B. Dumitrescu, R. Bregović, T. Saramäki, and R. Niemistö, "Low-delay nonuniform oversampled filterbanks for acoustic echo control," *Proc. European Signal Processing Conf. EUSIPCO-2006*, Florence, Italy, Sept. 2006, 4 pages.

LOW-DELAY NONUNIFORM OVERSAMPLED FILTERBANKS FOR ACOUSTIC ECHO CONTROL

Bogdan Dumitrescu[1], *Robert Bregović*[1], *Tapio Saramäki*[1] *and Riitta Niemistö*[2]

[1]Institute of Signal Processing
Tampere University of Technology
P.O.Box 553, 33101 Tampere, Finland
e-mail: bogdand,bregovic,ts@cs.tut.fi

[2]Nokia Research Center
P.O.Box 100, 33721 Tampere, Finland
riitta.niemisto@nokia.com

ABSTRACT

We propose an algorithm for designing nonuniform oversampled filterbanks with arbitray delay. The filterbank has uniform sections obtained by generalized DFT modulation; between the uniform sections, there are transition filters. There is no a priori constraint on the widths of transition filters channels, as in previous publications. The design algorithm is composed of three steps, in which a bank (analysis or synthesis) is optimized by solving convex optimization problems for finding the prototypes of uniform sections and the transition filters. In the first step, an orthogonal filterbank is designed, while in the other steps a bank is given and the other is optimized. We present an example of design suitable to subband processing of wideband speech signals.

1. INTRODUCTION

Subband adaptive filtering and particularly its applications to acoustic echo control have attracted considerable attention in the last decade. Oversampled filterbanks are used in this context, as the only possibility to have good out-of-band attenuation. Generalized DFT (GDFT) modulated uniform filterbanks (UFB) offer a low implementation complexity and several algorithms have been proposed recently for their design [5, 2, 8, 3]. Continuing the work from [3], in this paper we focus on the design of low-delay oversampled nonuniform filterbanks (NUFB) with a structure allowing efficient implementation.

There are three partly contradictory design requirements for filterbank design in high quality acoustic echo control and related speech enhancements. First, adaptive filtering can be carried out the more efficiently the lower the sampling rates in subchannels are. This suggests the use of UFBs whose number of channels is as high as possible for given delay and minimum stopband attenuation. Second, stopband attenuation of FB filters dictates cumulative alias in downsampling, which from the adaptive filtering point of view is noise. Thus, the higher the stopband attenuation, the better echo attenuation can be achieved until the level of background noise is reached. Third, in real-time applications, low delay is not only desirable but also required by standards.

NUFBs are more natural in subband speech processing because of human perception. Although it seems impossible in near future to design NUFBs that mitigate Bark-scale with affordable cost for real-time applications, it is desirable to have high frequency resolution in low band, because acoustic echo control is typically followed by and connected to other speech enhancement tasks such as noise reduction.

Speech signal has typically a spectrum that has a lowpass nature. Thus, strong low frequencies cumulate on weaker high frequencies in downsampling and high stopband attenuation is needed especially for the FB filters that correspond to high frequencies. Sufficient level of cumulative alias and delay can be obtained with NUFBs, where the frequency resolution provided by the filterbank is higher in low than in high frequencies.

The general structure of a NUFB with M channels is given in Figure 1. The NUFB is oversampled if the downsampling factors R_k, $k = 0 : M-1$, respect the relation

$$\sum_{k=0}^{M-1} \frac{1}{R_k} > 1. \tag{1}$$

The frequency responses of the filters in one bank (analysis or synthesis) have the idealized form presented in Figure 2. Subband speech processing requires that the filters $H_k(z)$, $F_k(z)$ have very good attenuation outside a band of width π/R_k and oversampling is the only way to fulfill this condition.

The input-output relation for the NUFB from Figure 1 is

$$Y(z) = T_0(z)X(z) + \sum_{k=0}^{M-1} \frac{F_k(z)}{R_k} \sum_{\ell=1}^{R_k-1} H_k(zW_{R_k}^{\ell})X(zW_{R_k}^{\ell}), \tag{2}$$

where $W_R = \mathrm{e}^{-j2\pi/R}$ and

$$T_0(z) = \sum_{k=0}^{M-1} \frac{1}{R_k} H_k(z)F_k(z) \tag{3}$$

is the distortion transfer function and determines the distortion caused by the overall system for the input signal. The terms of the double sum in (2) determine the effect of the aliased components $X(zW_{R_k}^{\ell})$ on the output signal.

The implementation complexity of a NUFB with independent filters $H_k(z)$, $F_k(z)$ is unacceptable in real-time applications; we describe in Section 2 a NUFB formed with sections of modulated uniform FB (UFB), joined with transition filters. In Section 3, we give an algorithm for designing low-delay NUFBs with such structures. In Section 4, we present an example of NUFB design.

2. FILTERBANK STRUCTURE

A low complexity NUFB consists of sections of several UFBs. Between consecutive sections lie transition filters. The number of sections, S, is usually very small, typically 2 or 3. The filters from the same section are obtained by GDFT modulation from a single prototype; so, their frequency responses are shifted versions of the frequency response of the

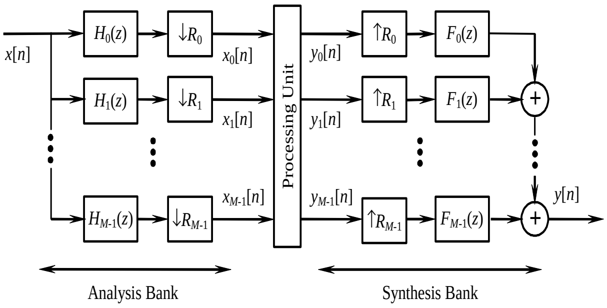

Figure 1: *Nonuniform filterbank.*

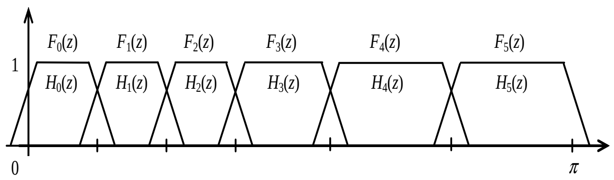

Figure 2: *Idealized frequency responses of the filters in the analysis bank.*

prototype. For example, the filters shown in Figure 2 can belong to a NUFB with 2 sections, the first with 3 filters and the second with 2 filters, joined by the transition filter $H_3(z)$.

Let us denote M_s, $s = 0 : S-1$, the number of channels of the UFB from which the filters in section s are extracted. Let m_s be the number of filters in section s; the number of channels of the NUFB is $M = \sum_{s=0}^{S-1} m_s + S - 1$. The width of channels in section s is $d_s = \pi/M_s$. Denoting $\tilde{d}_s$, $s = 1 : S-1$, the widths of transition channels, it follows that

$$\pi \sum_{s=0}^{S-1} \frac{m_s}{M_s} + \sum_{s=1}^{S} \tilde{d}_s = \pi.$$

Let $A_s(z)$, $s = 0 : S-1$, be the prototypes of the GDFT modulated UFBs. Let D be the overall delay of the NUFB. The impulse response of an analysis filter is

$$h_k[n] = a_s[n] e^{j\pi(k+\alpha_s)(n-D/2)/M_s}, \quad (4)$$

for some $s \in 0 : S-1$, $\alpha_s \in \mathbb{R}$. (The numbers α_s are determined by the position of the first filter in the section. For the first section we have $\alpha_0 = 1/2$.) Similar expressions hold for the synthesis filters, the prototypes being now $B_s(z)$, $s = 0 : S-1$. The transition filters have also complex impulse response, but not one obtained by modulation.

This NUFB structure was proposed by Princen [6] (using cosine modulation, i.e. real filters) and refined by Cvetkovic and Johnston [1], who gave also algorithms for large filters orders. All previous work dealt with orthogonal NUFBs, where the delay is equal to the order of the longest filter. We are interested here by the low-delay case, where D can have arbitrary values. Our structure is more general than that from [1] in yet another respect; there is no condition on the width of the transition channels, whereas in [1] this width is strictly defined by the width of channels in neighboring uniform sections.

3. DESIGN OF PROTOTYPE AND TRANSITION FILTERS

The filters $H_k(z)$, $F_k(z)$ have good filtering properties if they have good attenuation outside a band of width π/R_k. Also, this property means that the products $F_k(z)H_k(zW_{R_k}^{\ell})$, with $\ell \neq 0$, appearing in (2), are bounded by a small constant for all frequencies. As argued in [1, 3], it follows that the aliasing terms $X(zW_{R_k}^{\ell})$ from (2) have an almost negligible contribution to the output. To obtain a nearly perfect reconstruction (NPR) NUFB, only the condition

$$|T_0(e^{j\omega}) - e^{-jD\omega}| \leq \delta_d, \quad (5)$$

has to be imposed on the distortion transfer function, for all $\omega \in [0, 2\pi]$, where δ_d is a preset tolerance.

The design data are the orders of the prototypes and transition filters, the delay D and the tolerance δ_d. We seek to minimize the stopband energy of the filters, as explained below.

We follow the three-steps strategy employed for UFB design in [3]. First, an orthogonal filterbank is designed, usually of smaller order. Using the analysis bank thus obtained, the synthesis bank is optimized (the design data are used now). Finally, using the designed synthesis bank, the analysis bank is optimized. Each optimization consists of solving a convex problem, either semidefinite programming (SDP) or second-order cone programming (SOCP).

The simultaneous design of S prototypes and $S-1$ transition filters of a bank is difficult and so, in each step, we follow a progressive approach. First, the prototypes of the uniform sections are designed independently, using the algorithms from [3]. Then, the transition filters are designed one by one, by minimizing their stopband energy subject to the NPR constraint (5), imposed on a grid $\mathcal{G}$ of frequencies covering an interval $[\omega_{e1}, \omega_{e2}]$; typically, ω_{e1} is the middle of the last channel of the uniform section at the left of the transition filter and ω_{e2} is the middle of the first channel of the uniform section at the right of the transition filter. When computing the distortion transfer function (3), all available information is included, which means that an already designed transition filter is used in (5) when designing the other transition filters in the bank. We denote $\tilde{T}_0(z)$ the distortion transfer function obtained as in (3), with the not yet designed filters replaced by zero.

We describe below the design of a single transition filter of the analysis bank, denoted generically $U(z)$; its pair in the synthesis bank is $V(z)$. Let us denote ω_c the middle of the channel corresponding to this transition filter, R the downsampling factor for this channel and $\omega_{r1} = \omega_c - \pi/R$, $\omega_{r2} = \omega_c + \pi/R$ the edges of the ideal passband for the channel.

Optimization criterion. The stopband of the transition filter is $\mathcal{I}_s = [-\pi, \omega_{s1}] \cup [\omega_{s2}, \pi]$, where

$$\omega_{r1} \leq \omega_{s1} < \omega_{e1}, \;\; \omega_{r2} \geq \omega_{s2} > \omega_{e2}. \quad (6)$$

The stopband energy, which is the optimization criterion, has the expression

$$E_s = \frac{1}{2\pi} \int_{\mathcal{I}_s} |U(e^{j\omega})|^2 d\omega = \mathbf{u}^H \mathbf{\Phi} \mathbf{u}, \quad (7)$$

where $\mathbf{u}$ is the (complex) vector of coefficients of $U(z)$ and $\mathbf{\Phi}$ is a positive definite Hermitian Toeplitz matrix with the element on diagonal n defined by

$$\phi_n = \begin{cases} 1 - \frac{\omega_{2s}-\omega_{1s}}{2\pi}, & \text{if} \quad n=0, \\ \frac{j}{2\pi n}(e^{-j\omega_{s1}} - e^{-j\omega_{s2}}), & \text{otherwise.} \end{cases} \tag{8}$$

Orthogonal NUFB. In the first step of the overall algorithm, a complete orthogonal NUFB is designed. In this case, the synthesis filters are $F_k(z) = z^{-D}H_k^*(z^{-1})$ and so

$$T_0(e^{j\omega}) = e^{-jD\omega}\sum_{k=0}^{M-1}|H_k(e^{j\omega})|^2 \triangleq e^{-jD\omega}P_0(\omega). \tag{9}$$

We denote $\tilde{P}_0(\omega)$ a function defined as $P(\omega)$ above, but with the not yet designed filters replaced by zero. (We remind that the prototype filters are already designed when we start the transition filter design.) For a given frequency ω, the condition (5) becomes

$$1 - \tilde{P}_0(\omega) - \delta_d \le |U(e^{j\omega})|^2 \le 1 - \tilde{P}_0(\omega) + \delta_d. \tag{10}$$

So, the design problem becomes the minimization of the stopband energy (7), subject to the NPR constraint (10) for $\omega \in \mathcal{G}$. This optimization problem is convex in the coefficients of $G(e^{j\omega}) = |U(e^{j\omega})|^2$. Both (7) and (10) are linear in the coefficients of $G(z)$. Moreover, the coefficients of $G(z)$ are always a linear combination of the elements of a nonnegative definite matrix $\mathbf{Q}$ (more precisely, the coefficient g_n is the sum of the elements of $\mathbf{Q}$ on the n-th diagonal; see [4] for details). So, the design of the transition filter in an orthogonal NUFB consists of solving an SDP problem whose variable is $\mathbf{Q}$, followed by the complex spectral factorization of $G(z)$ that produces $U(z)$.

Biorthogonal NUFB. We assume that the synthesis bank is given (we are in step 2 or 3 of the overall algorithm described in the beginning of the section) and so $V(z)$ is known. The NPR condition (5) becomes

$$|\tilde{T}_0(e^{j\omega}) + U(e^{j\omega})V(e^{j\omega}) - e^{-jD\omega}| \le \delta_d. \tag{11}$$

Denoting $\mathbf{u} = \mathbf{u}_r + j\mathbf{u}_i$, where now $\mathbf{u}_r$ and $\mathbf{u}_i$ are real vectors, the inequality (11) can be transformed into the second-order cone inequality

$$\|\mathbf{A}_r(\omega)\mathbf{u}_r + \mathbf{A}_i(\omega)\mathbf{u}_i - \mathbf{b}(\omega)\| \le \delta_d, \tag{12}$$

where $\mathbf{A}_r(\omega)$, $\mathbf{A}_i(\omega)$ are known real matrices with two rows and $\mathbf{b}(\omega) \in \mathbb{R}^2$.

With $\mathbf{\Phi} = \mathbf{\Phi}_r + j\mathbf{\Phi}_i$, the stopband energy (7) of $U(z)$ can be expressed as

$$E_s = [\mathbf{u}_r^T\ \mathbf{u}_i^T]\begin{bmatrix} \mathbf{\Phi}_r & -\mathbf{\Phi}_i \\ \mathbf{\Phi}_i & \mathbf{\Phi}_r \end{bmatrix}\begin{bmatrix} \mathbf{u}_r \\ \mathbf{u}_i \end{bmatrix} \triangleq [\mathbf{u}_r^T\ \mathbf{u}_i^T]\mathbf{C}\begin{bmatrix} \mathbf{u}_r \\ \mathbf{u}_i \end{bmatrix}. \tag{13}$$

The minimization of the stopband energy subject to NPR constraints can be cast as the following SOCP problem

$$\begin{array}{ll} \min\limits_{u_r,u_i,\alpha} & \alpha \\ \text{s.t.} & \left\| \mathbf{C}^{1/2}\begin{bmatrix} \mathbf{u}_r \\ \mathbf{u}_i \end{bmatrix} \right\| \le \alpha \\ & (12),\ \omega \in \mathcal{G} \end{array} \tag{14}$$

Comments. Considering only the design of a pair of transition filters, the algorithm we propose has three additional parameters (besides those specified in the beginning of this section and the information given by the channel structure of the NUFB). One is the order N_0 of the initial orthogonal transition filter and the others are the frequencies ω_{e1}, ω_{e2} defining the interval on which the NPR constraint (5) is imposed. The appropriate choice of the values of the frequencies from (6) is important. Practically, we have taken ω_{s1}, ω_{s2} near the given edges ω_{r1}, ω_{r2} and selected ω_{e1}, ω_{e2} via a trial and error procedure.

As we make some approximations in imposing the NPR constraints, especially near the stopband edges of transition filters, there is a small price to pay: the uniform sections must have at least two filters (this is just a rule of the thumb; a careful analysis can be made, taking into account the relations between channel widths and down-sampling factors).

Section 1			Transition	Section 2		
m_0	M_0	d_0	$\tilde{d_1}$	m_1	M_1	d_1
24	48	$\pi/48$	$\pi/16$	7	16	$\pi/16$

Table 1: NUFB channel parameters.

4. EXAMPLE OF DESIGN

We illustrate the algorithm presented in the previous section with a NUFB with $S = 2$ uniform sections. The parameters describing the NUFB structure are given in Table 1. For simplicity, we have taken the transition channel width equal to the width of second section channels (we remind that such a condition is not necessary for our algorithm). The down-sampling factors are 32 for the first section and 10 for the second section and the transition filters. The filters lengths are 160 and 80 for the uniform sections prototypes and 150 for the transition filter; analysis and synthesis filters on the same channel have the same lengths. The NPR error from (5) is $\delta_d = 0.01$. The delay of the filterbank is $D = 128$ samples; for a sampling rate of 16 kHz, this corresponds to 8 ms.

The Matlab program implementing the design algorithm has been written with the help of the SDP library SeDuMi [7]. The design time for the NUFB with the above specifications is about 12 minutes, on a Pentium III PC at 1GHz. The design of the transition filters takes more than 10 minutes and thus is the most expensive part of the algorithm.

The frequency responses of the two uniform sections prototypes, transition filter and complete analysis bank are shown in Figures 3–6. The dashed vertical lines show the extent of the baseband. The stopband attenuations of the filters are given in Table 2; by A_s we denote the worst attenuation (typically near the baseband) and by A_m the best attenuation. The down-sampling factor of the transition channel can be raised to 12, case in which A_s becomes 65dB, i.e. still a convenient value. In Figure 6, the frequency axis is marked in kHz as for the use of the designed NUFB for processing wideband speech.

5. CONCLUSIONS

We have presented an algorithm for the design of NUFBs with uniform sections joined by transition filters. The algorithm consists of solving a succession of SDP and SOCP problems. Although the overall design problem is not convex, we have obtained filters with very good attenuations

	Section 1		Transition		Section 2	
	A_s	A_m	A_s	A_m	A_s	A_m
Analysis	44.6	75	84.3	101	75.2	93
Synthesis	43.6	77	83.9	103	74.5	95

Table 2: Stopband attenuations (in dB).

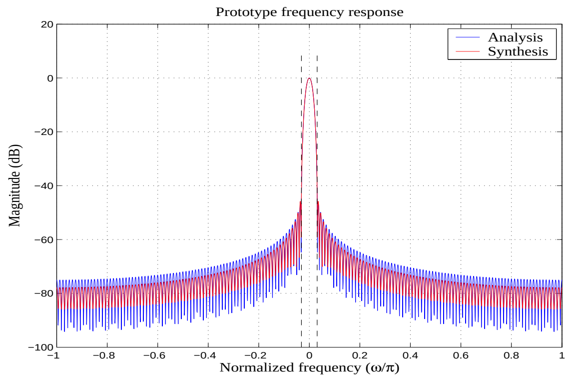

Figure 3: *Frequency response of the first prototype.*

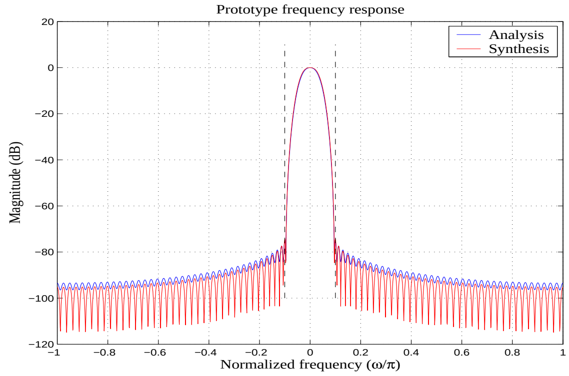

Figure 4: *Frequency response of the second prototype.*

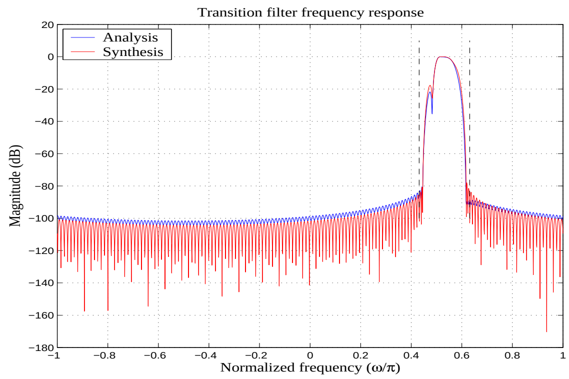

Figure 5: *Frequency response of transition filter.*

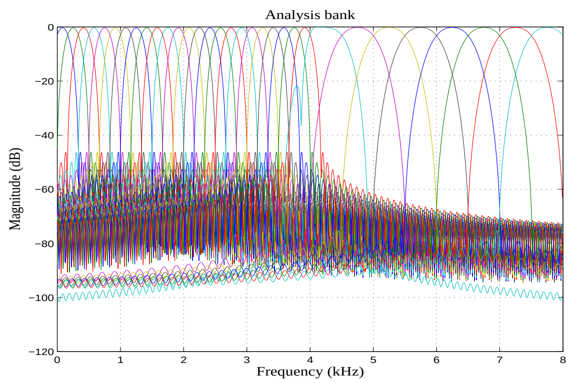

Figure 6: *Frequency response of all analysis filters.*

for NUFBs with delay appropriate to real-time processing of speech signals.

REFERENCES

[1] Z. Cvetkovic and J.D. Johnston. Nonuniform Oversampled Filter Banks for Audio Signal Processing. *IEEE Trans. Speech Audio Proc.*, 11(5):393–399, Sept. 2003.

[2] J.M. de Haan, N. Grbic, I. Claesson, and S.E. Nordholm. Filter Bank Design for Subband Adaptive Microphone Array. *IEEE Trans. Speech Audio Proc.*, 11(1):14–23, Jan. 2003.

[3] B. Dumitrescu, R. Bregović, and T. Saramäki. Simplified Design of Low-Delay Oversampled NPR GDTF Filterbanks. In *IEEE Conf. Acoustics, Speech, Sign.Proc. ICASSP*, volume 4, pages 525–528, Philadelphia, PA, USA, 2005.

[4] B. Dumitrescu, I. Tăbuş, and P. Stoica. On the Parameterization of Positive Real Sequences and MA Parameter Estimation. *IEEE Trans. Signal Proc.*, 49(11):2630–2639, Nov. 2001.

[5] K. Eneman and M. Moonen. DFT Modulated Filter Bank Design for Oversampled Subband Systems. *Signal Processing*, 81:1947–1973, 2001.

[6] J. Princen. The Design of Nonuniform Modulated Filter Banks. *IEEE Trans. Signal Proc.*, 43(11):2550–2560, Nov. 1995.

[7] J.F. Sturm. Using SeDuMi, a Matlab Toolbox for Optimization over Symmetric Cones. *Optimization Methods and Software*, 11-12:625–653, 1999. http://sedumi.mcmaster.ca.

[8] M.R. Wilbur, T.N. Davidson, and J.P. Reilly. Efficient Design of Oversampled NPR GDFT Filterbanks. *IEEE Trans. Signal Proc.*, 52(7):1947–1963, July 2004.