Improved Performance Analysis for Superimposed Pilot Based Short Channel Estimator

Toni Levanen, Jukka Talvitie and Markku Renfors
Department of Communications Engineering
Tampere University of Technology
P.O.Box 553, FIN-33101, Finland
Email: {firstname.lastname}@tut.fi

***Abstract*—In this paper we study the MSE performance of a short ML channel estimator in a discontinuous block fading channel using superimposed pilots. The earlier analytical MSE estimates that we have seen were not concerned with ideal feedback or discontinuous block-wise transmission. In addition, we are interested in the scenario where we use shorter channel estimator than the true channel length. In this paper, we present solutions for these modeling problems and obtain improved analytic MSE estimates.**
***Keywords*: analytical MSE limits; superimposed pilots**

I. Introduction

Currently, we live in the era of wireless digital communications and constantly explore for higher throughput in this challenging environment. Even though the physical layer throughput performance has increased rapidly in the past years, there often remains a significant overhead due to signalling (for system level communication) and training information (for channel estimation). In our study, we have concentrated on reducing the overhead required by the traditional training information, referred to as pilot symbols. Traditionally the pilot symbols are placed on specified slots in time or/and in frequency domain [1]. Another way to add training information to the transmitted signal is to directly add the pilot symbols on top of the information symbols, in time or frequency domain. For this reason, these pilots are often referred to as superimposed (SI) pilots [2]. By using SI pilots, we can improve the spectral efficiency by allowing the user information to occupy the whole spectral region designed for communications. The downside is that the user information interferes greatly with the pilot sequence and that the user data symbol to interference power ratio is decreased.

To overcome this problem of self interference, in [3] a cyclic pilot sequence structure was discussed. The main idea behind the cyclic pilot structure is to allow the utilisation of cyclic mean to improve the pilot to interference power ratio (PIPR) in the estimation process. Furthermore, in the same article optimal channel independent (OCI) training sequences were derived. We have also adopted the usage of OCI training sequences in our model because of their good properties.

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We have extended the model provided in [3] to our single carrier (SC) system model with filter bank (FB) based receiver structure, presented in [4]. The channel estimates are obtained in the time domain after which the sub-channel wise equalisation is performed in the frequency domain. We are using FB based receiver structure because it provides close to ideal linear equaliser performance, it has a good spectral containment properties and it is considered as a strong candidate for future wide area network communications.

In our system model the channel estimator length is smaller than the true channel length and this causes so called aliasing error in the cyclic mean calculation. The usage of short channel estimate is considered because when using cyclic pilot sequence we want to maximise the number of cycles and this leads us to compromise between cycle (estimator) length and estimation error. In addition, we can obtain complexity savings by intentionally using shorter channel estimator, if we can allow limited error floor increase in the channel estimator mean squared error (MSE) performance. In addition, we incorporate an ideal feedback for interference cancellation caused by the user data symbols and derive the estimator MSE also for this case.

This paper is organised as follows: in Section II the system model is introduced. In section III, the main concepts of the ML channel estimation are reviewed. Next, the main contributions of [3] and [5], that are utilised in this paper are reviewed in Section IV. Then we improve the MSE estimates for discontinuous block fading channel with short channel estimator in Section V and these results are tested with simulations in Section VI. Finally, in Section VII, conclusions and future topics are provided.

II. System Model

The system design originates from the uplink assumption. Thus, the complexity of the transmitting end is kept as small as possible, and most of the complexity is placed on the receiving end. We consider SC transmission because it has the benefits of lower peak to average power ratio (PAPR) and less strict frequency synchronization requirements, compared to multicarrier systems. The very simple block level design of the transmitter is given in Fig. 1. The transmitter contains only symbol mapper, pilot insertion and the transmitter pulse shape filter.

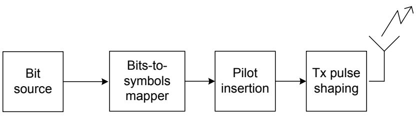

Fig. 1. Transmitter model.

The used channel model is ITU-R Vehicular A channel with about 2.5 μs delay spread and approximately 20 MHz bandwidth [6]. The delay spread has maximum delay of 39 symbols or 78 samples in the receiver, where 2 times oversampling is utilised in the frontend.

We assume perfect synchronization in frequency and time domain, ideal down conversion and 2 times oversampling of the received signal in the Rx block, as shown in Fig. 2. Based on these normal ideality assumptions, we can present the channel between transmitter and receiver as a 2 times oversampled discrete time equivalent channel as $h(k) = |h_{Tx}(t) \otimes h_{channel}(t) \otimes h_{Rx}(t)|_{t=kT/2}$, where $\otimes$ defines a continuous-time convolution. Thus, the received symbol $z(k)$ can be given as

$$z(k) = \sum_{m=0}^{M-1} h(m)s(k-m) + w(k), \tag{1}$$

where M is the channel length in samples, k is the time index for 2 times oversampled symbol sequence and $s(k)$ is the transmitted symbol which is zero if $k < 0$ or $k > 2L-1$, where L is the block length in symbols. Because of the oversampling $s(k) = d(k) = p_c(k) = 0$, when k modulus $2 = 1$. The noise term $w(k) = |h_{Rx}(t) \otimes v(t)|_{t=kT/2}$, where $v(t)$ is complex additive white Gaussian noise (AWGN), is simply modelled as AWGN without considering the correlation caused by the receiver pulse shape filtering with oversampling. We will see in section V that this simplification has a minor effect on the channel estimation MSE.

When we are using SI pilots, the transmitted symbols are normalised combination of user data symbols and pilot symbols, defined as

$$s(k) = \sqrt{1-\gamma}d(k) + \sqrt{\gamma}p_c(k), \tag{2}$$

where $d(k)$ represents a data symbol, $p_c(k)$ represents a symbol from the cyclic OCI pilot sequence and γ is power normalisation factor. The power normalisation factor, γ, is used to normalise the overall transmitted symbol power to unity. This way the average transmitted power is not increased because of the SI pilots. We assume that in the transmitter the user data and pilot signal have unity power, $\sigma_d^2 = \sigma_p^2 = 1$.

From the receiver frontend, the oversampled signal is provided for channel estimator and for analysis FB. After obtaining channel estimates, sub-channel wise equalisation is performed in the frequency domain. It should be noted that the equalisation is now performed within mildly frequency selective subbands. More details on the equaliser structure can be found from [4], [7] and references there in.

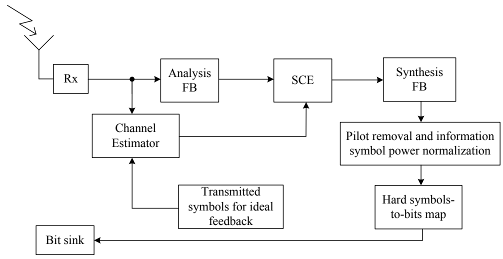

Fig. 2. Receiver model with IF.

After sub-carrier wise equalization (SCE), the subsignals are recombined in the synthesis FB, which also efficiently realises the 2 times sampling rate down conversion. After the synthesis FB, the pilot structure is removed from the received symbol sequence, and with SI pilots the symbol sequence is power normalised as

$$\hat{d}(k) = \frac{\hat{s}(k) - \sqrt{\gamma}p_c(k)}{\sqrt{1-\gamma}}. \tag{3}$$

After this the hard estimates are provided to the bit sink for error rate calculations and the transmitted symbols are provided for the ideal feedback (IF) loop in the receiver.

The IF loop in our receiver model implies that the correct channel response and the transmitted symbols are provided for interference cancellation (IC) operation before the cyclic mean calculation. In other words, we completely remove the interference caused by transmitted data symbols from the received data and use the distorted pilot symbol sequence for channel estimation. A more detailed block diagram of the channel estimator is provided in Fig. 3.

The IF loop provides us a lower bound on the MSE performance when using our channel estimator with hard symbol feedback. In this paper we do not consider iterative processing, but based on our practical experience we can achieve performance close to this lower bound with turbo coded system and 3 feedback iterations when using QPSK or 16-QAM modulation. For 64-QAM the performance is worse and clearly a more sophisticated system is required.

III. Channel Estimation

The channel estimation procedure is similar to the one presented in [3] and [5]. We find the maximum likelihood (ML) estimator for $\hat{\mathbf{h}}$, which minimises $\|\mathbf{z} - \mathbf{P_c}\hat{\mathbf{h}}\|^2$, where $\mathbf{P_c}$ is a matrix containing symbols from the cyclic pilot symbol vector $\mathbf{p}_c = [p_c(0)\ p_c(1) \ldots p_c(L-2)\ p_c(L-1)]^T$ and $\mathbf{z} = [z(0)z(1) \ldots z(L-1)]$ is the vector of received symbols.

We assume that the frame length, L, is an integer multiple of the length of one cycle, N_p, in the cyclic pilot structure $\mathbf{p}_c$. Now $L = N_c N_p$, where N_c is the number of cycles in the whole pilot vector. Thus, the cyclic pilot vector $\mathbf{p_c}$ is made of N_c copies of pilot vector $\mathbf{p}$, given as $\mathbf{p_c} = \tilde{\mathbf{I}}^T\mathbf{p}$, where $\tilde{\mathbf{I}} = [\mathbf{I}_{N_p}\ \mathbf{I}_{N_p} \ldots \mathbf{I}_{N_p}]$ is an $1 \times N_c$ block matrix and $\mathbf{I}_{N_p}$ is

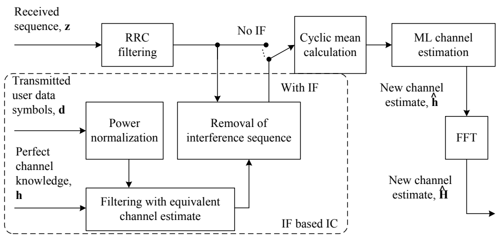

Fig. 3. SI channel estimator with interference cancellation using ideal feedback.

an $N_p \times N_p$ identity matrix. In addition, we define a matrix $\mathbf{P}$, that is a $N_p \times N_p$ cyclic matrix built from $\mathbf{p}$ as

$$\mathbf{P} = \begin{bmatrix} p(0) & p(N_p-1) & \dots & p(1) \\ p(1) & p(0) & \dots & p(2) \\ \vdots & \vdots & \ddots & \vdots \\ p(N_p-1) & p(N_p-2) & \dots & p(0). \end{bmatrix} \quad (4)$$

Now, we assume a continuous block wise transmission, and therefore $\mathbf{P_c} = \tilde{\mathbf{I}}^T\mathbf{P}$, which is an $N_c \times 1$ block matrix. If we also assume that the channel length M is equal to the cycle length N_p, we get [3], [5]

$$\hat{\mathbf{h}}_{without\ IF} = \frac{1}{Nc}\mathbf{P}^{-1}\tilde{\mathbf{I}}^T\mathbf{z} = \mathbf{P}^{-1}\hat{\mathbf{m}}_z, \quad (5)$$

where $\hat{\mathbf{m}}_z = [\hat{\mathbf{m}}_z(0)\ \hat{\mathbf{m}}_z(1)\dots\hat{\mathbf{m}}_z(N_p-1)]$, models the vector of cyclic means of the received samples, defined as

$$\hat{\mathbf{m}}_z(\iota) = \frac{1}{N_c}\sum_{\kappa=0}^{N_c-1} z(\iota + \kappa N_p). \quad (6)$$

After obtaining the channel estimate, a $4S$-point DFT of the channel estimate $\hat{\mathbf{H}} = DFT\{\hat{\mathbf{h}}\}$ is provided to the filter bank based channel equaliser. Here, S is the number of subbands in the synthesis bank, and in our simulations is set to be $S = 128$. A 3-tap complex FIR filter is used for SCE as in [7]. The equalisation structure following the channel estimator is not critical in a sense, because we are interested in the channel estimator MSE performance and not in frame or symbol error performance metrics.

IV. Theoretical MSE Limits with OCI Training Sequences

Following similar procedure as in [3] and [5], assuming OCI training sequence and by taking into account the filtered noise, the MSE of the channel estimator, $\sigma_e^2 = E\|\hat{\mathbf{h}}_{no\ feedback} - \mathbf{h}\|^2$, can be given as

$$\sigma_e^2 = \frac{(1-\gamma)\sigma_d^2 \sum_{i=0}^{N_p-1}\sigma_{h(i)}^2 + \sigma_v^2 \sum_{l=0}^{N_{Rx}-1}\sigma_{h_{Rx}(l)}^2}{N_c\gamma\sigma_p^2}, \quad (7)$$

where $\sigma_{h(i)}^2$ is the power of the ith equivalent channel tap, $\sigma_{h_{Rx}(l)}^2$ is the power of the lth receiver RRC filter tap and N_{Rx} is the length of the receiver RRC filter. Remember that here we have assumed that the channel length is equal to the length of one cycle in the cyclic pilot sequence.

Let us next concentrate on the IF case. If we consider the situation in the channel estimator with IF (see Fig. 3), the sampled sequence used in the channel estimation after IC is

$$\tilde{\mathbf{z}}_{IF} = \mathbf{Dh} + \mathbf{P_c h} + \mathbf{w} - \mathbf{Dh} = \mathbf{P_c h} + \mathbf{w}, \quad (8)$$

where $\mathbf{D}$ is a matrix of the transmitted symbols $\mathbf{d}$, and $\mathbf{w}$ is representing the filtered noise in the receiver. In the IF case, the interference caused by the user data is completely removed from the sequence. It follows that the channel estimate is then

$$\hat{\mathbf{h}}_{with\ IF} = \frac{1}{Nc}\mathbf{P}^{-1}\tilde{\mathbf{I}}\tilde{\mathbf{z}}_{IF} = \mathbf{h} + \mathbf{P}^{-1}\hat{\mathbf{m}}_w. \quad (9)$$

In this ideal case, assuming OCI pilot sequences, the MSE can be given as

$$\sigma_e^2 = E\|\hat{\mathbf{h}}_{ideal\ feedback} - \mathbf{h}\|^2 = \frac{\sigma_v^2}{N_c\gamma\sigma_p^2}\sum_{l=0}^{N_{Rx}-1}\sigma_{h_{Rx}(l)}^2, \quad (10)$$

where $E(\cdot)$ refers to a statistical expectation operator. We notice that this is the same error limit that was obtained for the data dependent superimposed training (DDST) in [5]. It is intuitive because both methods remove the interference caused by the user symbols, but in DDST the additional complexity is in the transmitting end. Also, the IF provides lower bound for the MSE with our channel estimator model for any hard feedback structure.

These error estimates are valid when the channel and the estimator have equal lengths, $N_p = M$, and when we have continuous transmission. The modeling problems arise if we consider a shorter channel estimator length than the true equivalent channel length, or when we consider discontinuous block wise transmission. In the following sections we discuss the modeling of these modifications in detail and provide analytic and simulated MSE results for comparison.

V. Improved Theoretical MSE Limits with OCI Training Sequences for Discontinuous Blockwise Transmission and Short Channel Estimator

Let us start with some comments on the modeling and effects of the oversampling in the receiver. Because we use two times oversampling in the receiver, we have $2L$ samples for each frame. In addition, the estimated portion of the channel has length $2N_p \leq M$. Also, the average power per sample has to be scaled by the oversampling factor, leading us to $\sigma_{d,o}^2 = \sigma_{p,o}^2 = 1/over = 0.5$ in the following equations, where $over = 2$ is the oversampling factor. We did not consider oversampling in Section III, where the earlier results were restated. To obtain oversampled versions of equations (7) and (10) we just have to scale the power terms with the oversampling factor and multiply the number of samples used in the summations with the oversampling factor.

In our considerations the channel estimator length is smaller than the true channel length and this causes error in the cyclic mean calculation. We provide an intuitive model for this error

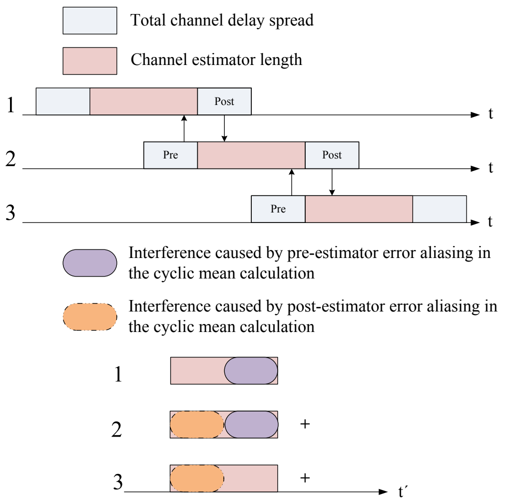

Fig. 4. Illustration of the error aliasing phenomenon when using short channel estimator. Here three copies of the cyclic pilot sequence are drawn, which have shorter length than the channel delay spread. The lower part of the figure models the summation in the cyclic mean calculation.

referred as the estimation error aliasing. This model is not accurate model of the error phenomenon, but it provides simple estimation means and relatively good estimation performance, as will be seen in the end of Section VI in Figures 6 and 7.

The basic concept of error aliasing is shown in Fig. 4. The idea is that we model the error caused by underestimating the channel response length by placing the expected channel amplitude response on top of each pilot cycle and assume that the error can be modelled as aliasing on top of the following or previous cycles. This can be thought as a simplified version of the true aliasing error caused by symbols spread in time because of the dispersive channel. As shown in Fig. 4, the pre-estimator portion of the channel response falls on top of the previous cycle and the post-estimator portion falls on top of the next cycle.

The usage of short channel estimator is considered because when using cyclic pilot sequence we want to maximise the number of cycles and this leads us to compromise between cycle (estimator) length and channel estimation error. In addition, we can obtain complexity savings by intentionally using shorter channel estimator if we can allow limited error floor increase in the channel estimator performance.

Thus, when considering discontinuous block wise transmission and short channel estimator, there are two main reason for MSE estimation inaccuracy, caused by 1) error aliasing because of shorter channel estimator length than the true channel delay spread, and 2) by modified pilot and data symbol matrices obtained in the receiver. Let us now define that the length of the true equivalent channel is $N_{channel}$, pre-estimator part of the true channel is N_{pre}, post-estimator part is N_{post} and the channel estimator length is $N_{estimator}$ where $N_{estimator} \leq N_{channel}$ and $N_{channel} = N_{pre} + N_{estimator} + N_{post}$.

Additionally, we utilise two different MSE error metrics. First is the estimation error of the short channel estimator, which is obtained by comparing the estimate to the estimated portion of the channel, $\sigma^2_{e,short} = E\|\tilde{\mathbf{h}} - \mathbf{h}_{I_{short}}\|^2$, where I_{short} corresponds to the indices related to the estimated part of the channel. This metric indicates how well the desired part of the channel is estimated. Second metric is the overall (more traditional) MSE, $\sigma^2_e = E\|\tilde{\mathbf{h}}_{extended} - \mathbf{h}\|^2$, which defines the total error between the true channel and the estimate. The difference to the first metric, $\sigma^2_{e,short}$, is equal to the sum of expected power of the channel taps outside the channel estimator. This term is defined as $\sigma^2_{e,modeling} = \sum_{i \in I_{pre,post}} E|\tilde{h}_{extended}(i) - h(i)|^2$, where $I_{pre,post}$ includes all indices not estimated by the channel estimator. Here $\hat{\mathbf{h}}_{extended}$ is the channel estimate which is extended with zeros to length $N_{channel}$. This metric can be used for symbol error rate analysis, which is one interesting future topic. Also, it is an indicator for the system designer on the compromise between improved interference cancellation through increased number of shorter cycles and increased modeling error caused by shorter channel estimate.

For ease of derivation and presentation, we assume that the length of pre-estimator part and post-estimator parts are shorter than the channel estimator length, $N_{pre} < N_{estimator}$ and $N_{post} < N_{estimator}$. The pre-estimator error is defined by $\mathbf{h}_{pre}$, where

$$\mathbf{h}_{pre} = \frac{N_c - 1}{N_c}[0 \ 0 \ \ldots \ 0 \ E|h(0)| \ E|h(1)| \ \ldots \ E|h(N_{pre}-1)|]^T. \tag{11}$$

If $N_{estimator} < N_{channel} - N_{pre}$, then we have post-estimator error aliasing, and the post-estimator error is equal to $\mathbf{h}_{post}$, where

$$\begin{aligned} \mathbf{h}_{post} = \ & \frac{N_c - 1}{N_c}[E|h(N_{pre} + N_{estimator} - 1)| \\ & E|h(N_{pre} + N_{estimator})| \ldots \\ & E|h(N_{channel} - 1)| \ 0 \ \ldots \ 0]^T. \end{aligned} \tag{12}$$

The normalisation term $(N_c - 1)/N_c$ is caused by the fact that in block-wise transmission there is no post-estimator error in the first term of the cyclic mean and there is no pre-estimator error in the last term of the cyclic mean calculation. With large number of copies this normalisation term has no significant meaning, and can be left out from the derivation. We can now define the aliasing error term $\mathbf{h}_{aliasing} = \mathbf{h}_{pre} + \mathbf{h}_{post}$, which contains pre- and post-estimator error and has length $2N_p$, which is equal to the channel estimator length in our considerations. Note that $\mathbf{h}_{aliasing}$ is now a deterministic vector.

The second error present in the simulated MSE, is caused by the approximation $\mathbf{P}_c \approx \tilde{\mathbf{I}}\mathbf{P}$. This model does not hold if we assume discontinuous block wise transmission, where there is nothing in the air before transmitting our own information block. In reality, $\mathbf{P}_c$ is made of $N_c - 1$ full copies of $\mathbf{P}$ and one copy of lower triangular matrix version of $\mathbf{P}$, referred as $\mathbf{P}_{LT}$, where everything above the main diagonal is set to zero,

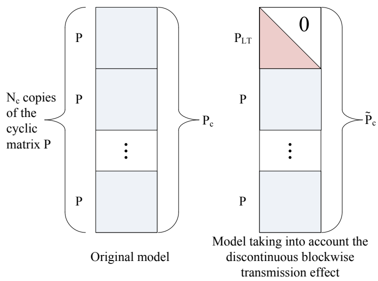

Fig. 5. Illustration of the modeling error caused by discontinuous blockwise transmission.

as shown in Fig. 5. This type of error becomes dominant as the number of cyclic copies is decreased, and is significant if only a few copies are available, e.g. less than 20 cyclic copies.

First, we have to modify the ML estimator to follow more accurately the model of discontinuous blockwise transmission. Therefore, we can easily modify the matrix $\mathbf{P}$ used in the ML estimator to incorporate the effect of the all zero portion in the $\mathbf{P}_{LT}$. Thus, the error between the approximation and model is the upper triangle part of $\mathbf{P}$, defined as

$$\mathbf{P}_{UT}(r,c) = \begin{cases} 0, & \text{if } r \geq c, \\ \mathbf{P}(r,c), & \text{otherwise.} \end{cases} \tag{13}$$

Based on this, we can generate the true ML channel estimator for the presented blockwise transmission scenario, by defining a new matrix $\tilde{\mathbf{P}} = \mathbf{P} - (1/N_c)\mathbf{P}_{UT}$, we obtain

$$\hat{\mathbf{h}} = \tilde{\mathbf{P}}^{-1}\tilde{\mathbf{m}}_z = \tilde{\mathbf{P}}^{-1}\left[\tilde{\mathbf{P}}\mathbf{h} + \mathbf{M_d}\mathbf{h} + \mathbf{m_w} + \tilde{\mathbf{P}}\mathbf{h}_{aliasing}\right]. \tag{14}$$

For this channel equaliser, we can derive the MSE limits with and without the IF in a similar manner as was done in [3] and [5]. While deriving the results, we have assumed that the products between $\mathbf{P}$ and $\mathbf{P}_{UT}$ generate diagonal matrices, which is not accurate but provides us a good approximation of the error weighting in MSE generated by the modified ML estimator structure. We obtain a weighting factor vector $\beta = \text{diag}\{\tilde{\mathbf{P}}^H\tilde{\mathbf{P}}\}$, where $\text{diag}\{\cdot\}$ generates a vector of the diagonal elements of the matrix inside the brackets. In other words, β represents the effect of the missing upper triangle portion of the first $\mathbf{P}$ matrix in $\mathbf{P}_c$ after the cyclic mean, and is defined as

$$\beta(m) = \gamma\sigma_p^2\left[N_p - \lfloor\frac{m}{over}\rfloor(2 - 1/N_c)/N_c\right], \tag{15}$$

where $m = 0, 1, \ldots, 2N_p - 1$ and $over = 2$ is the oversampling factor in the receiver frontend. For comparison, with two times oversampling, continuous transmission and with OCI sequences, $\mathbf{P}^H\mathbf{P} = \gamma\sigma_{p,o}^2 2N_p\mathbf{I}_{2N_p}$. In addition to prementioned weighting caused by missing pilots, the block wise assumption also affects the interference from user data. Clearly, if there is no earlier transmission, there is no interfering data either. This is taken into account in the MSE derivation as additional weighting factor vector, δ, of the user data related error, as

$$\delta(m,i) = \begin{cases} (1-\gamma)\sigma_{d,o}^2/N_c, & \text{if } m - i \geq 0 \\ (1-\gamma)\sigma_{d,o}^2(N_c - 1)/N_c^2, & \text{otherwise,} \end{cases} \tag{16}$$

where $m, i = 0, 1, \ldots, 2N_p - 1$. Finally, for the channel estimator given in (14) we obtain MSE estimates without IF as

$$\sigma_{e,short,without\ IF}^2 = \sum_{m=0}^{2N_p-1}\frac{1}{\beta(m)}\left\{\sum_{i=0}^{2N_p-1}\delta(m,i)\sigma_{h(i)}^2 + \sigma_w^2/N_c\right\} + \|\mathbf{h}_{aliasing}\|^2, \tag{17}$$

$$\sigma_{e,without\ IF}^2 = \sum_{m=0}^{2N_p-1}\frac{1}{\beta(m)}\left\{\sum_{i=0}^{2N_p-1}\delta(m,i)\sigma_{h(i)}^2 + \sigma_w^2/N_c\right\} + \|\mathbf{h}_{aliasing}\|^2 + \sigma_{e,modeling}^2. \tag{18}$$

In similar manner, for the IF iteration we get

$$\sigma_{e,short,with\ IF}^2 = \frac{\sigma_w^2}{N_c}\sum_{m=0}^{2N_p-1}\frac{1}{\beta(m)} + \|\mathbf{h}_{aliasing}\|^2. \tag{19}$$

$$\sigma_{e,with\ IF}^2 = \frac{\sigma_w^2}{N_c}\sum_{m=0}^{2N_p-1}\frac{1}{\beta(m)} + \|\mathbf{h}_{aliasing}\|^2 + \sigma_{e,modeling}^2. \tag{20}$$

Here, $\sigma_w^2 = \sigma_v^2/over\sum_{l=0}^{N_{Rx}-1}\sigma_{h_{Rx}(l)}^2$ models the power of the receiver pulse shape filtered noise with oversampling. The analytical MSE is slightly increased due to the missing pilot symbol information in $\tilde{\mathbf{P}}$, whereas the interference caused by the user data is slightly decreased.

VI. Simulation Results

In the presented simulations, the used constellation is 16-QAM and we used power factor $\gamma = 0.26$ to obtain the presented results. The power factor was chosen based on simulated BER results with and without IF, where the designer has to make compromise between the performance with and without IF. Higher pilot power improves the channel estimation performance but degrades the user data symbol power, which degrades the BER performance. In addition, the performance with IF always decreases if the pilot power is increased.

In Fig. 6, we have plotted the simulation based MSE values, new analytical MSE estimates without and with IF based on (17) and (19), and the old MSE estimates without and with IF given in (7) and (10). There are now $N_c = 120$ copies of a pilot sequence of length $2N_p = 64$ samples per transmitted block and the used block length is 7680 samples. The true equivalent channel length is 142 samples and it is estimated by 64 samples long estimator. Now there are 78 samples outside the estimator, from which the aliasing error can be defined. From these samples, 16 are pre-estimator and 62 are post-estimator. The new MSE estimates follow the simulated behaviour clearly better than the old ones, because of the

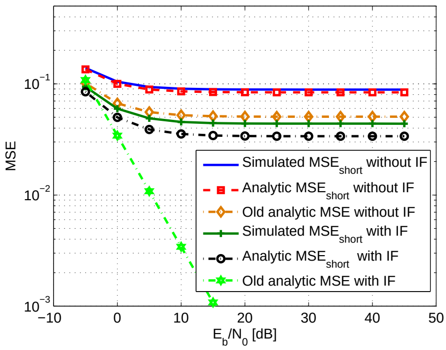

Fig. 6. Comparison between simulated and analytical MSE performance with 16-QAM constellation and 64-tap channel estimator.

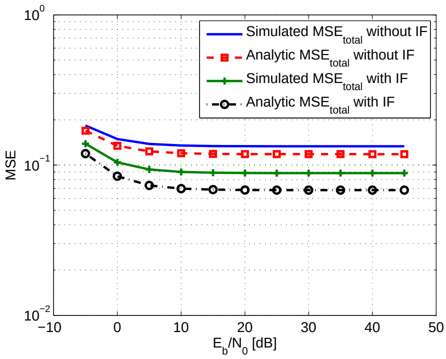

Fig. 7. Comparison between simulated and analytical total MSE performance with 16-QAM constellation and 64-tap channel estimator.

modified problem setup. Even though the error aliasing model is a simplified model, the analytic MSE estimates follow well the simulated ones.

In Fig. 7 we have plotted the simulated total MSE and the analytic MSE estimates based on (18) and (20), without and with IF, respectively. As expected, the total MSE is bigger than the MSE for the estimated portion of the channel. The analytic MSE is following the simulated values, but is slightly optimistic about the error value. Overall, the simulated values follow well the analytical ones up to $N_c = 120$, which corresponds to channel estimator length $2N_p = 64$. With channel estimator lengths shorter than this, the analytical estimates become very optimistic because of the simple error aliasing model.

VII. Conclusion

In this paper we have presented an interference cancelling receiver structure for SC communications with SI pilot based channel estimation and FB based channel equalisation. The interference cancelling structure is designed to shift the realisation complexity to the receiver side, e.g., to the base station. Channel equalisation is performed in frequency domain with FB based SCE, which has close to ideal linear equaliser performance.

We have restated the MSE limits for the ML channel estimator obtained in [3] and [5]. We showed that when we are interested in a discontinuous blockwise transmission in a block fading channel, we have to modify the cyclic matrix $\mathbf{P}$ to incorporate the assumption of the first transmission to the channel. In addition, we obtained a method to improve the MSE estimate in the cases when we are using a channel estimator, which is shorter than the true equivalent channel length. The presented analytical estimates follow well the simulated MSE values, and provide us important performance bounds which can be used, for example, when deriving the analytic symbol error performance of the presented receiver structure.

Furthermore, in future studies we will show that with iterative decision feedback structures using efficient channel coding we can achieve performance very close to the presented ideal feedback performance limits. This is the major motivating force behind this study. In addition, further studies will concentrate on obtaining the analytic symbol error rates for this channel estimation scheme in single antenna and multiantenna scenarios. Interesting throughput comparisons with traditional time domain multiplexed pilot symbols in single-input multiple-output channel are under preparation.

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