Improved Performance Bounds for Iterative IC LMMSE Channel Estimator with SI Pilots

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***Abstract*—In this paper an iterative, interference cancelling receiver structure for single carrier transmission using superimposed training is studied. We derive the analytical MSE limits for LMMSE channel estimator using perfect channel knowledge and show that the simulated values follow well the analytical ones. Then, we utilise these results to analyse the MSE performance of a combined ML-LMMSE channel estimator structure, where the initial channel estimates are obtained from the ML channel estimator.**

***Keywords*: iterative receiver, LMMSE estimator, superimposed pilots**

I. Introduction

Currently, we live in the era of wireless digital communications and constantly explore for higher throughput in this hostile environment. Even though the physical layer throughput performance has been increasing rapidly in the last years, there are, in addition, increased requirements for signalling and training information. In our study, we have concentrated on reducing the overhead required by the training information used for channel estimation, referred as pilot symbols. Traditionally the pilot symbols are placed on specified slots in time and/or in frequency [1]. Another way to add training information to the transmitted signal is to directly add the pilot symbols on top of the user data symbols. For this reason, these pilots are often referred as superimposed (SI) pilots [2]. By using SI pilots, we can improve the spectral efficiency by allowing the user information to occupy the whole spectral region designed for communications. The down side is that the user information interferes greatly with the pilot sequence and that the user data symbol power to interference power ratio is decreased.

To overcome this problem of self interference, in [3] a cyclic pilot sequence structure was discussed. The main idea behind the cyclic pilot structure is to allow the utilisation of cyclic mean to improve the pilot to interference power ratio (PIPR) in the estimation process. Furthermore, in the same article optimal channel independent (OCI) training sequences were derived which are also used in our system model. Also, the effect of DC bias was studied in [3], but it is not considered in this paper.

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We have extended the model provided in [3] to our single carrier (SC) model with filter bank (FB) based receiver structure, presented in [4]. The channel estimates are obtained in time domain after which the sub-channel wise equalisation (SCE) is performed in the frequency domain. The FB based receiver structure is used because it provides close to ideal linear equaliser performance, has good spectral containment properties and is considered as a strong candidate for future wide area network (WAN) communications. The performance of the maximum likelihood (ML) receiver in the interference cancelling (IC) receiver with single-input multiple-output reception was studied in [5]. By IC we mean that we model the interference caused by user data based on latest symbol and channel response estimates, and remove this interference from the received sequence before channel estimation procedures.

In our system model the channel estimator length is smaller than the true channel length and this causes so called aliasing error in the cyclic mean calculation. The usage of short channel estimate is considered because when using cyclic pilot sequence we want to maximise the number of cycles and this leads us to compromise between cycle (estimator) length and estimation error. In addition, we can obtain complexity savings by intentionally using shorter channel estimator if we can allow limited error floor increase in the channel estimator mean squared error (MSE) performance.

For this paper, we have implemented the linear minimum mean squared error (LMMSE) channel estimator and derived the analytical MSE performance for two different setups. First, we derive the MSE performance limits for ideal LMMSE, where we assume that LMMSE knows the channel response. The second setup considers ML-LMMSE channel estimator, in which we obtain apriori channel estimate from ML channel estimator to be used in the LMMSE channel estimator.

This paper is organised as follows: in Section II the system model is introduced. The derivation of the analytical MSE for the ideal LMMSE channel estimator is given in subsection III-A. Next, in subsection III-B, the MSE results derived for the ideal LMMSE estimator are used together with results from [5] to estimate the MSE performance of the combined ML-LMMSE receiver with ML apriori channel estimation. In the end, in Section IV, conclusions and future topics are provided.

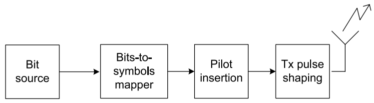

Fig. 1. Transmitter model.

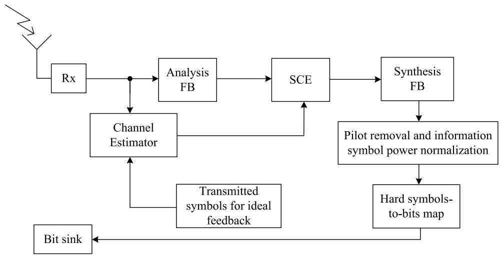

Fig. 2. Receiver model for analytical MSE derivations with IC based on IF.

II. System Model

The system design originates from the uplink assumption. Thus, the complexity of the transmitting end is kept as small as possible and most of the complexity is positioned to the receiving end. The very simple block level design of the transmitter is given in Fig. 1. The transmitter contains bit source, symbol mapper, pilot insertion and the transmitter pulse shape filter.

The used channel model is ITU-R Vehicular A channel with about 2.5 μs delay spread and approximately 20 MHz bandwidth [6]. The delay spread has maximum delay of 39 symbols or 78 samples in the receiver, where 2 times oversampling is utilised in the ideal frontend.

By ideal frontend we mean that we assume perfect synchronisation in frequency and time domain, ideal down conversion and 2 times oversampling of the received signal in Rx block. The receiver model used for deriving the MSE performance is presented in Fig. 2. This model contains only one receiving antenna, but the channel estimation procedure is simply copied to each reception branch with multiple antennas.

Based on these normal ideality assumptions, we can present the channel between transmitter and receiver as a 2 times oversampled discrete time equivalent channel as $h(k) = |h_{Tx}(t) \circ h_{channel}(t) \circ h_{Rx}(t)|_{t=kT/2}$, where $\circ$ defines a continuous time convolution. The overall equivalent channel has in our case a length of 142 samples. The received symbol $z(k)$ can be given as

$$z(k) = \sum_{m=0}^{M-1} h(m)s(k-m) + w(k), \tag{1}$$

where M is the channel length in samples, k is the time index for 2 times oversampled symbol sequence and $s(k)$ is a transmitted symbol, which is zero if $k < 0$ or $k > 2L-1$, where L is the frame length in symbols. The noise term $w(k) = |h_{Rx}(t) \circ v(t)|_{t=kT/2}$, where $v(t)$ is complex additive white Gaussian noise (AWGN), is simply modelled as AWGN without considering the correlation caused by the 2 times oversampled receiver pulse shape filtering. We will see in section III that this simplification has a minor effect on the channel estimation mean squared error (MSE).

When we are using SI pilots, the transmitted symbols are normalised combination of user data symbols and pilot symbols, defined as $s(k) = d(k) + p_c(k)$, where $d(k)$ represents a data symbol and $p_c(k)$ represents a symbol from the cyclic OCI pilot sequence. The transmitted user data symbol power is given as $\sigma_d^2 = 1 - \gamma$ and the pilot symbol power as $\sigma_{p_c}^2 = \gamma$, where γ is a power normalisation factor. Because of the oversampling, $s(k) = d(k) = p_c(k) = 0$ when k modulus 2 = 1. The power normalisation factor, γ, is used so that the overall transmitted symbol power is normalised to unity, $\sigma_s^2 = 1$.

From the receiver frontend, the oversampled signal is provided for channel estimator and for analysis FB. After obtaining channel estimate, SCE is performed in the frequency domain. It should be noted that the equalisation is now performed within mildly frequency selective subbands. More details on the equaliser structure can be found from [7], [4] and references there in.

After SCE the subsignals are recombined in the synthesis FB, which also efficiently realises the 2 times sampling rate down conversion. After the synthesis FB, the pilot structure is removed from the received symbol sequence, and with SI pilots the symbol sequence power has to be normalised. Next, we have a hard symbols-to-bits mapping and the hard bit estimates are provided to the bit sink for error rate calculations. In the derivation of the MSE limits, we consider only one pass through the channel estimator without feedback, and one pass with ideal feedback (IF). Channel estimation with IF assumes that the transmitted symbols and the true equivalent channel is known by the receiver in the IC process. In other words, IF perfectly removes the interference caused by the user data symbols and provides us a lower bound for the MSE for any hard IC feedback scheme.

III. Channel Estimation

In this paper we assume that we use a short channel estimator, which has shorter length than the true channel response length. We consider this setup because with cyclic SI pilots, the channel estimation performance does not depend only on the length of the estimator but also on the number of cyclic copies per transmitted frame. Therefore, there exists an optimal compromise between the pilot cycle length and the number of cycles per frame. Of course, other parameters, like pilot power allocation factor γ affect the performance of the channel estimator, but the overall system parameter optimisation problem is greatly bigger and is not considered in this paper. Further more, we assume discontinuous block wise transmission, which leads us to modify the pilot symbol matrix present in the equations and also creates an additional weighting for the user symbol error related parameters. These phenomenons are discussed in more detail in [5].

First, we clarify the matrix notation used in this paper for analytical MSE derivations. We assume that the frame length is $L = N_c \times N_p$, where N_c is the number of cyclic copies and N_p is the length of cyclic OCI pilot sequence. One OCI pilot cycle is given as $\mathbf{p} = [p(0)\ p(1)\ \dots\ p(N_p-1)]^T$. The cyclic pilot sequence added on top of the transmitted frame is given as $\mathbf{p}_c = \tilde{\mathbf{I}}\mathbf{p} = [\mathbf{p}\ \mathbf{p}\ \dots \mathbf{p}]^T$, where $\tilde{\mathbf{I}} = [\mathbf{I}\ \mathbf{I}\ \dots \mathbf{I}]^T$ is an $N_c \times 1$ block matrix built from $N_p \times N_p$ identity matrices. The pilot matrix $\mathbf{P}_c = \tilde{\mathbf{I}}\mathbf{P} = [\mathbf{P}\ \mathbf{P}\ \dots \mathbf{P}]^{\mathbf{T}}$ is an $N_c \times 1$ block matrix built from $N_p \times N_p$ cyclic pilot matrices $\mathbf{P}$.

Because we assume a discontinuous block wise transmission, we have to take into consideration the effect of discontinuous transmission in the pilot matrix $\mathbf{P}_c$. As it was shown in [5], we can obtain accurate estimator for discontinuous block wise transmission by defining a new matrix $\tilde{\mathbf{P}} = \mathbf{P} - (1/N_c)\mathbf{P}_{UT}$, where $\mathbf{P}_{UT}$ is an upper triangular matrix built from $\mathbf{P}$, and is defined as

$$\mathbf{P}_{UT}(r,c) = \begin{cases} 0, \text{ if } r \geq c, \\ \mathbf{P}(r,c), \text{ otherwise.} \end{cases} \quad (2)$$

We use a simple method to incorporate the additional error caused by the short channel estimator defined as error aliasing [5]. The basic idea is that the additional estimation error caused by short channel estimator is approximated by error aliasing in the cyclic mean calculations. Basically, we model the error by adding the expected amplitude response on top of each pilot cycle and study the channel tap powers aliasing on top of previous or following pilot cycles. This model causes optimistic behaviour estimates when channel estimators with length less than half of the true channel response are used. In our case, we use 96 tap channel estimator to estimate the 142 taps long true equivalent channel response. This choice of channel estimator length is arguable, because most of the equivalent channel taps outside the channel estimator reach are close to zero and are caused by the double RRC filtering in the system. Thus, the channel behavior is well estimated with our short channel estimator.

The deterministic error vector $\mathbf{h}_{aliasing} = \mathbf{h}_{pre} + \mathbf{h}_{post}$, is the aliasing error vector and consists of pre-estimator and post-estimator aliasing error vectors. The $\mathbf{h}_{pre}$ contains the expected channel amplitude response taps aliasing on top of the previous pilot cycles and the vector $\mathbf{h}_{post}$ contains the expected channel amplitude response taps aliasing on top of the following pilot cycles. This phenomenon is discussed in more detail in [5].

When defining the LMMSE channel estimator, we want to minimise the expected value of the squared error, $E\{|\mathbf{h}_{I_{short}} - \hat{\mathbf{h}}_{I_{short}}|^2\}$, where I_{short} is the set of indices which are estimated by the short channel estimator. Thus, the presented MSE estimates provide only the MSE for the estimated part of the equivalent channel response. The total MSE can be easily approximated by adding the summed power of the expected channel amplitude response outside the estimator, as was done in [5]. We define $\tilde{\mathbf{h}} = \mathbf{h}_{I_{short}}$ as the estimated part of the true equivalent channel. If we now make the assumptions that the noise and the total interference experienced by the pilot sequence is AWGN, channel taps are i.i.d. and have zero mean, i.e. $E\{\mathbf{h}\} = \mathbf{0}$, the LMMSE estimator can be simplified to [8]

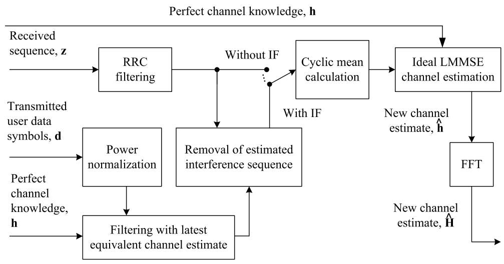

Fig. 3. Ideal LMMSE channel estimator with IC based on IF.

$$\hat{\mathbf{h}} = (\sigma^2 \mathbf{C}_{\tilde{\mathbf{h}}}^{-1} + \tilde{\mathbf{P}}_c^H \tilde{\mathbf{P}}_c)^{-1} \tilde{\mathbf{P}}_c^H \mathbf{z}, \quad (3)$$

where $(\cdot)^H$, refers to a Hermitian transpose operation, and

$$\sigma^2 = \begin{cases} \sigma_w^2 + \|\tilde{\mathbf{h}}\|^2 \sigma_d^2 \text{ , without IF and} \\ \sigma_w^2 \text{ , with IF} \end{cases} \quad (4)$$

is the total interference power experienced by the pilot sequence. The channel covariance matrix $\mathbf{C}_{\tilde{\mathbf{h}}}$, contains the apriori information of the channel tap values. By the assumption of independent tap coefficients, it becomes diagonal, i.e., $\mathbf{C}_{\tilde{\mathbf{h}}} = diag\{|\tilde{h}(0)|^2, |\tilde{h}(1)|^2, \cdots, |\tilde{h}(overN_p - 1)|^2\}$. By assuming the cyclic OCI training sequence, the LMMSE estimator can be reduced to

$$\hat{\mathbf{h}} = (\frac{\sigma^2}{N_c} \mathbf{C}_{\tilde{\mathbf{h}}}^{-1} + \tilde{\mathbf{P}}^H \tilde{\mathbf{P}})^{-1} \tilde{\mathbf{P}}^H \hat{\mathbf{m}}_z. \quad (5)$$

Based on [5], we can approximate the matrix product term $\tilde{\mathbf{P}}^H \tilde{\mathbf{P}}$, concerning the new pilot matrix $\tilde{\mathbf{P}}$, as

$$\tilde{\mathbf{P}}^H \tilde{\mathbf{P}} \approx \text{diag}\left\{ \sigma_p^2 \left[N_p - \lfloor \frac{i}{over} \rfloor (2 - 1/N_c)/N_c \right] \right\}, \quad (6)$$

where $i = 0, 1, \dots N_p - 1$ is the main diagonal index, diag$\{\cdot\}$ refers to a diagonal matrix and $over = 2$ is the oversampling factor. Further on, we will refer to one diagonal element of this matrix product as $\beta(i)$. Now, we can rewrite (5) as,

$$\hat{\mathbf{h}} = \frac{\tilde{\mathbf{P}}^H \tilde{\mathbf{P}}}{\frac{\sigma^2}{N_c} \mathbf{C}_{\tilde{\mathbf{h}}}^{-1} + \tilde{\mathbf{P}}^H \tilde{\mathbf{P}}} \tilde{\mathbf{P}}^{-1} \hat{\mathbf{m}}_z, \quad (7)$$

we notice that everything preceding the inverse of the pilot matrix $\tilde{\mathbf{P}}$ can be presented by a diagonal matrix $\mathbf{X}_{diag}$, defined by its diagonal elements as

$$\mathbf{X}_{diag}[i,i] \quad = \frac{\beta(i)}{\sigma^2/(N_c \sigma_{\tilde{h}(i)}^2) + \beta(i)}, \quad (8)$$

where $\sigma_{\tilde{h}(i)}^2$ is the power of the ith apriori channel tap estimate. The channel estimator can be stated as $\hat{\mathbf{h}} = \mathbf{X}_{diag} \tilde{\mathbf{P}}^{-1} \hat{\mathbf{m}}_z$. It is important to notice that the diagonal elements of $\mathbf{X}_{diag}$ are real and deterministic.

After obtaining the channel estimates a $4S$-point DFT of the channel estimate obtained $\hat{\mathbf{H}} = DFT\{\hat{\mathbf{h}}\}$, where S is the number of subbands in the synthesis bank, and in our simulations is set to be $S = 128$. The SCE is performed based on the frequency domain equivalent channel estimates, and a 3-tap complex FIR filter is used for SCE as in [4].

A. Theoretical MSE Limits for Ideal LMMSE Channel Estimator with OCI Training Sequences

Here we derive the analytic MSE threshold for the ideal LMMSE channel estimator, for which a block diagram is shown in Fig. 3. For ideal LMMSE channel estimator we assume that the LMMSE estimator has the true channel as apriori information. Also, in Fig. 3 the channel estimate and the user data symbols used for IC are the true ones. This setup is referred as estimation with IF.

The cyclic mean estimate vector of the received samples can be written in a matrix format as $\hat{\mathbf{m}}_z = \tilde{\mathbf{P}}\tilde{\mathbf{h}} + \mathbf{M}_d\tilde{\mathbf{h}} + \hat{\mathbf{m}}_w + \tilde{\mathbf{P}}\mathbf{h}_{aliasing}$, where $\hat{\mathbf{m}}_w$ is the cyclic mean estimate of the noise samples and $\mathbf{M}_d$ is a matrix of the cyclic mean estimates of the user data symbols and has a similar structure as $\mathbf{P}$.

Now, we can rewrite the channel estimator as

$$\hat{\mathbf{h}} = \mathbf{X}_{diag}[\tilde{\mathbf{h}} + \tilde{\mathbf{P}}^{-1}\mathbf{M}_d\tilde{\mathbf{h}} + \tilde{\mathbf{P}}^{-1}\hat{\mathbf{m}}_w + \mathbf{h}_{aliasing}]. \quad (9)$$

If we now define the estimation error, $\mathbf{e} = \hat{\mathbf{h}} - \tilde{\mathbf{h}}$, we can derive the MSE for the LMMSE estimator to be

$$\begin{aligned} \sigma_e^2 &= E\{\|\mathbf{e}\|^2\} = trE\{\mathbf{e}\mathbf{e}^H\} = \cdots \\ &= trE\{(\mathbf{X}_{diag} - \mathbf{I})\tilde{\mathbf{h}}\tilde{\mathbf{h}}^H(\mathbf{X}_{diag} - \mathbf{I})^H\} \\ &+ trE\{\mathbf{X}_{diag}\tilde{\mathbf{P}}^{-1}\mathbf{M}_d\tilde{\mathbf{h}}\tilde{\mathbf{h}}^H\mathbf{M}_d^H\tilde{\mathbf{P}}^{-H}\mathbf{X}_{diag}^H\} \\ &+ trE\{\mathbf{X}_{diag}\tilde{\mathbf{P}}^{-1}\hat{\mathbf{m}}_w\hat{\mathbf{m}}_w^H\tilde{\mathbf{P}}^{-H}\mathbf{X}_{diag}^H\} \\ &+ trE\{\mathbf{X}_{diag}\mathbf{h}_{aliasing}\mathbf{h}_{aliasing}^H\mathbf{X}_{diag}^H\}. \end{aligned} \quad (10)$$

All the cross terms inside the expectation are evaluated to zero, because of the normal independence assumptions between user data, channel response and additive noise. Because we have assumed discontinuous transmission, there is an additional weighting factor δ for the interference caused by the user data, as was shown in [5]. This weighting factor includes the knowledge of missing interfering user symbols in the beginning of the transmission. The weighting factor is defined as

$$\delta(m,i) = \begin{cases} \sigma_d^2/(overN_c) & \text{, if } m - i \geq 0 \\ \sigma_d^2(N_c - 1)/(overN_c^2) & \text{, otherwise,} \end{cases} \quad (11)$$

where $m, i = 0, 1, 2, \ldots, overN_p - 1$ and $over = 2$ is the oversampling factor used in the receiver frontend. Based on the above derivation, the MSE of the ideal LMMSE estimator can now be written as

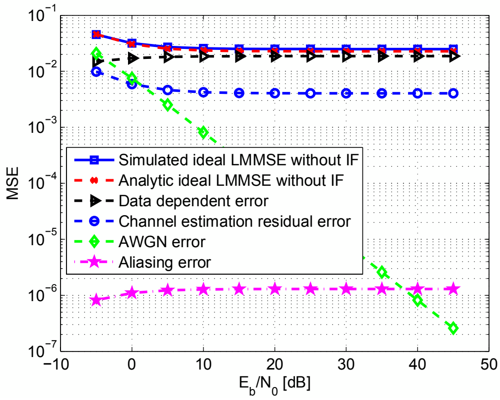

Fig. 4. Analytic and simulated MSE with 16-QAM constellation for ideal LMMSE channel estimator without IF.

$$\begin{aligned} \sigma_{e,i-LMMSE}^2 &= \sum_{m=0}^{2N_p-1} \frac{1}{(\frac{\sigma^2}{N_c}\sigma_{\tilde{h}(m)}^{-2} + \beta(m))^2} \\ &\times \left[\frac{\sigma^4}{N_c^2\sigma_{\tilde{h}(m)}^2} + \left(\sum_{i=0}^{2N_p-1} \sigma_{\tilde{h}(i)}^2\delta(m,i) \right.\right. \\ &\left.\left. + \frac{\sigma_w^2}{N_c} + |h_{aliasing}(m)|^2\beta(m) \right) \beta(m) \right]. \end{aligned} \quad (12)$$

In the given MSE equation above, the summation indices go to $2Np - 1$ because of the 2 times oversampling. In (12), the first term of the four additive terms is referred as the residual channel estimation error, second term is related to the interference caused by the user data, third term is the error caused by the AWGN noise and the fourth term is the aliasing error caused by the usage of shorter channel estimator than the true equivalent channel.

For the simulated results presented in this paper, we have channel estimator of length 96 samples, while the true equivalent channel length is 142 samples, as in [5]. Therefore, the presented MSE results model the error between the channel estimate and the estimated portion of the true equivalent channel. The overall error between the whole equivalent channel and short channel estimate can also be easily approximated by adding the channel power outside the channel estimator to the obtained MSE, as discussed in [5].

The used frame length is 3840 symbols (7680 samples). Thus, we have 80 copies of OCI pilot sequence in top of each frame. For the presented MSE results, we assumed $\gamma = 0.26$ for the 16-QAM constellation. The AWGN noise power was assumed to be known in the receiver.

In Fig. 4 we have presented the simulated and analytic MSE performance of the presented ideal LMMSE channel estimator without IF. In Fig. 4, the different error sources are plotted with separate lines. The simulated MSE follows closely the analytic

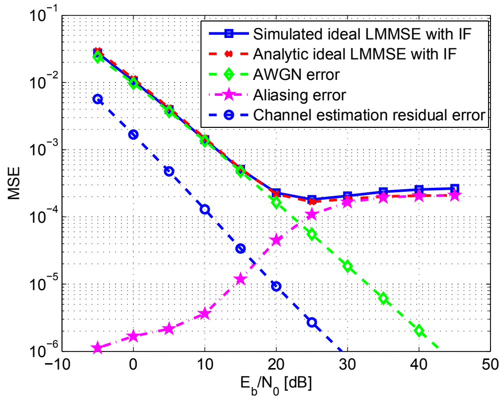

Fig. 5. Analytic and simulated MSE with 16-QAM constellation for ideal LMMSE channel estimator with IF.

one. We can see that the error performance is dominated by the interference caused by user data, as was to be expected. In all presented figures, x-axis follows logarithmic E_b/N_0 values, where E_b/N_0 is the bit energy to one sided noise power spectral density ratio.

In Fig. 5, the performance of the ideal LMMSE channel estimator with IF is presented. The IF is studied because it provides the lower bound for the MSE for any hard symbol feedback scheme. The MSE for the ideal LMMSE estimator with IF is given as

$$\sigma^2_{e,i-LMMSE,IF} = \sum_{m=0}^{2N_p-1} \frac{1}{(\frac{\sigma^2}{N_c}\sigma^{-2}_{\tilde{h}(m)} + \beta(m))^2} \left[\frac{\sigma^4}{N_c^2 \sigma^2_{\tilde{h}(m)}} + \left(\frac{\sigma_w^2}{N_c} + |h_{aliasing}(m)|^2 \beta(m) \right) \beta(m) \right]. \quad (13)$$

Based on the definition, this result does not include the error caused by the user data symbols because they are ideally removed. Interesting in the results is that the aliasing error is greatly enhanced in the IF case at high E_b/N_0 values. This causes an clear error floor in the MSE, which is similar as with ML channel estimation (see [5] for comparison). The aliasing error approaches the squared norm of the vector modelling aliasing, $\|\mathbf{h}_{aliasing}\|^2$, because with IF at high E_b/N_0 values the multiplier of the term $|h_{aliasing}(m)|^2$ approaches one.

B. Theoretical Limits for Combined ML-LMMSE Channel Estimator with OCI Pilot Sequences

Now, we assume that before the LMMSE channel estimator we have an ML channel estimator [5], which is used to provide the apriori channel estimates used by the LMMSE estimator. The channel estimator structure is shown in Fig. 6 and is referred throughout this paper as combined ML-LMMSE channel estimator.

We utilise the equations (12) and (13) to obtain the MSE estimates for the combined structure. Here we assume that the error and the channel estimate from the ML channel estimator are independent and that the average estimation error is evenly spread over all estimated channel taps. Thus, we can approximate the power of each estimated channel tap by

$$E|\hat{h}_{ML}(i)|^2 = \sigma^2_{\tilde{h}(i)} + \frac{\sigma^2_{ML,\mathbf{e}}}{overN_p}, \quad (14)$$

where $\sigma^2_{ML,\mathbf{e}}$ is the MSE of the ML channel estimator. It is defined as [5]

$$\sigma^2_{ML,e} = \sum_{m=0}^{2N_p-1} \frac{1}{\beta(m)} \left[\sum_{i=0}^{2N_p-1} \delta(m,i)\sigma^2_{\tilde{h}(i)} + \sigma_w^2/N_c \right] + \|\mathbf{h}_{aliasing}\|^2, \quad (15)$$

without IF, and as [5]

$$\sigma^2_{ML,e,IF} = \frac{\sigma_w^2}{N_c} \sum_{m=0}^{2N_p-1} \frac{1}{\beta(m)} + \|\mathbf{h}_{aliasing}\|^2, \quad (16)$$

with IF. These equations are used to derive the results shown in Figures 7 and 8.

Based on these assumptions, the equations (12) and (13) can be rewritten as

$$\sigma^2_{e,ML-LMMSE} = \sum_{m=0}^{2N_p-1} \frac{1}{(\frac{\sigma^2}{N_c}(\sigma^2_{\tilde{h}(m)} + \frac{\sigma^2_{ML,e}}{overN_p})^{-1} + \beta(m))^2} \times \left[\frac{\sigma^4}{N_c^2}(\sigma^2_{\tilde{h}(m)} + \frac{\sigma^2_{ML,e}}{overN_p})^{-1} + \left(\sum_{i=0}^{2N_p-1} \sigma^2_{\tilde{h}(i)}\delta(m,i) + \frac{\sigma_w^2}{N_c} + |h_{aliasing}(m)|^2\beta(m) \right) \beta(m) \right], \quad (17)$$

and

$$\sigma^2_{e,ML-LMMSE,IF} = \sum_{m=0}^{2N_p-1} \frac{1}{(\frac{\sigma^2}{N_c}(\sigma^2_{\tilde{h}(m)} + \frac{\sigma^2_{ML,e,IF}}{overN_p})^{-1} + \beta(m))^2} \times \left[\frac{\sigma^4}{N_c^2}(\sigma^2_{\tilde{h}(m)} + \frac{\sigma^2_{ML,e,IF}}{N_p})^{-1} + \left(\frac{\sigma_w^2}{N_c} + |h_{aliasing}(m)|^2\beta(m) \right) \beta(m) \right]. \quad (18)$$

The difference between the ideal LMMSE and ML-LMMSE with IF is that, although we fully remove the interference caused by user data with IF, the ML-LMMSE channel estimator uses the channel estimates obtained from ML channel estimator as apriori estimates and incorporates the estimation error related to ML channel estimation. Therefore, this provides us a lower bound for the MSE performance in a coded system with hard symbol feedback when using the ML-LMMSE channel estimator.

In Fig. 7, the MSE of the combined ML-LMMSE channel estimation structure without IF feedback is provided. It shows

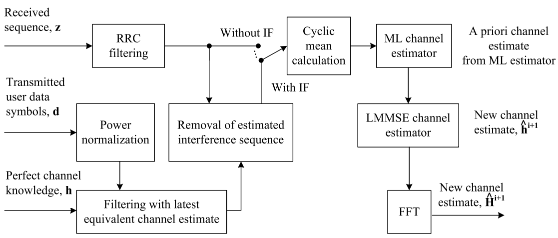

Fig. 6. Combined ML-LMMSE channel estimator with IC based on IF.

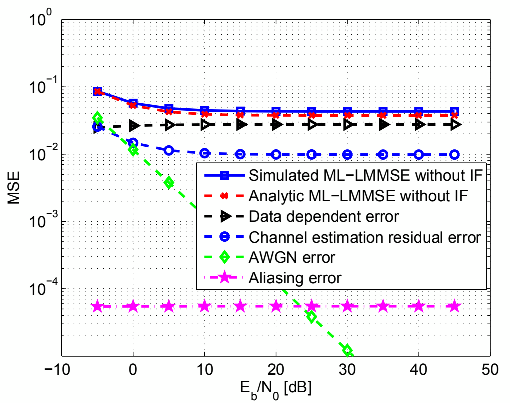

Fig. 7. Analytic and simulated MSE with 16-QAM constellation for the combined ML-LMMSE channel estimator without IF.

that the used approximations provide good MSE estimation accuracy. The MSE is clearly higher than with ideal LMMSE, as was expected. Also, the channel estimation residual error is increased because of the ML estimation error. Even so, it does not dominate the MSE and is clearly lower than the interference caused by the user data symbols.

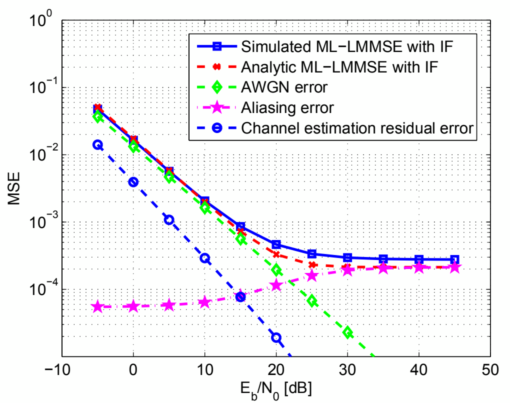

Fig. 8. Analytic and simulated MSE with 16-QAM constellation for the combined ML-LMMSE channel estimator with IF.

In Fig. 8, the performance of the combined channel estimator with IF is shown. Interestingly, we notice that with IF we can achieve almost the same performance as with ideal LMMSE. This indicates that through iterative IC processing we can achieve good performance.

IV. Conclusion

In this paper we have derived the MSE performance limits for the ideal LMMSE channel estimator and for the combined ML-LMMSE channel estimator, which is based on LMMSE using ML channel estimates as apriori information. The simulated MSE values follow well the presented analytical ones.

Based on the results, we can state that even though the performance of the combined channel estimation structure is degraded by the accuracy of the ML channel estimator, the ML-LMMSE solution can provide performance improvements through modest increment in the realisation complexity with higher order constellations. The channel estimation complexity is roughly doubled, but compared to the complexity of the whole reception chain, including e.g. some iterative channel decoding algorithm, this increase is minor.

The SI pilot structure studied in this paper provides interesting opportunities for future ad-hoc and device-to-device communications. In the future work, throughput performance comparison versus traditional time domain multiplexed pilots and analytical limits for the symbol error rate at the FB based maximum ratio combiner output are also of great interest.

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