob shown in the image. Note that the inner contour line of 0.95 probability is due to the shape of the radar antenna pattern in elevation; this phenomenon is not significant when appraising the proposed RCS model.

shown in Fig. 8. This equivalence is achieved by setting the mean RCS value in the calculation to 7.6 dBsm or 5.7 m^2, which corresponds to the expected value of the histogram produced with the simplified version of the model shown in Fig. 4.

The P_d has been produced with a computation tool that performs the calculation as a spatial grid of points considering the terrain and its altitude. However, the influence of the surrounding terrain is minimal, as the radar has been placed on a hilltop surrounding relatively flat terrain and we consider a target flying high enough not to be shadowed by the ground. In addition, the target elevation angle is not considered in the P_d calculation, as the RCS model considered corresponds to straight and steady flight and represents aspect angles near $\theta = 0°$.

IV. DISCUSSION

In this paper, we proposed an approach for representing the target RCS conveniently for radar performance prediction. The results presented in the previous section visualize and demonstrate its ability to capture the dynamic nature of the RCS. Especially, the differences between Fig. 6, Fig. 7, and Fig. 8 demonstrate the benefits of the proposed approach to considering the target aspect angle. However, the validity of the RCS representation in terms of its realism is dependent on the validity of the underlying RCS information that is used to compile the histogram.

The proposed approach requires the deterministic RCS of the target of interest, which can be produced through computer simulation or measurement. Naturally, such information is required for the operating frequencies and polarizations corresponding to the radar systems of interest. The proposed model does not pose limitations on the selection of these values, as
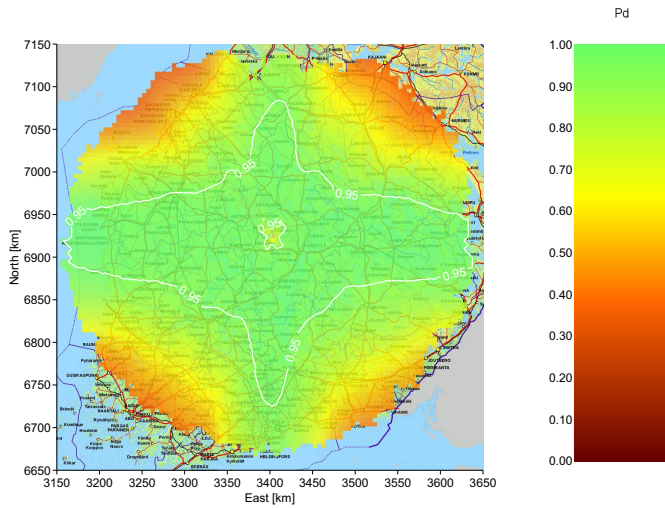
Fig. 7. The P_d of the Cessna 182 using the proposed model dependent on the aspect angle and the target is heading north. The radar is located on the ground at the center of the colored blob shown in the image. Note that the inner contour line of 0.95 probability is due to the shape of the radar antenna pattern in elevation; this phenomenon is not significant when appraising the proposed RCS model.

Fig. 8. The P_d of the Cessna 182 according to the Swerling I model. The radar is located on the ground at the center of the colored blob shown in the image. Note that the inner contour line of 0.95 probability is due to the shape of the radar antenna pattern in elevation; this phenomenon is not significant when appraising the proposed RCS model.
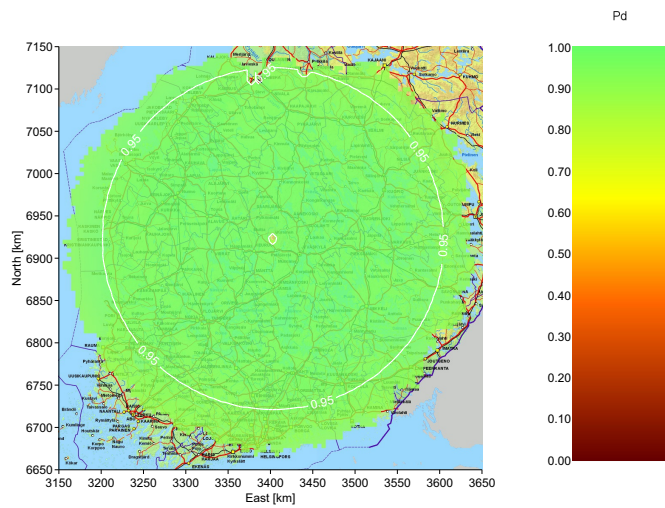
it is a higher-level model using the RCS values simply as source information. In this paper we considered the proposed model in relation to a monostatic surveillance radar but the model could be easily extended for different objectives, e.g. to consider bistatic RCS.

In radar performance prediction, it is often beneficial to consider various targets of different shapes to gain insight into the characteristics of the system. The proposed approach offers a sound premise for representing the target RCS accurately for such analysis. The accurate modeling of the RCS of targets, and thus their probabilities of detection, also allows more accurate modeling of the tracking performance. Such models provide more reliable data that can be used, e.g., to identify weaknesses in radar coverage or to evaluate the performance of alternative configurations for a radar network.

We presented two alternative versions of the proposed RCS model: a simplified one lacking the dependence on the aspect angle and one including this dependence. As can be seen in Fig. 6 and Fig. 7, the P_ds produced with these parallel RCS models differ significantly from each other. This is expected since the result produced with the simplified model represents the target in a generalized manner regardless of its heading, whereas the result including the dependence on the aspect angle represents a target with a straight flight path and a fixed heading.

We also presented the P_d computed with the Swerling I model setting its mean RCS value to the expected value of the histogram produced with the simplified version of the proposed method. We would assume this to produce a similar result compared with the histogram but, as can be seen by comparing Fig. 8 and Fig. 6, this is not the case. With a different choice for the mean RCS value, more similar results may have been produced but justification for such a choice would be questionable. This illustrates the convenience of the proposed method, as the need for parametrization is minimal.

The proposed method offers a convenient way of representing the target RCS suitable for different objectives in radar performance prediction. The model may be advantageous applied to algorithms relating e.g. to tracking, recognition, or the path planning of stealth aircraft. Its principle of considering the target RCS as a stochastic variable can be utilized for analysis, e.g. related to non-cooperative target recognition (NCTR) when estimating the RCS of a particular target observed at a known or estimated aspect angle. Such an approach is beneficial especially when limited information about the target reflectivity is at disposal and does not enable the use of more sophisticated techniques such as synthetic aperture radar (SAR) or micro-doppler analysis.

V. CONCLUSION

In this paper, we proposed a model for representing the RCS of complex targets in radar performance prediction that considers the stochastic nature of the target aspect angle. The presented examples illustrate the underlying principles and the presented results visualize its convenience and benefits in capturing the dynamics of the target RCS.

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