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An Efficient Method for Designing Low-Delay Nonuniform Oversampled *M*-channel Filterbanks

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ABSTRACT

This paper proposes an efficient method for designing oversampled nearly perfect reconstruction (NPR) *M*-channel nonuniform filterbanks (NUFBs) with an arbitrary delay. NUFBs considered in this paper are generated by joining two or more uniform sections with a transition filter (TF) in between every two adjacent sections. The uniform sections are generated by using the generalized DFT (GDFT) modulation of one prototype filter per section and are designed by using semidefinite programming and/or second-order cone programming. The complex-valued TFs are designed by utilizing the frequency-sampling technique (FST) for designing FIR filters. The performance of the proposed design method is shown by means of an example.

1. Introduction

Before processing an audio signal, due to the nature of the human hearing system, it is desired to split the signal, by means of a filterbank (FB), into subbands with channel widths following the Bark scale [1]. However, due to the complexity of the Bark scale, it is not possible, at least for now, to efficiently design as well as implement, in real time applications, a FB with such channels. Moreover, the delay that such a FB is allowed to add to the overall system is, in most cases, limited by standards. Therefore, the current research in this area concentrates on searching for compromise solutions between the complexity of the design (implementation) and the subband division. For example, the recent research has shown that various classes of oversampled nearly perfect reconstruction (NPR) uniform filterbanks (UFBs) perform very well in connection with typical speech enhancements application, like, adaptive filtering, echo cancellation, etc. [2]–[5]. However, due to the nature of psychoacoustic signals it is to expect that nonuniform filterbanks (NUFB) are a better choice.

In order to simplify the design and implementation of NUFBs, that is, to make a compromise between the above requirements, the commonly used NUFBs are designed by using one or more UFBs, with filters in a UFB generated by utilizing a generalized DFT (GDFT) or cosine modulation of one or more prototype filters. There are two main classes of such NUFBs introduced so far. The first class contains FBs that are generated by merging adjacent channels of one UFB [6]–[8]. The second class contains FBs that are generated by joining two or more UFBs, with transition filters (TFs) between two adjacent UFBs [9]–[11]. This paper concentrates on the second class as the one giving the highest flexibility, from the above two, with respect to the choice of the stopband attenuations, filter orders, and decimation factors in the subbands. Moreover, in comparison with [9] and [10] where linear-phase prototype filters have been used for generating the filters in the FB, in this paper low-delay filters are used like in [11] in order to generate FBs having a better selectivity then the ones in [9] and [10] for a given FB delay.

The authors of this paper have presented in [11] a method for designing NUFBs belonging to the class under consideration. However, in that method, a lot of time is used for designing the TF(s). This is due to the fact that in [11] a rather complex design procedure for the TFs has been used. The goal of this paper is to show that the TFs can be designed more efficiently by using the frequency-sampling technique (FST) used for designing FIR filters [12], [13]. The TFs generated in this way satisfy the design requirements with the overall design time being significantly reduced, as it is shown by means of an example. The design of filters belonging to the UFBs is identical to the design presented in [11], [14].

The outline of this paper is as follows: Section 2 gives the basic relations for the FBs under consideration. Section 3 gives the proposed design method, whereas the efficiency of the proposed method is shown by means of an example in Section 4.

Notations: When referring to a filter, $h[n]$, $H(z)$, and $H(e^{j\omega})$ denote the filter impulse response, transfer function and frequency response, respectively. Filters related with the analysis {synthesis} FB are denoted by h {f}, with superscript and subscript indexes specifying the channel. Filter orders and decimation factors are denoted by N and R, respectively. When referring to the NUFB, D denotes the FB delay in samples, M stands for the number of channels, and S for the number of uniform sections used to build the NUFB. Symbols referring to a channel have

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subscript k for $k=0,1,...,M-1$. When referring to the FBs in the uniform sections, M_s and m_s, for $s=1,2,...,S$ denote the overall number of channels of the UFB used in section s and the actual number of channels in section s of the NUFB, respectively. Symbols referring to a channel in a section have superscript (s) and subscript l for $s=1,2,...,S$ and $l=0,1,...,M_s-1$. Prototype filters used for building the UFBs are denoted by a subscript p. When referring to the TFs, the subscript t is used and the superscript (s), for $s=2,3,...,S$, defines the TF position. Additional notations are given as they appear in the paper.

2. Nonuniform Filterbanks: Basic Relations

The block diagram of a NUFB is given in Figure 1. In the case of NUFBs under consideration, the FB is build from two or more UFBs (uniform sections) with one TF between each two uniform sections. This is illustrated in Figure 2 for a 9-channel FB built from two uniform sections (S_1 and S_2) and one TF (t_2). Here, $M_1=12$, $m_1=6$, $M_2=6$, and $m_2=2$. Figure 2 also illustrates some of the notations given in the introduction.

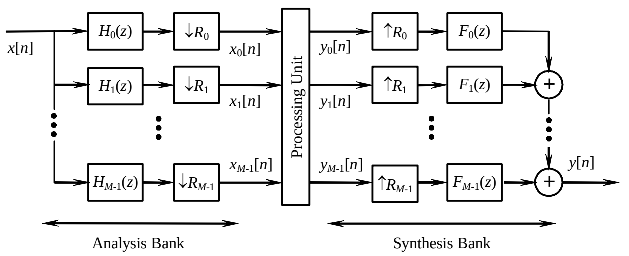

Figure 1. *M*-channel filterbank.

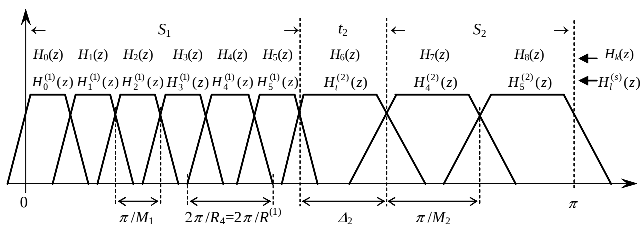

Figure 2. *M*-channel nonuniform filterbank for $M=9$ with two uniform sections and one transition filter.

In order to cover the overall spectra of the real-valued input signal, the following criterion must be satisfied:

$$\pi\sum_{s=1}^{S}\frac{m_s}{M_s}+\sum_{s=2}^{S}\Delta_s=\pi\ , \tag{1}$$

with Δ_s being the transition bandwidth of the sth TF.

For a FB to be oversampled, the following relation between the decimation factors R_k has to be satisfied:

$$\sum_{k=0}^{M-1}\frac{1}{R_k}>1\,. \tag{2}$$

Moreover, in order to minimize the aliasing errors, the bandwidth of the channel filters has to be smaller than $2\pi/R_k$ as illustrated in Figure 2.[1] In this case, the alias errors are at the level of the filter stopband attenuations, and as such can be ignored.

By restricting the filters building the FB in the above manner, the FB input-output relation can be stated as[2]

$$Y(z)\approx T_0(z)X(z)\,, \tag{3}$$

with

$$T_0(z)=\sum_{k=0}^{M-1}\frac{1}{R_k}H_k(z)F_k(z) \tag{4}$$

being the FB distortion transfer function. The goal now is to design the filters in the analysis and synthesis FB in such a way to obtain a NUFB satisfying the NPR property, that is, $y[n]\approx x[n-D]$.

In order to simplify the design and implementation of the FBs under consideration, the filters in each uniform section are derived by using the GDFT modulation of two real-valued prototype filters, one for the analysis and one for the synthesis side. The GDFT modulation used in this paper is defined as

$$h_l^{(s)}[n]=h_p^{(s)}[n]e^{j\pi(l+1/2)(n-D/2)/M_s} \tag{5a}$$

$$f_l^{(s)}[n]=f_p^{(s)}[n]e^{j\pi(l+1/2)(n-D/2)/M_s} \tag{5b}$$

for $n=0,1,...,N_h^{(s)}$ or $n=0,1,...,N_f^{(s)}$, $l=0,1,...,M_s-1$, and $s=1,2,...,S$.

The relation between the filters in the UFBs, $H_l^{(s)}$, and the filters in the NUFB, H_k, is illustrated in Figure 2.

3. Proposed Design Method

For the given FB requirements, channel division, decimation factors, stopband attenuations, FB delay, and distortion error, the NUFB can be designed by using the following two step procedure:

Step 1: Separately design the corresponding S different NPR UFBs satisfying the desired requirements (See Section 3.1).

Step 2: Design the complex-valued TFs to fill in the gaps between two adjacent uniform sections (See Section 3.2).

The methods for designing the UFBs and the TFs are given in the next two sections.

3.1 Design of Prototype Filters for Uniform Filterbanks

For each of the S uniform sections building the NUFB, one UFB is designed by solving the following optimization problem:

Minimize:

[1] It is assumed that signals in the subbands are complex-valued. Discussion on how to use NUFBs with real-valued subbands can be found in [15].

[2] In (3), the equality is not strict due to the alias errors that are ignored, in the given input-output function, as discussed earlier. See also [9], [14].

$$\int_{(1+\rho_s)\pi/M_s}^{\pi} \left|H_p^{(s)}(e^{j\omega})\right|^2 d\omega + \int_{(1+\rho_s)\pi/M_s}^{\pi} \left|F_p^{(s)}(e^{j\omega})\right|^2 d\omega \qquad (6a)$$

subject to:

$$\left|T_0^{(s)}(e^{j\omega}) - e^{-jD\omega}\right| \le \delta_d \quad \text{for} \quad \omega \in [0, \pi], \qquad (6b)$$

with

$$T_0^{(s)}(e^{j\omega}) = \sum_{l=0}^{M_s-1} \frac{1}{R^{(s)}} H_l^{(s)}(e^{j\omega}) F_l^{(s)}(e^{j\omega}) . \qquad (6c)$$

Here, δ_d is the desired NPR error of the NUFB and $T_0^{(s)}(e^{j\omega})$ is the frequency response of the distortion transfer function of the designed UFB. The unknowns in the above optimization problem are the coefficients of the prototype filters. The factor ρ_s is used to specify the stopband edge and has to be chosen such that it satisfies the following expression:

$$(1+\rho_s)\frac{\pi}{M_s} \le \frac{\pi}{R^{(s)}} . \qquad (7)$$

The above problem can be efficiently solved by using standard convex optimization algorithms (SDP, SOCP) as it has been shown in [14]. In order for the generated FB to satisfy the design specifications (stopband attenuation, FB distortion, etc.), the filter orders have to be selected as discussed in [14].

3.2 Design of Transition Filters

The TFs have to be designed in such a way that the *S*–1 gaps between the uniform sections designed in Step 1 are filled out. These filters must have the required attenuation in the filter stopbands, that is, on intervals $[-\pi, \omega_{r1}^{(s)}]$ and $[\omega_{r2}^{(s)}, \pi]$, and minimize the distortion error, evaluated on an interval $[\omega_{e1}^{(s)}, \omega_{e2}^{(s)}]$. Here, $\omega_{r1}^{(s)}$ and $\omega_{r2}^{(s)}$ are given as

$$\omega_{r1}^{(s)} = \omega_c^{(s)} - (1+\rho^{(s)})\pi / R^{(s)} \qquad (8a)$$

$$\omega_{r2}^{(s)} = \omega_c^{(s)} + (1+\rho^{(s)})\pi / R^{(s)} , \qquad (8b)$$

with $\omega_c^{(s)}$ denoting the centre of the *s*th TF and the parameter $\rho^{(s)}$ determining the TF width. The interval $[\omega_{e1}^{(s)}, \omega_{e2}^{(s)}]$ on which the NPR condition is enforced is typically selected in such a way to include the passbands of the filters adjacent to the designed transition one as well as to satisfy $\omega_{e1}^{(s)} \le \omega_{r1}^{(s)}$ and $\omega_{r2}^{(s)} \le \omega_{e2}^{(s)}$. This is illustrated in Figure 3. Moreover, this paper shows that for the NUFBs under consideration, it is enough to design one TF and use it on the analysis and synthesis side, that is, $F_t^{(s)}(z) = H_t^{(s)}(z)$ for $s = 2, 3, ..., S$.

In order to design the TF efficiently, in this paper, the FST for designing FIR filters is utilized [12], [13]. The main idea of the FST is to characterize the desired transfer function in the frequency domain, sample it in *N*+1 points, with *N* being the desired filter order, optimize few frequency domain samples (typically transition band samples), and then generate the filter coefficients by an inverse DFT.

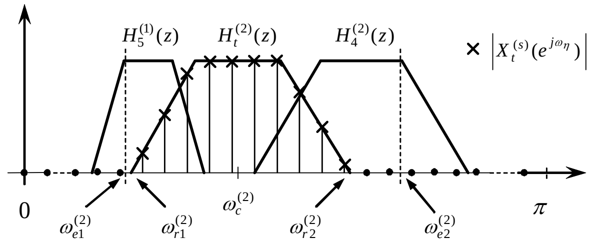

Figure 3. Characteristic frequencies and sampled frequency response for designing a transition filter (see also Figure 2).

In the case of TFs under consideration, the sampled frequency response can be constructed as

$$\tilde{H}_t^{(s)}(e^{j\omega_\eta}) = \begin{cases} X_t^{(s)}(e^{j\omega_\eta}) & \text{for} \quad \omega_\eta \in [\omega_{r1}^{(s)}, \omega_{r2}^{(s)}] \\ 0 & \text{elsewhere} \end{cases} \qquad (9a)$$

with

$$\omega_\eta = \pi \, \eta / (N_{ht}^{(s)} + 1) \qquad (9b)$$

for $\eta = 0, 1, ..., N_{ht}^{(s)}$. Here, ω_η is the discrete sampling frequency and $X_t^{(s)}(e^{j\omega_\eta})$ are the unknown complex-valued samples (see Figure 3). The number of these samples is

$$N_x^{(s)} = \left\lfloor (N_{ht}^{(s)} + 1)(\omega_{r2}^{(s)} - \omega_{r1}^{(s)}) / \pi \right\rfloor . \qquad (10)$$

The complex-valued impulse response coefficients are evaluated by applying the inverse DFT on the sampled frequency response:

$$h_t^{(s)}[n] = \frac{1}{N_{ht}^{(s)} + 1} \sum_{\eta=0}^{N_{ht}^{(s)}} \tilde{H}_t^{(s)}(e^{j\omega_\eta}) e^{j\omega_\eta} \qquad (11)$$

for $n = 0, 1, ..., N_{ht}^{(s)}$.

By taking into account the above discussion, the design of one TF for the NUFB under consideration can be performed by using the following three step procedure:

First, a partial filter bank transfer function is generated as

$$T_d(z) = \sum_k \frac{1}{R_k} H_k(z) F_k(z) \qquad (12)$$

for $k = 0, 1, ..., M-1$, excluding *k*'s corresponding to the not yet designed TFs. Such transfer function has a big distortion at frequencies corresponding to the gaps between the uniform sections.

Second, the initial values for the unknown samples of the TF transfer function can be generated as

$$\hat{X}_t^{(s)}(e^{j\omega_\eta}) = \sqrt{\left|1 - \left|T_d(e^{j\omega_\eta})\right|\right|} \, e^{-j\omega_\eta \frac{D}{2}} \qquad (13)$$

for $\eta = 0, 1, ..., N_{ht}^{(s)}$, $\omega_\eta \in [\omega_{r1}^{(s)}, \omega_{r2}^{(s)}]$, and $T_d(e^{j\omega})$ being defined by (12). In (13), the square root is used due to the relation $F_t^{(s)}(z) = H_t^{(s)}(z)$. Moreover, the exponential term is added in order to make the delay of the TF appropriate for use in the NUFB. The order $N_{ht}^{(s)}$ can be selected as the higher order between the filter orders of the adjacent uniform sections, that is,

$$N_{ht}^{(s)} = \max\left\{N_h^{(s-1)}, N_h^{(s)}, N_f^{(s-1)}, N_f^{(s)}\right\}. \quad (14)$$

Third, the following optimization problem has to be solved:

Minimize:

$$\max\left\{ W_s \left| H_t^{(s)}(\boldsymbol{X}_t^{(s)}, e^{j\omega}) \right| \right\} \quad \text{for } \omega \in [-\pi, \omega_{r1}^{(s)}] \cup [\omega_{r2}^{(s)}, \pi] \quad (15a)$$

subject to:

$$\left| T_0(\boldsymbol{X}_t^{(s)}, e^{j\omega}) - e^{-jD\omega} \right| \le \delta_d \quad \text{for } \omega \in [\omega_{e1}^{(s)}, \omega_{e2}^{(s)}] \quad (15b)$$

with

$$T_0(\boldsymbol{X}_t^{(s)}, e^{j\omega}) = T_d(e^{j\omega}) + H_t^{(s)}(\boldsymbol{X}_t^{(s)}, e^{j\omega})^2 / R^{(s)} \quad (15c)$$

$$\boldsymbol{X}_t^{(s)} = \begin{bmatrix} X_t^{(s)}(e^{j\omega_1}) & X_t^{(s)}(e^{j\omega_2}) & \dots & X_t^{(s)}(e^{j\omega_{N_x^{(s)}}}) \end{bmatrix}^T \quad (15d)$$

$$W_s = \begin{cases} 10^{A_{s-1}/20} & \text{for } \omega \in [-\pi, \omega_{r1}^{(s)}] \\ 10^{A_s/20} & \text{for } \omega \in [\omega_{r2}^{(s)}, \pi]. \end{cases} \quad (15e)$$

Here, the vector $\boldsymbol{X}_t^{(s)}$ contains the N_x complex-valued unknown frequency samples and W_s is the weighting function ensuring different attenuations in the left and right TF stopband, with A_s being the desired minimum attenuation in the sth uniform section. In order to make things more clear, in (15a)–(15e), $\boldsymbol{X}_t^{(s)}$ is added as an argument of every variable that depends on this values.

The above optimization problem can be solved quickly by using any available unconstrained optimization routine, for example, the function `fminimax` included in the Optimization Toolbox [17] provided by MathWorks, Inc. with initial values provided by (13). In each step, (9a) and (11) are used to derive the TF impulse-response coefficients out of the optimization variables $\boldsymbol{X}_t^{(s)}$. This coefficients are then used to evaluate (15a) and (15b) on a dense frequency grid.

The following points regarding the above optimization problem should be emphasized. First, as objective function, in (15a), a minimax criterion is used instead of the LS criterion used in [11]. This has turned out to be more convenient when using `fminimax` for solving the above problem. The results generated in this way are similar to the ones that would be generated by using the LS design. This is partially because of the fact that, due to a limited number of unknowns, the control over the stopband is limited. Second, the number of complex-valued unknowns has been considerably reduced, from $N_{ht}^{(s)} + N_{ft}^{(s)}$ when designing the TFs directly[3], to N_x when using the proposed approach. This reduction in the number of unknowns considerably reduces the design time. Third, the filter order given by (14) is always high enough to ensure a solution satisfying the design requirements. If desired, then the filter order can be reduced until the minimum one is found.

After deriving the coefficients of one TF, the procedure is repeated for the other TFs, one at a time. When evaluating (12) the already designed TFs are included.

4. Example

To illustrate the efficiency of the proposed method, a NUFB with $S = 2$ uniform sections will be designed. The specifications are the same as in the example in [11] and are given in TABLE I. The delay of the FB is $D = 128$ samples and the required NPR error is $\delta_d = 0.01$.

TABLE I EXAMPLE NUFB CHANNEL PARAMETERS

	m_s	M_s	Δ	R_s	A_s	N
S_1	24	48	$\pi/48$	32	45	159
T_f	-	-	$\pi/16$	10	45, 75	149
S_2	7	16	$\pi/16$	10	75	79

By evaluating (10) it turns out that $N_x = 15$. This leaves the design with 15 complex valued unknowns for designing the TF. The proposed design algorithm has been implemented in matlab using the Matlab Optimization Toolbox [17] and the SDP library SeDuMi [16]. The design time for the NUFB with the above specifications is about 2.5 minutes (about 90 sec for the UFB sections and about 60 sec for the TF) on a Pentium III PC at 1GHz. When using the method proposed in [11], the overall design requires 12 minutes with the design of the TF filter taking 10.5 minutes. In this case, the overall design time is reduced with respect to the method proposed in [11] by a factor of 4.

The normalized frequency response of the TF, analysis FB, and the distortion transfer function are shown in Figure 4, Figure 5, and Figure 6, respectively. The distortion transfer function is evaluated by using (4). As can be seen from the figures, the designed TF, as well as the FB, satisfies the design requirements.

[3] In [11], although two different transition filters are used, they are designed iteratively, one at a time.

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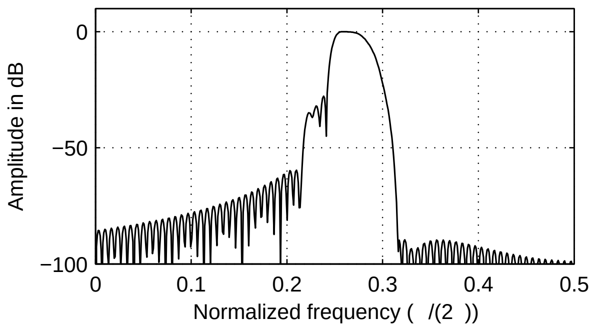

Figure 4. Frequency response of the transition filter.

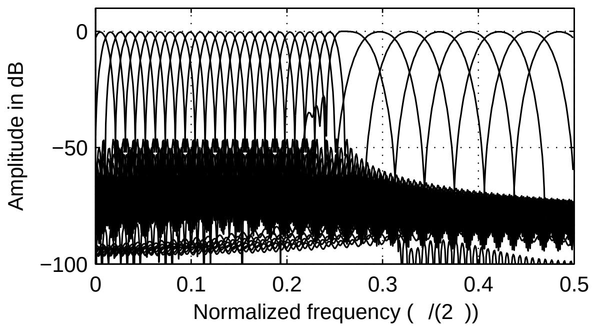

Figure 5. Frequency response of the analysis filterbank.

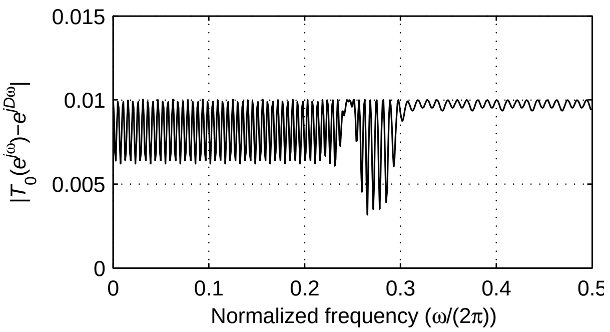

Figure 6. Filterbank distortion.

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