

# On the Complexity Reduction of Polyphase Linear Phase FIR Filters with Symmetric Coefficient Implementation

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**Abstract**—The multiple constant multiplication (MCM) implementation of polyphase decomposed finite impulse response (FIR) filters can be formulated as (1) one MCM block with all coefficients, (2) one MCM block for each subfilter, or (3) as a matrix MCM block. The coefficient symmetry of linear phase FIR filters, however, is not exploited in the later two formulations. In this paper, we investigate the MCM implementation of polyphase structures with coefficient symmetry, where each subfilter MCM block is synthesized separately. We show that by using the proposed implementation it is possible to reduce the number of adders compared with currently existing methods without increasing the storage elements. The proposed techniques can also be extended to the matrix MCM block implementation.

## I. INTRODUCTION

Polyphase structures have wide applications in multirate systems such as decimators, interpolators and filter banks [1]. The advantage of a polyphase structure is the efficiently reduced arithmetic complexity and data storage. Several researches [2]–[4] have investigated the multiplierless implementation of the polyphase structure to further reduce the computational complexity. In these techniques, the filter coefficient multiplications are collectively considered as a multiple constant multiplication (MCM) problem, where the multiplication of one data with several constant coefficients is realized by using shared shifter and addition blocks.

The storage efficient polyphase implementations for decimators (in transposed direct form) and interpolators (in direct form), denoted as Type I and Type II structures [5], are shown in Fig. 1(a) and Fig. 1(b), respectively. In these figures a 22nd order filter is decomposed to its 5 polyphase representation. Either structure of Type I or Type II has its own transposed counterpart implementation, denoted as Type III and Type IV structures, respectively. Although in Type III and Type IV structures, a single MCM block could be shared by all polyphase subfilters [6], it requires a factor  $M$  more storage elements than that required by Type I and Type II structures in an  $M$  polyphase implementation. In [2], [3], the MCM implementations of the above 4 structures are investigated. It is shown that although Type III and IV structures use less adders [2], [3], the silicon areas are larger than that of Types I and II due to the large number of storage elements. In the implementation of Type I and II structures in [2], [3], the

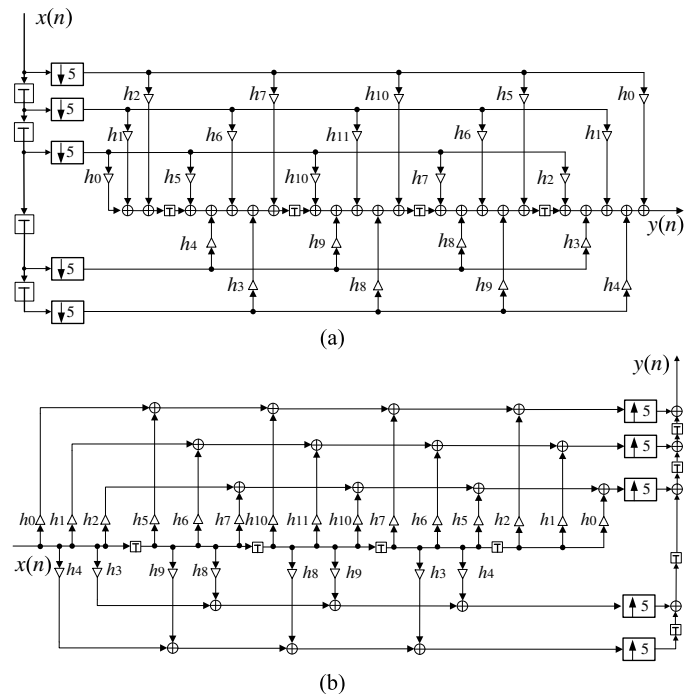


Fig. 1. Polyphase structure for (a) Type I polyphase for decimator and (b) Type II polyphase for interpolator. T is a unit delay.

problem is formulated as one MCM block for each subfilter, or as a matrix MCM block for all subfilters. However, the coefficient symmetry of the linear phase FIR filters is not exploited in either approach.

Recent studies [7]–[9] show that the coefficient symmetry can be restored in the polyphase implementation of Type I and Type II structures without any increase in storage elements. In this paper, the MCM implementation of the polyphase structure for Type I and Type II are investigated, when the coefficient symmetry is exploited. In the proposed technique, each subfilter uses one separate MCM block. The number of adders used for the synthesis of polyphase subfilters, as well as the adder depth in the implementation of decimators, of the proposed implementation is compared with the conventional polyphase implementations, where the coefficient symmetry

is not utilized. Results show that the number of adders used in synthesizing the proposed polyphase filter is significantly reduced compared with the conventional implementations [2], [3] of Type I and Type II structures with a slight increase in adder depth. The proposed technique could also be extended to formulate the problem using a matrix MCM block.

## II. POLYPHASE STRUCTURES WITH RESTORED COEFFICIENT SYMMETRY

Assume that a symmetric FIR filter of length  $N$  has the  $z$ -transform given by

$$H(z) = \sum_{n=0}^{N-1} h(n)z^{-n}, \quad (1)$$

where  $h(n) = h(N-1-n)$  for  $n = 0, 1, 2, \dots, N-1$ . When  $H(z)$  is decomposed into  $M$  polyphase subfilters,  $P$  subfilters have filter length  $\lceil N/M \rceil$  and the other  $M-P$  subfilters have filter length  $\lfloor N/M \rfloor$ , where  $P = N - \lfloor N/M \rfloor M$ .  $\lceil x \rceil$  denotes the smallest integer greater than or equal to  $x$  and  $\lfloor x \rfloor$  denotes the largest integer less than or equal to  $x$ .  $H(z)$  can thus be written as

$$H(z) = \sum_{m=0}^{P-1} z^{-m} H_m^L(z^M) + \sum_{m=P}^{M-1} z^{-m} H_m^S(z^M), \quad (2)$$

where  $H_m^L(z)$  and  $H_m^S(z)$  are the  $z$ -transforms of the subfilters with lengths of  $\lceil N/M \rceil$  and  $\lfloor N/M \rfloor$ , respectively, and are given by

$$\begin{aligned} H_m^L(z) &= \sum_{n=0}^{\lceil N/M \rceil} h(m+nM)z^{-n} \\ H_m^S(z) &= \sum_{n=0}^{\lfloor N/M \rfloor} h(m+nM)z^{-n}. \end{aligned} \quad (3)$$

It has been shown in [7]–[9] that the subfilters of  $H_m^L(z)$  for  $m = 0, 1, \dots, \lfloor P/2 \rfloor - 1, \lfloor (P+1)/2 \rfloor, \dots, P-1$  and  $H_m^S(z)$  for  $m = P, P+1, \dots, P + \lfloor (M-P)/2 \rfloor - 1, P + \lfloor (M-P+1)/2 \rfloor, \dots, M-1$  are asymmetric. This implies that the subfilter  $H_m^L(z)$  has symmetric coefficients only when  $P$  is odd and  $m = \lfloor P/2 \rfloor$ , whereas subfilter  $H_m^S(z)$  has symmetric coefficients only when  $M-P$  is odd and  $m = P + \lfloor (M-P)/2 \rfloor$ . However, it is further shown in [8], [9] that the subfilter pairs of  $H_m^L(z)$  and  $H_{P-1-m}^L(z)$  have mirror image coefficients for  $m = 0, 1, \dots, \lfloor P/2 \rfloor - 1$ . Similarly,  $H_m^S(z)$  and  $H_{M-1-m}^S(z)$  have mirror image coefficients for  $m = P, P+1, \dots, P + \lfloor (M-P)/2 \rfloor - 1$ . Thus, if transforms

$$h'_m(n) = [h_m(n) + h_{P-1-m}(n)]/2 \quad (4)$$

$$h'_{P-1-m}(n) = [h_m(n) - h_{P-1-m}(n)]/2 \quad (5)$$

for  $m = 0, 1, \dots, \lfloor P/2 \rfloor - 1, n = 0, 1, \dots, \lceil N/M \rceil$ , and

$$h'_m(n) = [h_m(n) + h_{M-1-m}(n)]/2 \quad (6)$$

$$h'_{M-1-m}(n) = [h_m(n) - h_{M-1-m}(n)]/2 \quad (7)$$

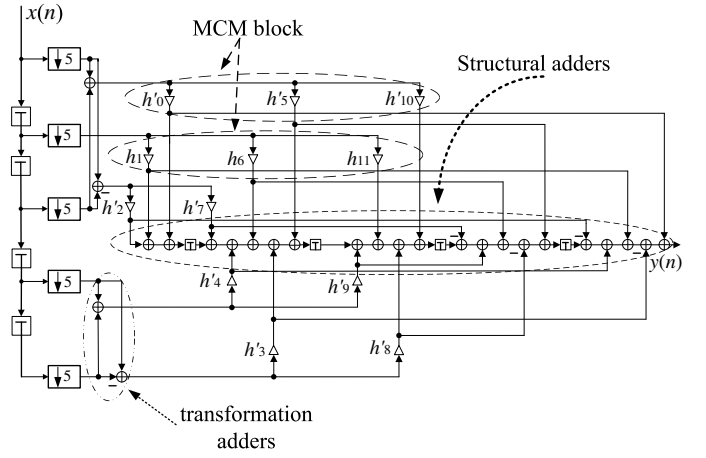


Fig. 2. The implementation of Type I polyphase by exploiting the coefficient symmetry. T is a unit delay.

for  $m = P, P+1, \dots, P + \lfloor (M-P)/2 \rfloor - 1, n = 0, 1, \dots, \lfloor N/M \rfloor$  are employed,  $H(z)$  in (2) can be rewritten as

$$\begin{aligned} H(z) &= \sum_{m=0}^{\lfloor P/2 \rfloor - 1} z^{-m} \left[ (1 + z^{-(P-1-2m)}) H_m^{\prime L}(z^M) \right. \\ &\quad \left. + (1 - z^{-(P-1-2m)}) H_{P-1-m}^{\prime L}(z^M) \right] \\ &+ \sum_{m=P}^{P + \lfloor (M-P)/2 \rfloor - 1} z^{-m} \left[ (1 + z^{-(M-1-2m)}) H_m^{\prime S}(z^M) \right. \\ &\quad \left. + (1 - z^{-(M-1-2m)}) H_{M-1-m}^{\prime S}(z^M) \right] \\ &+ \text{symmetric } H_m^L(z) \text{ and } H_m^S(z) \text{ terms,} \end{aligned} \quad (8)$$

where

$$H_m^{\prime L}(z) = \sum_{n=0}^{\lceil N/M \rceil} h'_m(n) z^{-n} \quad (9)$$

for  $m = 0, 1, \dots, P-1$  and

$$H_m^{\prime S}(z) = \sum_{n=0}^{\lfloor N/M \rfloor} h'_m(n) z^{-n} \quad (10)$$

for  $m = P, P+1, \dots, M-1$ , and the symmetric  $H_m^L(z)$  and  $H_m^S(z)$  terms are given by

$$\begin{cases} 0 & \text{if both } P \text{ and } M-P \text{ are even;} \\ z^{-\lfloor P/2 \rfloor} H_{\lfloor P/2 \rfloor}^L(z^M) & \text{if } P \text{ is odd and } M-P \text{ is even;} \\ z^{-(P + \lfloor (M-P)/2 \rfloor)} H_{P + \lfloor (M-P)/2 \rfloor}^S(z^M) & \text{if } P \text{ is even and } M-P \text{ is odd;} \\ (11) + (12) & \text{if both } P \text{ and } M-P \text{ are odd.} \end{cases} \quad (11)$$

In (9) and (10),  $H_m^{\prime L}(z)$  and  $H_m^{\prime S}(z)$  have either symmetric or anti-symmetric coefficients due to the transformations (4)–(7).

Based on the above transformations, the polyphase structure of the decimator in Fig. 1(a) can be realized by exploiting the

coefficient symmetry as shown in Fig. 2. In this example, the number of longer subfilters, i.e.,  $P$  is equal to 3 while the number of shorter subfilters  $M - P$  is equal to 2. Since  $P$  is odd whereas  $M - P$  is even,  $H_1^L(z)$  has symmetric coefficients while all the other subfilters have asymmetric coefficients. By applying the transformations (4)–(7), each subfilter has either symmetric or anti-symmetric coefficients and thus can be implemented as shown in Fig. 2. In the implementation of the decimator in Fig. 2, each subfilter employs an MCM block to synthesize the subfilter coefficients, as indicated. The adders along the delay chain are called structural adders in the transposed direct form filters, as shown in Fig. 2. In addition to the MCM block adders and structural adders, it is also noted from Fig. 2 that extra adders are required next to the down samplers to add or subtract the signals, corresponding to the transformations (4)–(7). The number of the extra adders, referred to as transformation adders as indicated in Fig. 2, is given by  $2(\lfloor P/2 \rfloor + \lfloor (M - P)/2 \rfloor)$ .

The Type II interpolator implementation that utilizes the coefficient symmetry is a transposed structure of that in Fig. 2. The total number of adders for Type I decimator and Type II interpolator are the same when a filter is decomposed to  $M$  polyphase subfilters. However, in the structure of Type II interpolator, the adders are not differentiated as the MCM block adders and structural adders.

### III. COMPLEXITIES

As is discussed in Section II, for Type I decimator and Type II interpolator the total number of adders for the proposed structures are the same. Therefore, in this section only the Type I decimator structure in Fig. 2 is used to compare with the conventional Type I implementation in Fig. 1(a). Since the structural adders for both conventional and proposed Type I implementations are the same, the total number of adders except the structural adders are compared. Hence, in the conventional Type I implementation, the number of MCM block adders is defined as the number of adders used to implement the multipliers, whereas in the proposed Type I implementation, the number of MCM block adders is defined as the number of adders used to implement the multipliers plus the transformation adders. In addition, the adder depths of the two implementations are also compared.

Since there have been many MCM algorithms concerning the reduction of adders in MCM block [10]–[15], the Rag- $n$  algorithm in [13] is used in this paper to synthesize the MCM block of each subfilter.

#### A. Fixed Filter Length with Varying $M$

In this example, a linear phase FIR filter with the length of 150 is synthesized using *remez.m* in Matlab and rounded arbitrarily to 13 bit precision, excluding the sign bit. The filter is then decomposed into  $M$  subfilters with  $M$  varying from 2 to 16. The MCM block of each subfilter is synthesized using Rag- $n$  for the conventional and the proposed Type I decimator implementations. The curves in Fig. 3 show the adder count in the synthesis of all the MCM blocks of the polyphase filters.

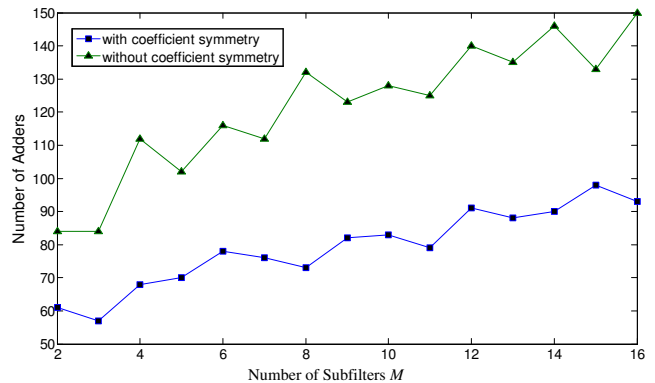


Fig. 3. Total number of adders used to realize the MCM blocks with varying  $M$  and fixed filter length

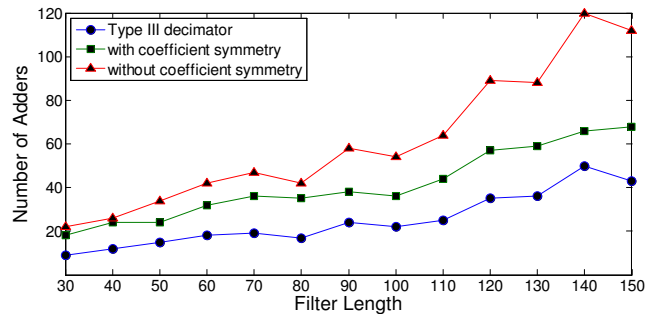


Fig. 4. Total number of adders used to realize the MCM blocks with varying filter lengths and fixed  $M = 4$ .

In addition, for the lower curve, i.e., the proposed Type I decimator in Fig. 2, the number of the transformation adders is also included. It is observed that as  $M$  increases, the number of adders for both of the implementations increases. This is because the number of coefficients within each subfilter decreases as  $M$  increases, resulting in less sharing of the partial products among the coefficients in each MCM block. Also, for the proposed implementation, the number of transformation adders increases with  $M$ . However, the total number of adders used to implement the MCM blocks for the proposed Type I implementation is approximately one third less than that of the conventional Type I implementation. Clearly, in terms of number of adders, the proposed implementation brings significant reduction.

#### B. Fixed $M$ with Varying Filter Length

In this example, the MCM blocks of the polyphase decomposition of 13 linear phase FIR filters with different lengths ranging from 30 to 150 are synthesized in the same way as that of the examples in Section III-A. Each filter is decomposed into 4 subfilters. The number of adders used to synthesize the MCM blocks for both implementations is shown in Fig. 4. In addition, the number of adders used to synthesize an entire MCM block for Type III decimator is also shown for reference. It can be seen from Fig. 4 that as the filter length increases, the reduction of adders by exploiting the coefficient symmetry becomes more significant. This can be explained

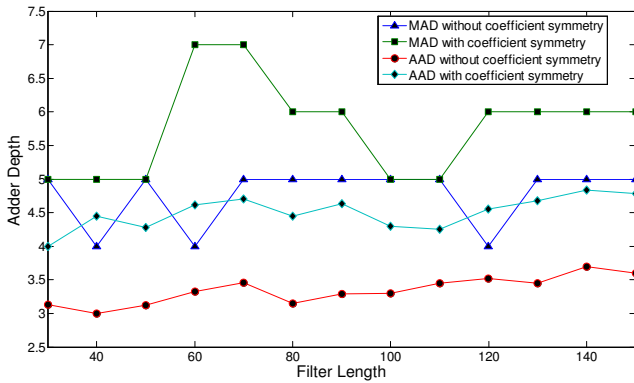


Fig. 5. MAD and AAD of conventional and proposed Type I decimators with varying filter length and fixed  $M = 4$ .

by assuming that applying the MCM algorithms results in the same reduction factor for both implementations, i.e., the number of adders used to synthesize the conventional and proposed structure is  $N/C$  and  $0.5N/C$ , respectively, where  $N$  is the filter length and  $C$  is the reduction factor. Obviously, the difference between the MCM blocks of the two structures, in terms of the number of adders, is proportional to  $N$ .

It can also be seen that synthesizing an entire MCM block of Type III implementation always results in less adders than synthesizing the MCM blocks of each subfilter separately in Type I implementation. However, as stated in introduction, Type III implementation requires a large number of storage elements and therefore uses more silicon area even than the Type I implementation without utilizing the coefficient symmetry. Furthermore, it has been shown in [3], [16] that the Type I implementation can be formulated as a matrix MCM algorithm, and the required number of adders is much fewer than that of the implementation where each subfilter uses a MCM block separately. Future research could investigate the matrix MCM formulation in Type I polyphase implementation, while the filter coefficient symmetry is exploited.

### C. Adder Depth

Fig. 5 shows the comparison of the maximum adder depth (MAD) and average adder depth (AAD) of the conventional and proposed Type I decimator implementations with  $M = 4$ . The adder depth is defined as the number of adders that a signal goes through before reaching a delay element. The corresponding structural adders are organized to minimize the adder depth. While MAD decides the throughput of the circuit, the power consumption is very much related to AAD. It can be seen that by exploiting coefficient symmetry, the MAD and AAD of the proposed structure is generally longer than the original polyphase structure because it is shown in Fig. 2 that the actual input signals of the subfilters are the sum and difference of the down sampled signals instead of the original down sampled ones. As reported in [17], larger AAD usually results in higher power dissipation and lower throughput of the circuit. However, since the proposed structure can achieve more than one third MCM block adders reduction, the actual

power consumption of the circuit due to the overall effect of increased adder depth and reduced adders is not clear in the system level. Future work will be conducted in a lower design level to investigate the power consumption of different Type I implementation structures.

## IV. CONCLUSION

In this paper, we investigate the MCM implementation of Type I and Type II polyphase structures, in which the coefficient symmetry is exploited. The mirror image nature of the subfilters are utilized to construct pairs of new subfilters with symmetric and anti-symmetric coefficients. The coefficient symmetry within each new subfilter results in more than one third reduction in MCM block adders compared with the conventional polyphase implementations. In spite of the slight increase in the adder depth, the reduction in the number of adders is significant.

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