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COMPUTATIONAL MODELLING OF HIGH-CYCLE FATIGUE USING A CONTINUUM BASED MODEL

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Summary. In this paper a computational implementation of continuum based transversally isotropic fatigue model is described. The key idea of the continuum based HCF-model is the moving endurance surface where the movement is described by a back stress type tensor, the evolution of which is described by a rate type equation. Furthermore, damage accumulation is also governed with a rate type evolution equation. The model is implemented in the Abaqus FE-program using the user material subroutine. Two strategies to perform a fatigue analysis are compared in a standard cycling loading case. The first analysis reflects the procedure used in a standard fatigue computation. In the second analysis type the effect of evolving damage fields on fatigue life is investigated.

1 INTRODUCTION

Fatigue of materials under variable loads is a complicated physical process which can even result in catastrophic failure of engineering components. It is characterized by nucleation, coalescence and stable growth of cracks. Nucleation of cracks starts from stress concentrations near persistent slip bands, grain interfaces and inclusions[1,2,3].

In high-cycle fatigue, the macroscopic behavior of the material is primarily elastic, while in the low-cycle fatigue regime considerable macroscopic plastic deformations take place. Transition between low- and high-cycle fatigue occurs between $10^3 - 10^4$ cycles. In recent years, it has been observed that fatigue failures can occur at very high fatigue lives $10^9 - 10^{10}$, below the previously assumed fatigue limits.

In this paper only high-cycle fatigue modelling is considered. Many different approaches have been proposed to model the high-cycle fatigue behaviour which can roughly be classified into stress invariant, or average stress based and critical plane approaches. In those approaches damage accumulation is usually based on cycle-counting, which makes their use questionable under complex load histories[4,5].

A different strategy for high-cycle fatigue modelling was proposed by Ottosen et al.[4]. In their approach, which could be classified as evolutionary, the concept of a moving endurance surface in the stress space is postulated together with a damage evolution equation. The endurance surface is expressend in terms of the second invariant of the reduced deviatoric stress tensor where the center of the surface is defined by a deviatoric back stress tensor, as is done similarly in kinematic plasticity models. Therefore, the load history is memorized by the back-stress tensor. In this model arbitrary stress states are treated in a unified manner for different loading histories, thus avoiding cycle-counting techniques.

In the present paper, some experiences of the finite element implementation of the transversally isotropic fatigue[6] model is given. The model is based on the idea by Ottosen et al.[4].

2 MODEL FORMULATION

The continuum fatigue model developed in[6] is briefly described. It is based on the assumption that a material exhibit loading condition dependent endurance limits within which no damage results under cyclic loading. Ottosen et al.[4] proposed a moving endurance surface in stress stress space to account for these limits. The kye idea of the transversally isotropic model is to split the stress tensor into the longitudinal and transverse parts $\boldsymbol{\sigma} = \boldsymbol{\sigma}_{\rm L} + \boldsymbol{\sigma}_{\rm T}$, where the transverse component is obtained from

$$\boldsymbol{\sigma}_{\rm T} = \boldsymbol{P}\boldsymbol{\sigma}\boldsymbol{P} = \boldsymbol{\sigma} - \boldsymbol{\sigma}\boldsymbol{B} - \boldsymbol{B}\boldsymbol{\sigma} + \sigma_b \boldsymbol{B}, \tag{1}$$

where $\boldsymbol{P}$ is the projection tensor, $\boldsymbol{P} = \boldsymbol{I} - \boldsymbol{B}$, and $\boldsymbol{B} = \boldsymbol{b} \otimes \boldsymbol{b}$ is the structural tensor for transverse isotropy, The unit vector $\boldsymbol{b}$ designates the privileged longitudinal direction. The endurance surface for transversely isotropic fatigue has the form[6]

$$\beta = \frac{1}{S_{\rm T}} \left\{ \bar{\sigma} + A_{\rm L} I_{\rm L1} + A_{\rm T} I_{\rm T1} - [(1-\zeta)S_{\rm T} + \zeta S_{\rm L}] \right\} = 0, \tag{2}$$

where the linear invariants of the longitudinal and transverse stress tensors are

$$I_{\rm L1} = {\rm tr}\, \boldsymbol{\sigma}_{\rm L} = I_4 = {\rm tr}\,(\boldsymbol{\sigma}\boldsymbol{B}), \qquad I_{\rm T1} = {\rm tr}\, \boldsymbol{\sigma}_{\rm T} = {\rm tr}\, \boldsymbol{\sigma} - {\rm tr}\,(\boldsymbol{\sigma}\boldsymbol{B}). \tag{3}$$

Endurance limits at zero mean stress in the longitudinal and transverse directions are denoted as $S_{\rm L}$ and $S_{\rm T}$, rspectively. Non-dimensional positive parameters $A_{\rm L}$ and $A_{\rm T}$ are related in a constant amplitude cyclic loading to the slope in the Haigh diagram. Moreover, $\bar{\sigma}$ in (2) is the effective stress defined in terms of the second invariant of the reduced deviatoric stress $\boldsymbol{s} - \boldsymbol{\alpha}$, with $\boldsymbol{\alpha}$ being the back stress type tensor, as

$$\bar{\sigma} = \sqrt{\tfrac{3}{2}(\boldsymbol{s} - \boldsymbol{\alpha}) : (\boldsymbol{s} - \boldsymbol{\alpha})}, \tag{4}$$

where $\boldsymbol{s} = \boldsymbol{\sigma} - \frac{1}{3}{\rm tr}\,(\boldsymbol{\sigma})\mathbf{I}$ is the deviatoric stress tensor, $\mathbf{I}$ stands for the identity tensor. The scalar variable ζ reflects the average loading direction and is defined as

$$\zeta = \left(\frac{\boldsymbol{\sigma}_{\rm L} : \boldsymbol{\sigma}_{\rm L}}{\boldsymbol{\sigma} : \boldsymbol{\sigma}} \right)^n = \frac{2I_5 - I_4^2}{2I_2}, \tag{5}$$

where the invariants $I_2 = \frac{1}{2}\text{tr}\,\boldsymbol{\sigma}^2, \quad I_5 = \text{tr}\,(\boldsymbol{\sigma}^2\boldsymbol{B})$.

The endurance surface, $\beta = 0$, moves in the stress space driven by the back stress which memorizes the load history. Contrarily to plasticity theory, the stress states out of the endurance surface, $\beta > 0$, are allowed. The final model component needed before specifying the damage formulations is the evolution law for the back stress tensor. For this end, a hardening rule similar to Ziegler's kinematic hardening rule in plasticity theory is adopted, i.e.

$$\dot{\boldsymbol{\alpha}} = C(\boldsymbol{s} - \boldsymbol{\alpha})\dot{\beta}, \tag{6}$$

where C is a non-dimensional material parameter, and the dot denotes time rate.

In the original formulation by Ottosen et al.[4] a scalar damage variable is chosen to descibe the material deterioration. In the present study the chosen evolution equation for the damage D is

$$\dot{D} = \frac{K}{1-D}\exp(L\beta)\dot{\beta}, \tag{7}$$

where K and L are material parameters to be estimated from experiments. It differs by the factor $1/(1-D)$ from the one used by Ottosen et al.[4]. From the evolution equation (7) it can be concluded that damage and the backstress only developes when the stress state is moving away from the endurance surface, that is $\beta \geq 0$ and $\dot{\beta} > 0$.

3 EXAMPLE

Above model has been implemented in the finite element program Abaqus using the user material subroutine. As an example, a plate in plane strain conditions with a circular inclusion under uniaxial cyclic loading is considered. Diameter of the inclusion is 40 % of the side length of the domain. Due to symmetry only one quarter is discretized. Analytical solution for an infinite plate with a circular inclusion is given by Sezewa and Nishimura[7]. The surprizing result of their solution is that irrespectively of the stiffness difference bewteen the base material and the inclusion, all the stressess in the inclusion are constant along a radius but not along an azimuthal direction.

In the present analysis the material parameter have been determined for the AISI-SAE 4340 steel and they are $S_\text{L} = S_\text{T} = S_0 = 490$ MPa, $A_\text{L} = A_\text{T} = A = 0.225, C = 0.11, K = 1.46{\cdot}10^{-5}$ and $L = 8.7$. Elastic properties for the base material are $E = 210$ GPa and $\nu = 0.3$, and $E = 375$ GPa$, \nu = 0.22$ for the aluminium oxide inclusion, respectively. Only the base material is accounted for in the fatigue analysis. Uniform normal stress along the upper boundary is prescribed as $\sigma_y = S_0 \sin \omega t$.

Damage variable fields in the base material after the first and 500th cycle are shown in Fig. 1. As it can be noticed, the most damaging areas change during the material degradation due to the backstress as a driving force of damage. Therefore fatigue analysis based on single cycle stress history and linear accumulation of damage does not result in correct life-time prediction. The fatigue life based only on the first cycle stress history at the most critical point is $2.6 \cdot 10^6$ cycles. However, if it is determined from the damage history of the first 500 cycles, the lifetime is only $1.5 \cdot 10^4$ cycles. Another interesting observation is that a region of pronounced damage does not solely emerge around the inclusion but also inside the base material roughly at distance one third of the inclusion's radius from its interface.

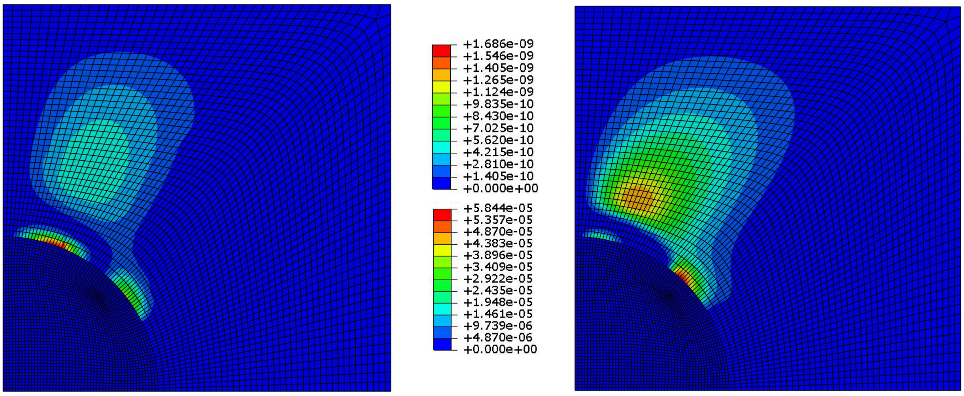

Figure 1: Damage field in the base material after cycle 1 (lhs) and 500 (rhs). Notice the difference in scale, the upper color bar refers to the first cycle and the lower one to the cycle 500.

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