A Simple Closed-Form Analysis of Clapp Oscillator Output Power Using a Novel Quasi-Linear Transistor Model

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***Abstract*— Clapp oscillator output power is analyzed in closed form using a novel quasi-linear transistor model where the transadmittance is a function of base-to-emitter voltage amplitude. The function is easy to invert which makes the analysis considerably simpler than any other previously published closed-form approach. The proposed technique is validated by transistor and oscillator measurements and with harmonic balance simulation at 100 MHz.**

***Index Terms*— Nonlinear circuits, Nonlinear network analysis, Oscillators, Output power.**

I. Introduction

Oscillator output power has been studied for decades. Much of the progress is reviewed, *e.g.*, in [1] and [2]. Both analytical and numerical techniques have been developed with varying presumptions, complexity, generality, and accuracy. Simple analytical techniques offer the best insight into oscillator operation and trade-offs. This paper proposes such a new technique.

We show that, by using a novel quasi-linear transistor model and conventional linear circuit analysis, Clapp oscillator output power can be calculated in closed form. The proposed technique is now applied (but not restricted) to the Clapp oscillator (Fig. 1) which is probably one of the most abundant oscillators, at least, if we accept that the usual crystal oscillators and Colpitts oscillators are actually sub-classes of Clapp. The proposed technique falls on the middle ground between the elementary linear techniques, frequently quoted in textbooks [3], and, those more advanced nonlinear techniques that, however, are often too complicated to provide useful insight. The present technique is significantly simpler than any of the previously published closed-form approaches [1]. Nevertheless, its prediction power can be comparable to that of a commercial *harmonic balance simulator*, as this paper demonstrates.

II. Overview of the Output Power Analysis

First the transistor is modeled at a chosen bias point (V_{CEQ}, I_{CQ}) as will be explained in the next chapter. Then values are assigned for the oscillator's "embedding" C_1, C_2, R_L,..., shown in Fig. 1. One component value should not be assigned, *e.g.*, C_r because it will be calculated according to the desired oscillation frequency $\omega_0/2\pi$.

Nodal analysis is then applied to the oscillator circuit that consists of the quasi-linear transistor model and the embedding. The determinant of the circuit's Y-matrix is set to zero, which is the condition for self-sustained oscillation [5]:

$$\det(Y) = 0. \tag{1}$$

From this condition, the amplitude-dependent large-signal transadmittance magnitude $|y_m|$ and the unassigned value, say C_r, are solved at ω_0. Having $|y_m|$ solved, next, base-to-emitter voltage amplitude $\hat{v}_{be}$ can be calculated as their interdependence (2) is under control. Finally, the load voltage amplitude $\hat{v}_L$ and output power are calculated using the dependence of $\hat{v}_L$ on $\hat{v}_{be}$, as given by (5). Chapter IV exemplifies the technique.

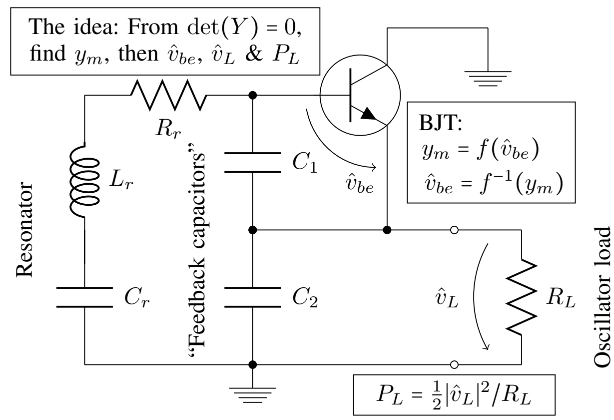

Fig. 1. Schematic diagram of a Clapp oscillator [4] with biasing omitted. Impedance matching from a true external load (50 Ω) is assumed to be embedded in the load R_L. The LC-resonator's equivalent series resistance is described by R_r.

III. The Quasi-Linear Transistor Model

For simplicity, the transistor is modeled now with a quasi-linear model: an equivalent circuit where only one element value is nonlinear. A key-enabler of the present technique is a simple yet sufficiently realistic transistor model. It is well known that there are various amplitude-limiting (or "power-saturating") nonlinear mechanisms in all transistors. In this analysis we assume that the nonlinearity of $|y_m|$ is dominating over all other mechanisms.

We propose a new model for the large-signal transadmittance $y_m = |y_m|e^{j\phi}$. One can expect that $|y_m|$ should

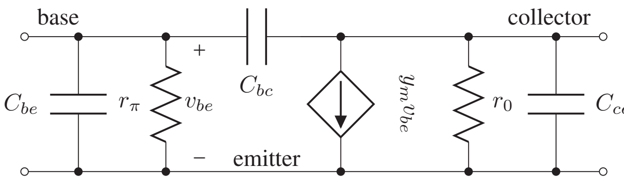

Fig. 2. The transistor model used in this analysis.

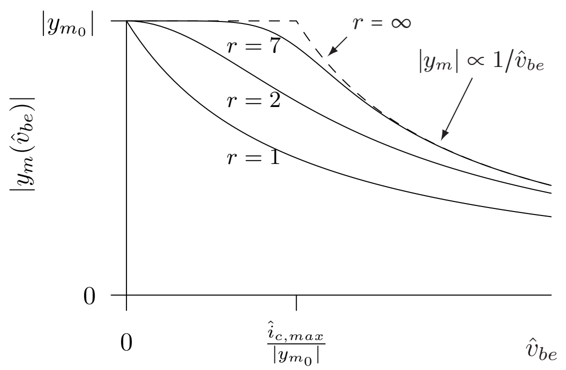

Fig. 3. Transadmittance magnitude according to (2).

behave according to the following function of $\hat{v}_{be}$

$$|y_m(\hat{v}_{be})| = \frac{|y_{m0}|}{\left[1 + \left(\frac{\hat{v}_{be}|y_{m0}|}{\hat{i}_{c,max}}\right)^r\right]^{1/r}} \tag{2}$$

where y_{m0} is the small-signal transadmittance while r and $\hat{i}_{c,max}$ are empirical fitting parameters. The argument ϕ can be regarded independent of amplitude, as discussed below in more detail. As far as we know, this simple model has not been presented earlier. However, the model is obvious as $|y_m| \approx |y_{m0}|$ for small $\hat{v}_{be}$, and, $|y_m| \approx \hat{i}_{c,max}/\hat{v}_{be}$ for large $\hat{v}_{be}$, implying that the current amplitude $\hat{i}_c = |y_m|\hat{v}_{be}$ eventually saturates to a constant or "clipped" value $\hat{i}_{c,max}$, independent of $\hat{v}_{be}$. The parameter r shapes the transition from the small-signal range (where $|y_m| \approx |y_{m0}|$) to the $1/\hat{v}_{be}$-dependency range, as shown in Fig. 3. Typically $r \approx 2$. The transition would be abrupt for $r = \infty$. For the purpose of calculating the oscillator output power in closed form, it is necessary that (2) can be solved for $\hat{v}_{be}$.

We measured the S-parameters of six different BJT devices as a function of network analyzer power (−20 . . . −2 dBm) at 100 MHz. Equivalent-circuit (Fig. 2) element values were extracted for all of them. Amplitude-dependency was simply ignored and small-signal values, those measured at −20 dBm, were adopted for all elements except $|y_m|$. Instead, $|y_m|$ was modeled using (2) whose parameters were obtained through least squares fitting. Fig. 4 shows that (2) describes the $|y_m|$-behavior of all tested transistors very well. What comes to ϕ, its typical variation was only 5°, while the maximum was 20°, over the entire test power range, which gives reasons to regard ϕ constant. This keeps the oscillator analysis simple.

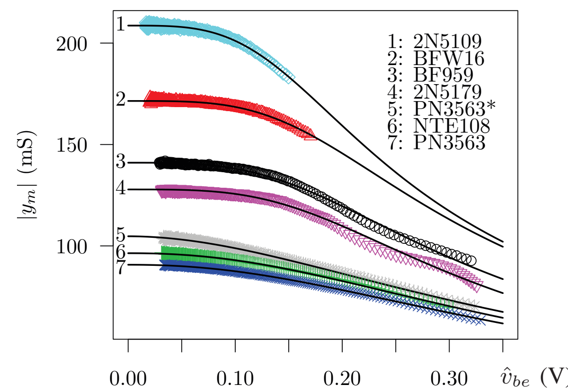

Fig. 4. Measured (circles) and modeled (solid line) $|y_m|$ of six different BJT devices at $V_{CEQ} = 5$ V and $I_{CQ} = 20$ mA. Two samples of PN3563 were measured; number 5, indicated with an asterisk (*) is the one used in the example calculation.

IV. Example of Power Calculation at 100 MHz

This example uses PN3563. A sample of PN3563 had $|y_{m0}| = 104$ mS, $\phi = -64°$ $r = 2$, and $\hat{i}_{c,max} = 30.6$ mA at the dc operating point $V_{CEQ} = 5$ V and $I_{CQ} = 20$ mA.

We lump the transistor's parasitic capacitances C_{be} and C_{ce} into C_1 and C_2, respectively: $C_1' = C_1 + C_{be}$ and $C_2' = C_2 + C_{ce}$, in hope of making the results less specific for this particular transistor. Further, we take C_1' and C_2' as *variables* in order to demonstrate the prediction power of the present theory. For the resonator we take $L_r = 500$ nH, which provides reactance of 314 Ω and high unloaded quality factor Q_u at 100 MHz. Typical Q_u for airwound inductors is 40. Therefore, we take $R_s = 314/40 \approx 8$ Ω. We arbitrarily take the load resistance as $R_L = 250$ Ω.

Next, the Y-matrix of the oscillator shown in Fig. 5 is constructed and its determinant is set to zero:

$$\det(Y) = \begin{vmatrix} Y_1 + j\omega C_{bc} + Y_r & -Y_1 \\ -Y_1 - y_m & Y_1 + Y_2 + y_m \end{vmatrix} = 0 \tag{3}$$

at $\omega = \omega_0$ with $Y_1 = 1/r_\pi + j\omega C_1'$, $Y_2 = 1/R_L + 1/r_0 + j\omega C_2'$, and $Y_r = 1/(R_r + j\omega L_r + {}^1\!/_{j\omega C_r})$. Both unknowns, $|y_m|$ and C_r can be solved from (3). Only the magnitude of y_m (not phase) needs to be solved, since ϕ is assumed constant and equal to the small-signal value. To keep the oscillation frequency at 100 MHz, capacitance of C_r must range from around 25 pF down to some 5 pF for C_1' and C_2' capacitances ranging from 5 to 200 pF. For small C_1' and C_2', large C_r are needed[1] since L_r is fixed.

After applying some algebra on (3), it turns out that $|y_m|$ is the smaller[2] of the real, positive roots of the polynomial

$$a|y_m|^2 + b|y_m| + c \tag{4}$$

[1]With large C_r the circuit is no longer a canonical Clapp oscillator where C_r is much smaller than C_1' or C_2'. Instead, the circuit will approach a Colpitts configuration, where C_r works merely as a dc block.

[2]For some reason, as yet unresolved, the larger root was always larger than $|y_{m0}|$, which is unphysical, and it was therefore ignored. Complex and negative roots imply that oscillation conditions are not met.

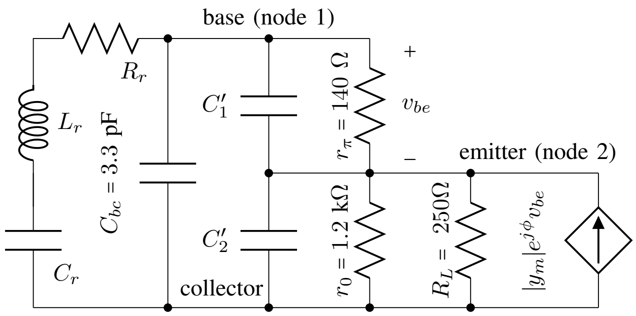

Fig. 5. The Clapp oscillator of Fig. 1 redrawn. The transistor is replaced by its equivalent circuit (Fig. 2). Transistor's capacitances C_{be} and C_{ce} are lumped into C_1' and C_2', respectively.

whose coefficients a, b, and c can be written in terms of the parameters appearing in Fig. 5. Having $|y_m|$ solved, then $\hat{v}_{be}$ can be solved from (2) and the output power from

$$P_L = \frac{\hat{v}_L^2}{2R_L} = \left|\frac{Y_1 + y_m}{Y_2}\right|^2 \frac{\hat{v}_{be}^2}{2R_L} \tag{5}$$

where $\hat{v}_L$ is the amplitude of the voltage across R_L. Fig. 6 shows the calculated power as a function of C_1' and C_2'.

V. Comparison to Measurement and Simulation

This 100-MHz oscillator was also built and tested. The output power of the fundamental frequency component was measured with a spectrum analyzer for over 20 different (C_1, C_2)-combinations. E12-series radial-leaded ceramic disc capacitors were selected so that the ratio C_1'/C_2' was always as close to 4 as possible. Impedance matching from 50 Ω to 250 Ω comprised a 16-pF series capacitor and a 200-nH air-wound shunt inductor.

The oscillator circuit was simulated with the harmonic balance simulator of Agilent ADS 2011.01. In the absence of a nonlinear model for PN3563, the ADS library model of an electrically very similar transistor, 2N918, was used. In the simulation, the oscillation frequency was kept to 100 MHz, within ±5% error, by adjusting C_r. Ideal models were used for all passive components except the resonator inductor for which the loss resistance R_r was, again, taken into account. The present theory agrees well with measurements and simulations, as Fig. 7 demonstrates.

VI. Conclusion

We have proposed a *novel quasi-linear transistor model* and shown that, by using this model together with conventional linear circuit theory, Clapp oscillator output power can be analyzed in *closed form*. The results offer *significant insight* into oscillators by showing that the amplitude-dependency of transadmittance magnitude, alone, can explain the oscillation amplitudes and, consequently, output power. This technique is *considerably simpler* than any of the previously published closed-form approaches, yet it agrees well with measurements and simulation.

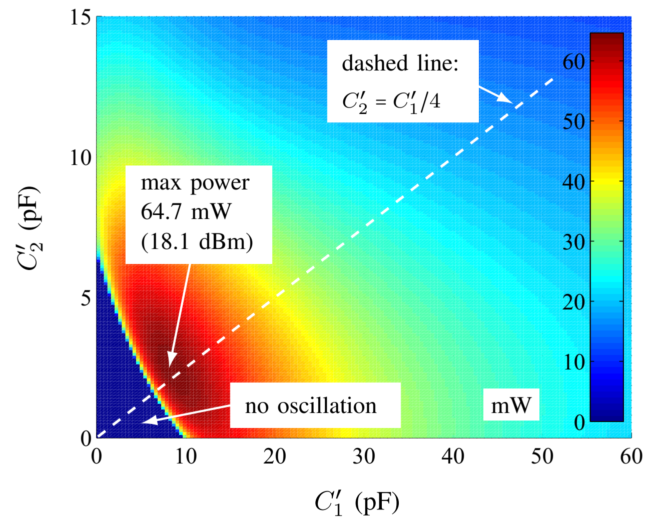

Fig. 6. Calculated oscillator output power P_L (mW) at 100 MHz as a function of C_1' and C_2'. Data along the dashed line appears in log-scale in Fig. 7.

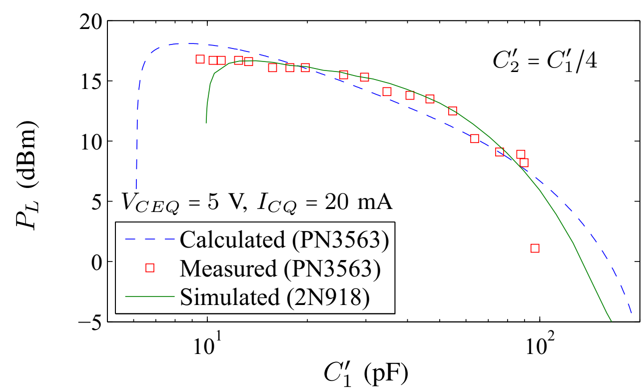

Fig. 7. Calculated, measured, and simulated output power P_L.

For further verification, the technique should be tested for different load resistances R_L. It should also be studied how well the proposed transadmittance model (2) would suit devices such as HBT's, MOSFET's, and HEMT's. The technique can easily be extended to other popular oscillators [1], [6], with modifications to (3), (4), and (5).

References

[1] A. Grebennikov, *RF and microwave transistor oscillator design*. Wiley, 2007.

[2] A. El-Tager, E. Abdallah, and A. Abdel-Nazir, "Design of microwave oscillators using linear, quasi-linear and nonlinear methods," in *1999 Asia Pacific Microwave Conference*, vol. 2, Nov. 1999, pp. 464–467.

[3] D. Pozar, *Microwave Engineering*, 4th ed. John Wiley & Sons, 2011, pp. 604 – 627.

[4] J. K. Clapp, "An inductance-capacitance oscillator of unusual frequency stability," *Proc. IRE*, vol. 36, no. 3, pp. 356–358, Mar. 1948.

[5] A. Cote, "Matrix analysis of oscillators and transistor applications," *IRE Trans. Circuit Theory*, vol. 5, no. 3, pp. 181–188, Sep. 1958.

[6] A. Hajimiri and T. Lee, "Design issues in CMOS differential LC oscillators," *IEEE J. Solid-State Circuits*, vol. 34, no. 5, pp. 717–724, 1999.