ORIGINAL PAPER



Structural Compatibility of Infrastructures Utilizing Alternative Earth Construction Materials

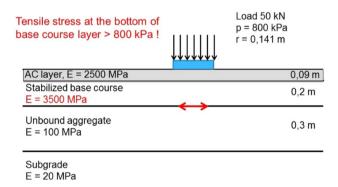
Pauli Kolisoja 100 · Antti Kalliainen 1

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Abstract

This paper presents analysis of the structural behavior of road pavements in which alternative construction materials are replacing the traditional ones in some of the structural layers. The analysis is consider important since from the structural performance point of view many of the alternative materials have mechanical properties far different from those of the traditional road construction materials, especially unbound aggregates, and as a consequence of that, the empirically calibrated design rules applied and adjusted for the normally utilized pavements solutions are not valid any more. The analysis is exemplified by means of four different low volume road pavement structures that are in line with the existing design guidelines in Finland. The mechanical behavior of these structures is analyzed using three different approaches: semi-empirical Oedemark design approach, multi-layer linear elastic analysis and finite element analysis. The obtained calculation results indicate clearly that if a low volume road structure containing a high stiffness layer made e.g. of stabilized fly ash is resting on soft subgrade soil, tensile stresses up to 1 MPa may be developed. Therefore, the performance and respective distress mechanisms of the structure are likely to be very different from those of a traditional solution. As a key conclusion from the analysis, need for a new concept, structural compatibility, was identified. It would help in drawing due attention to the mechanical behavior of alternative materials when they are used in replacing the traditional ones in road structures exposed to repeated heavy traffic loads.

Graphic Abstract



Keywords Road pavement · Alternative material · Mechanical behavior · Structural compatibility

□ Pauli Kolisoja pauli.kolisoja@tuni.fi

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Statement of Novelty

Structural design of road pavements made of traditional construction materials is normally based on design rules that have by time been adjusted to the local ambient conditions and available types of construction materials. However,



Tampere University, P.O. Box 600, 33101 Tampere, Finland

when various types of alternative earth construction materials have increasingly been taken into use, the empirically calibrated design approaches are not valid anymore, because replacing one structural layer in a road or field structure with a material having fundamentally different mechanical properties may change the overall performance of the whole structure. In this paper the importance of this issue is demonstrated with different calculation approaches normally used in mechanistic design of pavement structures and based on the analysis a new concept, structural compatibility, is suggested.

Introduction

Structural solutions that are applied in constructing road pavement structures exposed to repeated traffic loads have typically been developed over a long time span. Empiricism has also played a big role when structural solutions have by time been adjusted to the local ambient conditions and available types of construction materials. Long-term feedback obtained from the actual performance of pavement structures has resulted in operative solutions even though the applied design approaches may have been somewhat vague from a theoretical point of view.

When various types of alternative earth construction materials have increasingly been taken into use in constructing road and field structures, the status quo described above has changed. The mechanical properties, especially the stiffness and strength of these alternative materials, may be quite different from those of the traditional construction materials such as sand, gravel, and crushed rock aggregates. Replacing one of the structural layers in a road or field structure with a material having fundamentally different mechanical properties may therefore change the overall performance of the whole structure. This means, of course, that the empirically calibrated design approaches are not valid as such anymore.

From the mechanistic pavement analysis it is well known that under a wheel load acting on the road surface a large stiffness difference in between two structural layers on top of each other results in the development of tensile stresses at the bottom of the stiffer layer. In the case of an asphalt concrete layer resting on top of an unbound base course layer, this is one of the fundamental distress mechanisms against which the mechanistic design of a road pavement is normally made. Therefore, it is fairly evident that if we replace an unbound road pavement layer either with a very stiff material (e.g. a self-cementing or cement-stabilized layer of fly ash) or a material with very low stiffness (e.g. a layer of tire shreds), tensile stresses tend to develop in places different from the traditional type of road structure. Correspondingly, the critical distress mechanisms that are decisive regarding the service life of the structure will change as well.

Loading effects caused by heavy vehicle wheels moving on top of road structures are severe in many respects:

- Contact pressure between a truck tire and road surface has typically an intensity of about 800 kPa, in the case of old generation single tires even up to 1000 kPa. In comparison to the contact stresses normally allowed, for instance, under the footings of normal house construction, these values may be up to threefold.
- Wheel loads have a moving nature. In the literature, this has already been shown in the early 1990s to have a markedly more damaging effect on the road structure in comparison to a static load or even a cyclic load staying in place. This especially concerns the rutting behavior of unbound layers of road structures [1].
- One more characteristic feature of traffic loads is that they are repetitive. During the lifetime of a heavily trafficked road, even the heaviest wheel loads can be repeated hundreds of thousands or even millions of times over a certain point of road pavement.

Considering that in addition to the traffic loads, road infrastructures are exposed to the varying effects of weather and seasons—rain, heat, freezing, thawing, etc.—and the fact that roads must mainly be built using locally available construction materials, it is evident that the structural design of road pavements has a critical importance on their service life.

Since ancient times, roads have been built very much based on empirical design rules that have later on been supplemented by calculatory elements and experimental road tests. One of the very best-known examples among the later ones is the extensive AASHO (American Association of State Highway Officials) road test carried out in the U.S.A already in the early 1960s [2]. From that time dates back also the so-called "fourth power rule" according to which the damaging effect of a wheel or axle load increases by a power of four when the wheel or axle load increases a certain amount. For instance, if the axle load rises from the typical design value of 100 kN (10 tons) by 20%, the damaging effect of that axle is assumed to more than double (1.2^4 = 2.0736).

Developments made in computer technology since AASHO have enabled new types of design approaches to be introduced, because it has become possible to analyze the prevailing stresses and strains in road pavement exposed to a wheel load. Most typically, these analyses have utilized the so-called multi-layer linear elastic (MLLE) theory, in which each structural layer of road pavement is assumed to have a constant stiffness, both with regard to the compressive and tensile stresses the layer is experiencing. Since especially the unbound granular layers are known to have stress-dependent stiffness, multi-layer



analysis tools enabling the modeling of this important feature have also been introduced e.g.[3].

More advanced possibilities for analyzing the mechanical behavior of road pavement materials and traffic-loaded road pavements have been opened up by the introduction of easy-to-use numerical analysis tools based on Finite Element Method (FEM) e.g. [4]. and Discrete Element Method (DEM) e.g.[5, 6]. Both of these approaches have, of course, their own strengths and weaknesses. With regard to the FEM approach, an important feature is that it treats all the structural layer materials as continuums, because of which the representativeness of the structural model fundamentally depends on the available types of material models. In the DEM approach, this limitation is avoided by modeling the interactions between each individual grain separately, so ideally the model should be able to reproduce the actual behavior of a granular material layer inherently consisting of a large number of individual grains interacting with each other. The very large number of grains contained in a road structure imposes, however, a severe limitation on accomplishing a truly realistic representation of reality with the DEM approach.

One characteristic feature that concerns all the so-called mechanistic or analytical pavement design approaches utilizing either the multi-layer analyses or the more advanced FEM approaches is that even if we are able to determine the mobilizing stresses and strains in a traffic-loaded road pavement, we still need to know "how much is much." In other terms, what are the allowable distresses during one load application that correspond to the combination of repeated traffic loads and environmental conditions during the lifetime of the structure to be designed? This is the reason why empirical knowledge and calibration are required, even with the mechanistic design approaches. Excellent sources for this type of verification data with regard to traditional types of

pavement structures have been the Minnroad test, carried out in the state of Minnesota in the U.S.A since the 1990s, and various types of Accelerated Pavement Test facilities used in a number of countries around the world during the last few decades e.g.[7–9].

Concerning pavement structures containing alternative construction materials, a big challenge is that many of these materials are available only locally and/or in limited quantities in comparison to the traditional types of construction materials: natural and crushed rock aggregates. That is why it is not economically feasible to carry out very extensive and long-lasting experimental loading test campaigns to find out their long-term performance in different types of potential utilization applications. Therefore, even if some empirical testing and verification is inevitably needed, utilization of the available numerical analysis tools is of utmost importance in developing a better understanding of the performance and failure mechanisms of these non-traditional pavement structures and thus to enable sustainable utilization of alternative construction materials in different types of road pavement applications. The aim of this paper is to exemplify the structural analysis of a few pavement structures containing alternative construction materials and to discuss the meaning of the obtained results.

Materials and Methods

Analyzed Pavement Structures

A schematic picture of road pavement structures in which utilization of alternative construction materials could be considered is shown in Fig. 1. The provided examples are in line with recommended values of design parameters recommended in the national design guidelines in Finland [10,

	Structure 1	Structure 2	Structure 3	Structure 4	
Wearing course	Soft AC	Soft AC	Soft AC	Soft AC	50 mm
Base course	Fly ash stabilized BC	Bitumen stabilized BC	Crushed rock	Crushed rock	150 mm
Sub-base course	Sand/Gravel	Sand/Gravel	Fly ash I	Fly ash II	300 mm
Subgrade	Type a / b / c	Type a / b / c	Type a / b / c	Type a / b / c	

Fig. 1 Schematic picture of the road pavement structures to be analyzed

11] and related structural solutions typically used in Low Volume Road (LVR) types of applications not conforming, however, the overall structural layer thickness requirements set for higher quality roads based on the design against frost action.

Because the stiffness of subgrade soil underlying a road pavement has in earlier studies e.g. [12]. been observed to have a marked effect on the stresses and strains that are mobilizing into an infrastructure under the action of traffic load, subgrade stiffness was included as an additional variable into the analysis. Consequently, three different subgrade stiffness values were used, one representing very soft subgrade soil conditions, one subgrade soil with a medium high stiffness value, and one with high stiffness subgrade soil conditions.

Analysis Methods and Tools

Semi-empirical Design Approach Based on Odemark Method

The semi-empirical design approach that is based on the calculation method introduced by Odemark [13] is widely used for the structural design of road and street pavements in Finland. The key idea of this approach is that the overall stiffness of a pavement structure is a measure of its long-term load-carrying capacity (i.e. bearing capacity). Therefore, basically the only critical design parameter is the stiffness of each structural layer material, in addition to which the stiffness value of underlying subgrade soil is of course required.

In the design guidelines for road and streets with different traffic volumes, the respective target values for the overall stiffness of the pavement structure are given [10] After having that, the Odemark method is used to calculate the overall stiffness of the whole pavement structure by summing up the contributions of each structural layer one by one, starting from the bottom of the pavement structure and continuing up to the road surface. In practical terms, this approach means that the higher the target stiffness value is, the stronger the pavement structure, consisting of thicker layers of better quality materials, you need to design.

When it comes to the recommended stiffness values for different types of structural layer materials, empiricism plays a big role in the design approach described above. The recommended values are, in part, based on back-calculations of Plate Loading Test (PLT), but in addition to that it can be stated that the recommended values have by time been calibrated against the observations made from actual performance of real road pavements on a long time span. As far as traditional types of pavement structures and construction materials are utilized in building a road pavement, the empirical calibration makes the design approach operative,

even though the overall stiffness as a true measure of the long-term load-carrying capacity of a pavement structure as such can be highly questioned.

The stiffness values used in connection with the Odemark structural design method for the pavement structures given in Fig. 1 are summarized in Table 1.

Multi-layer Linear Elastic Analysis

Multi-layer analyses carried out in this research were accomplished using the program BISAR-PC originally delivered by Shell [14]. Characteristic features of the software are typical for the most of multi-layer linear elastic (MLLE) analysis tools; they include:

- Wheel loads are applied on top of the pavement structure using a set of circular contact areas, each having a constant contact pressure;
- each structural layer has a constant thickness, horizontal upper and lower boundaries and infinite length in the horizontal direction
- subgrade soil is described as an infinite half-space with a horizontal upper boundary
- all structural layer materials are isotropic, and their stiffness values are constant, independent of the prevailing stress conditions
- all structural layer materials have infinite strength, both with regard to compressive and tensile stresses, i.e. no tension cut-off property is included into the model
- structural layer materials do not have any weight of their own in the analysis.

The stiffness values used in the multi-layer linear analysis were intentionally kept the same as those used in connection with the Odemark design approach (Table 1). In addition to the stiffness values, multi-layer linear elastic analysis carried out using BISAR-PC software requires as an input the value of a Poisson's ratio for each structural layer material. In the

Table 1 Stiffness values (MPa) used in applying the Odemark structural design method

	Structure 1	Structure 2	Structure 3	Structure 4
Wearing course	1400	1400	1400	1400
Base course	3500	700	150	150
Sub-base course	100	100	1500*	600*
Subgrade a / b / c	20 / 50 / 100	20 / 50 / 100	20 / 50 / 100	20 / 50 / 100

^{*}The stiffness value of 600 MPa is recommended in the design guidelines, while 1500 MPa is believed to represent better the true physical behavior of the material in question



current analysis a constant value of $\upsilon = 0.35$ was selected for all of the structural layer materials and $\upsilon = 0.5$ for the subgrade soil.

Finite Element analysis

FEM analyses carried out in this study were accomplished using PLAXIS 3D (version 2017) software. Dimensions of the model were 5 m by 5 m, while the total thickness of structural and subgrade layers was 5.5 m. The element type used for modeling the structural layers of road pavements as well as subgrade soil was a ten-node tetrahedral element. The circular load was given as uniformly distributed pressure. The material models used were as follows: Hardening Soil (HS) for the structural layers of the road consisting of traditional aggregates, the Mohr-Coulomb (MC) model in undrained conditions for the subgrade soil, and the Linear Elastic (LE) model for the asphalt layer, fly ash, or bitumen-stabilized base course and fly ash layers in the sub-base course. Materials modeled as linear elastic had the same material parameters as those used in the MLLE calculations (Table 1). The undrained state of subgrade in MC was defined with undrained shear strength, s_n ($s_n = 20$ kPa for type a subgrade, 35 kPa for type b subgrade, and 70 kPa for type c subgrade). The applied material parameters for HS materials are summarized in Table 2.

Other Aspects of Structural Design

Because the aim of the current study is primarily to analyze the mechanical behavior of pavement structures under wheel load action, other important aspects influencing the actual service life of bound structural layers, such as variation in material quality, shrinkage cracking behavior or resilience against uneven frost heave or consolidation settlement, are not considered here.

Results

Overall Stiffness of Pavement Structures

The overall stiffness of analyzed pavement structures determined using the three parallel calculation methods have been

compared in Fig. 2. In the case of the Odemark calculation approach, the overall stiffness is directly the output of calculation as such, while in connection with MLLE and FEM analyses, stiffness values have been derived based on the intensity of applied surface load and respective surface deflection according to Eq. 1. The main principal difference between the results thus obtained is that the two later ones correspond to an evenly distributed surface load (i.e. flexible loading plate), while in connection with the Odemark approach, the result corresponds obviously more to the PLT type of loading, i.e., rigid loading plate under which load distribution is not uniform:

$$E = \left[\frac{2 \times (1 - v^2) \times \sigma \times r}{d} \right]$$
 (1)

where E is the overall stiffness of structure (MN/m²), ν is Poisson's ratio (-), σ is contact pressure (MN/m²), r is the radius of the loading plate (mm), and d is deflection (mm).

Based on Fig. 2, it is evident that the overall stiffness of a pavement structure in not a unique value, but it clearly depends on the calculation method that is used in determining it. In addition, with this thin pavement structures the overall stiffness determined on the top of a road structure markedly depends also on subgrade stiffness, as Fig. 2 indicates.

In the case of Structure number 3, the Odemark approach seems to give consistently higher, up to 30%, overall stiffness values in comparison to those derived from the results of MLLE and FEM. The main reason for this is assumed to be that Odemark approach has originally been derived for analyzing structures in which the stiffness is increasing layer by layer from the bottom to the top of structure, while MLLE and FEM are more robust in the analysis of any type of layered structures. The same phenomenon can at least partly explain the slight inconsistency of results obtained for Structure number 1, even though in this case the influence of subgrade stiffness seems to be important as well.

Correspondingly, FEM modelling results in slightly lower overall stiffness values compared to the MLLE method. Most likely, this arises from the elasto-plastic material models used in FEM analyses. Layers constructed with traditional aggregate materials do not have any tensional capacity in FEM simulations. This leads to somewhat larger and

Table 2 Material parameters used for unbound aggregate layers modeled with HS material model. The meaning of each parameter is explained in more detail by Brinkreve [4]

Parameter	c'	φ′	Ψ	E ₅₀ ^{ref}	E _{oed} ref	E _{ur} ref	v_{ur}	m	p ^{ref}	K ₀ ^{nc}	f
Unit	kPa	0	0	MPa	MPa	MPa	_	_	kPa	_	-
Base course	10	45	15	150	140	300	0.2	0.5	100	0.320	0.9
Sub-base	3	40	10	100	100	200	0.2	0.5	100	0.361	0.9



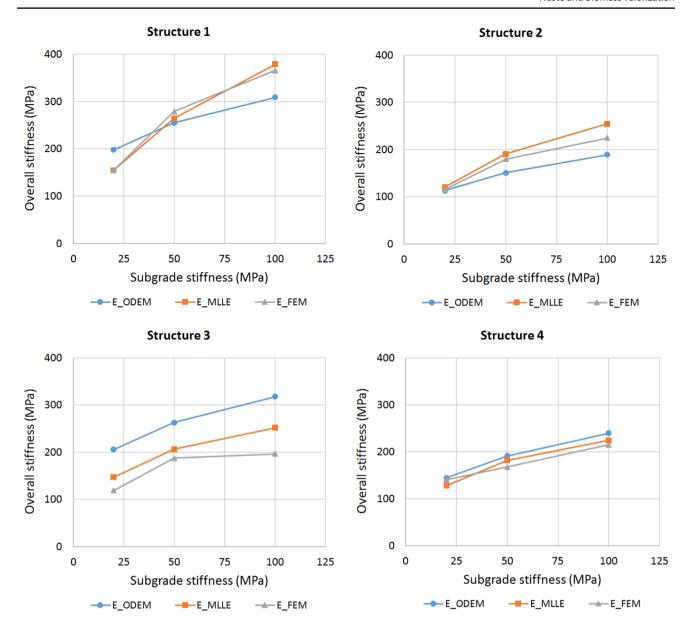


Fig. 2 Comparison of the overall stiffness values determined for Structures 1 to 4

partially permanent deformations, and thus slightly lower overall stiffness values.

Tensile Stresses Inside of Pavement Structures

Because the overall stiffness determined from top of a road structure is only a very robust measure of the mechanical behavior of the whole road pavement, it is worthwhile to investigate the stresses that are mobilized inside the structure as well. Considering the long-term performance of a pavement structure, one of the critical distresses is the intensity of tensile stress and/or strains at the bottom of any stiff structural layer that is resting on the top of a more flexible material layer. In connection with mechanistic pavement

analyses, this type of distress is normally considered as the critical one regarding the service life of an asphalt concrete layer resting on top of an unbound base course layer.

Figure 3 compares the tensile stresses calculated using the MLLE and FEM approaches at the bottom of the base course layer for Structures 1 and 2 and at the bottom of the sub-base layer for Structures 3 and 4, respectively. Because the Odemark approach does not enable the internal stresses of a pavement structure to be evaluated, it is not included in this comparison.

As Fig. 3 reveals, large tensile stresses are developing into the stiff structural layers included in the analyzed pavement structures. Clearly, the highest these tensile stresses under the 50 kN surface load, up to 1,5 MPa, are mobilized



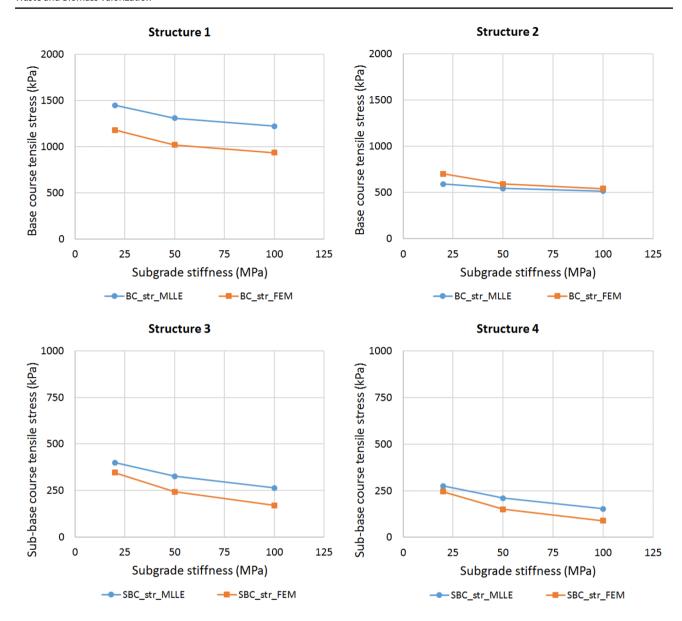


Fig. 3 Comparison of the values of tensile stresses at the bottom of base/sub-base course layers determined for Structures 1 to 4

in Structure 1 in which the stiffness of base course layer is very high, while in Structure 2 the respective tensile stresses are less than half of those mobilized in Structure 1. For a bituminous material, the mobilization of tensile stresses is not likely to be a very critical issue due to material's ductility, but in the case of a more fragile fly ash-stabilized base course material, repetitive application of this high tensile stresses is likely to result in gradual fracturing of base course layer.

In Structures 3 and 4, in which a stiff material layer is located deeper below the road surface, the mobilized tensile stresses are not as high as in Structure 1, but especially on soft subgrade soil conditions, they are still of the order of hundreds of kilopascals.

Discussion

The results of calculations carried out with three parallel modeling approaches and summarized in Figs. 2 and 3 indicate clearly that the overall stiffness of a pavement structure is not a unique quantity, but it markedly depends both on the calculation method and the subgrade soil on which the structure is located. In Fig. 3, it could be observed that high tensile stresses are developing into the structural layers that are stiffer than the underlying components of the structure. The simple Odemark calculation approach does not enable assessment of these internal distresses of the pavement structure, while in the MLLE modeling approach, the stresses and strains mobilizing into the loaded pavement structure can be



obtained at selected observation points. The most complete picture of the mobilized distresses can be obtained when the FEM approach is used in analyzing the overall performance of a traffic-loaded pavement structure.

The distribution of tensile stresses was examined more closely from the FEM simulation results. Cross sections from simulations under a PLT type of loading (subgrade type a) were examined here and are illustrated in Fig. 4. As Fig. 4 indicates in the fly ash-stabilized base course layer of Structure 1, much higher tensile stresses seem to be mobilized in comparison to the bitumen-stabilized Structure 2. In addition, in Structure 1, remarkably high tensile stresses prevail in a large area practically throughout the whole base course layer.

When the fly ash material is positioned into the sub-base course layer (Structures 3 and 4), the tensile stresses are significantly lower in magnitude. However, an almost continuous tensioned zone seems to develop from the bottom of the layer under the loaded area to the top of the sub-base course layer on both sides of the load.

Both types of tensile stress zones in the fly ash materials may affect the long-term behavior of the layer. Fly ash is stiff but brittle material. If the magnitude of tensile stress is relatively high, such as in these structures, it is questionable that the fly ash layers can withstand these tensile stresses without cracking under the repeated wheel loads during the service life of the road structure.

Conclusions

If structural layers made of alternative construction materials, e.g. with especially high or low stiffness values, are used in a pavement structure to replace the traditional types of materials, it is important to realize that the overall performance of the whole pavement structure is likely to change. Therefore, the failure mechanisms that are decisive regarding the service life of the structure may also be quite different from those that are relevant to a traditional type of road structure. Recognition of these critical failure mechanisms is, however, not straightforward, especially if the structural analysis is made using the empirically based design approaches that are, as such, applicable for more traditional types of pavement structures.

Based on the structural analyses exemplified in this study it can be concluded that:

Even such a simple quantity as the overall stiffness of a pavement structure is far from being a unique value, but it depends on the calculation approach that is used in determining it. In addition, it naturally also depends to a great extent on the conditions, especially the type of subgrade soil, on which a certain type of pavement structure is located.

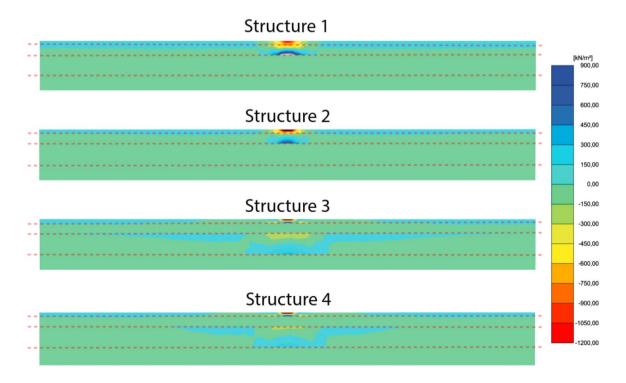


Fig. 4 Comparison of horizontal stress distributions in cross-sectional direction as obtained from FEM analyses carried out for Structures 1 to 4 resting on subgrade type a. Structural layer boundaries are indicated in cross sections by dotted lines



- Tensile stresses clearly exceeding 1 MPa may mobilize in a Low Volume Road type of pavement structure loaded by a heavy wheel load if the base course is made of very stiff material.
- If and when these high tensile stresses lead to cracking
 of the base course layer, load distribution capacity of the
 base course layer will markedly reduce, which in turn
 results in the increase of stresses and unevenness of stress
 distribution in the underlying layers.
- A much more complete picture of the prevailing stresses and strains within a traffic-loaded pavement structure can be obtained if the analysis is made using more sophisticated analysis tools such as the FEM. In comparison to the performance evaluation made based on a simple quantity such as the overall stiffness of a pavement structure, it enables better recognition of the critical distresses and related failure mechanisms that are characteristic for non-traditional types of pavement structures.

Even though the use of the FEM approach enables the development of a thorough understanding of the structural behavior of basically any type of non-traditional pavement structure, at least two major challenges still remain:

- Identification of the potentially critical distresses and failure mechanisms is not enough without a knowledge about the allowable level of stresses or strains that are repeated a certain number of times during the service life of a road pavement at the critical points of the structure to be designed. By using the terminology of mechanistic pavement analyses, better knowledge of the fatigue models of alternative pavements construction materials should be developed.
- In spite of the great developments made in the user-friendliness of modern Finite Element software tools, they are still too complicated to be used in the routine structural design of pavement structures. Therefore, simplified design tools still incorporating the recognition of critical failure mechanisms typical for the non-traditional types of pavements structures are required.

Based on the above, it is evident that quite a bit of work is still ahead before these two main challenges have been tackled. One of the first steps on this way is to understand that utilization of the alternative types of construction materials may change the overall behavior and related failure mechanisms of a road pavement. People who are used to work with alternative construction materials are familiar with the concept of chemical compatibility. In addition to that, the authors of this paper would like to introduce the concept of structural compatibility. Keeping this concept in mind, due attention will, it is hoped, also be drawn to the mechanical behavior of these alternative materials when they are used

in replacing the traditional ones in road structures exposed to heavy traffic loads.

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