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# The influence of quantization of optimal FTN signals with PAM on correlation properties and level of out-of-band emissions

A S Ovsyannikova<sup>1</sup>, S B Makarov<sup>1</sup>, S V Zavjalov<sup>1</sup> and D Solomitckii<sup>2</sup>

<sup>1</sup> Peter the Great St. Petersburg Polytechnic University, Polytechnicheskaya 29, St. Petersburg 195251, Russia

<sup>2</sup> University of Tampere, Korkeakoulunkatu 7, 33720 Tampere, Finland

E-mail: anny-ov97@mail.ru

**Abstract.** Usage of faster than Nyquist signaling (FTN) in practice is limited by the complexity of the formation and detection devices. Optimal FTN pulses may be formed via digital non-recursive filters. One of the important parameters that affect the performance of such devices is the number of quantization levels. In this paper, the influence of the number of quantization levels for optimal FTN signals with pulse-amplitude modulation on spectral characteristics, correlation properties and the probability of error detection is investigated. It is shown that at least 64 quantization levels should be chosen to provide almost potential bit error rate performance and additional spectral quantization noise about  $-40$  dB for volumes of channel alphabet up to 8. Decreasing the volume of channel alphabet allows reducing the requirements for the number of quantization levels.

## 1. Introduction

The practical usage of faster than Nyquist (FTN) signals [1-3], which provide transmission at the rate higher than the "Nyquist limit" without significant energy losses, is limited because of the complexity of formation and detection devices. In existing telecommunication, systems FTN signals with duration  $T_s$  longer than duration  $T$  of one transmitted bit are formed via linear filtration [1]. In this case, the requirements for given frequency bandwidth and the level of out-of-band emissions are satisfied. The limiting factor is a significant complication of formation (algorithmic complexity of high-order filter design) and detection algorithms [4] for such signals when demands for a reduction rate of out-of-band emissions increase.

Optimal FTN signals [5-6] have specified time and frequency parameters (occupied frequency bandwidth  $\Delta F$ , the reduction rate of out-of-band emissions). The main characteristics of energy spectrum  $|S(f)|^2$  of a random sequence of optimal pulses  $s_{opt}(t)$  with duration  $T_s$  are determined by solving the optimization problem in accordance to the certain optimality criterion [5-10]. Optimal FTN signals may be formed with the use of non-recursive digital (discrete) filters with impulse analog response  $K(t)=s_{opt}(T_s - t)$ . Implementation complexity of such non-recursive filters depends on the number of quantization levels of optimal FTN signal samples, the number of samples and memory volume.

Reducing the number of quantization levels of samples leads to deterioration of correlation properties of optimal FTN signals and, as a result, to bit error rate (BER) performance degradation. In addition to this, occupied frequency bandwidth widens and reduction rate of out-of-band emissions



decreases. On the other hand, the performance of formation devices is improved and hardware and algorithmic costs on the realization of these digital modems may be reduced.

In this paper, it is proposed to consider the influence of the number of quantization levels for optimal FTN signals with pulse amplitude modulation (PAM) on spectral characteristics of random sequences and their correlation properties and to estimate probabilities of error detection. The research is focused on multi-level PAM and the number of quantization levels which is enough to use in order to satisfy the requirements for spectral and energy characteristics of optimal FTN signals.

## 2. Simulation model

Let us consider random sequence  $y(t)$  of  $N$  optimal FTN pulses  $s_{opt}(t)$  with PAM, energy  $E_{opt}$  and duration  $T_s=LT$  transmitted at the symbol rate  $R$ :

$$y(t) = \sqrt{E_{opt}/T} \sum_{n=-N/2}^{N/2} c_j^{(n)} s_{opt}(t - nT/R). \quad (1)$$

For FTN signals with the volume of channel alphabet  $M$  values of  $c_j^{(n)}$  in (1) are determined by the next formula (2):

$$c_j^{(n)} = \frac{M - 2j + 1}{M - 1}, j = 1 \dots M. \quad (2)$$

For example, for  $M=2$  we have  $j=1,2$  and  $c_1^{(n)} = 1; c_2^{(n)} = -1$ . If  $M=4$ , then  $j=1,2,3,4$  and  $c_1^{(n)} = 1; c_2^{(n)} = 0.33; c_3^{(n)} = -0.33; c_4^{(n)} = -1$ . The values of  $c_j^{(n)}$  are uniformly distributed in the from  $-1$  to  $+1$  and have equal probabilities for each  $n$ .

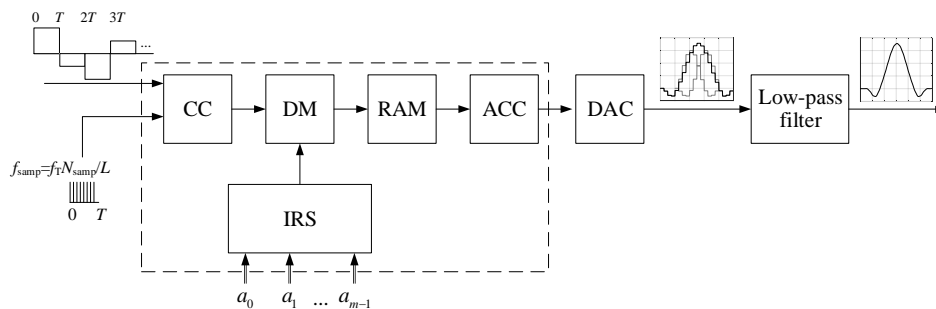
Increasing the duration  $T_s=LT$  of pulses  $s_{opt}(t)$  allows reducing occupied frequency bandwidth  $\Delta F$ . However, when  $R > 2/T$ , significant intersymbol interference (ISI) between pulses appears. Higher symbol rate  $R$  causes a greater number of adjacent optimal FTN pulses which influence on the shape of the current transmitted pulse. The shapes of optimal FTN pulses  $s_{opt}(t)$  are obtained as a result of solving optimization problem [5-6]. The optimization problem is solved by maximizing the reduction rate of out-of-band emissions of a random signal sequence (1) with constraints on the pulse energy  $E_{opt}$ , pulse duration  $T_s$ , correlation coefficient  $K_0$  or Euclidean distance for limited signal sequence. The last two constraints take into account the level of ISI. Thus, we get coefficients of Fourier series [5-6, 11]  $a_0, \dots, a_{m-1}$  which determine optimal pulse shape  $s_{opt}(t)$ .

Spectral and energy characteristics of such optimal FTN signals are well-known. Nevertheless, discretization and quantization caused by digital formation and detection devices lead to degradation of  $s_{opt}(t)$ . Reducing the number of quantization level results in quantization noise and spectrum distortion while reducing the number of samples becomes the reason of interference between spectrum replicas. At the same time using a minimum number of quantization levels in formation and detection devices allows achieving the best possible performance of such devices. Now we have to estimate the influence of quantization levels for optimal FTN signals with PAM on spectral characteristics of random sequences and their correlation properties and to estimate error probabilities using the digital non-recursive filter.

The block diagram of the digital formation of optimal FTN signals with PAM (volume of channel alphabet  $M=4$ ) is presented in figure 1. This diagram is based on the digital non-recursive filter (shown by the dashed line). The filter performs discrete convolution which may be expressed in digital form the next way (3):

$$s_{opt}^{[q]}[k] = \Delta t \sum_{i=0}^p c^{[l]}[k-i] K^{[l]}[i], \quad (3)$$

where  $c^{[j]}[k-i]$  is a  $j$ -bit number corresponding to one of the values of channel symbol  $c_j^{(n)}$  in (1). The sequence of  $l$ -bit numbers is defined by impulse response  $K^{[l]}[i]$  with length  $P$  of the digital non-recursive filter. On the filter output, the sequence of  $s_{opt}^{[q]}[k]$   $k$ -bit numbers is calculated as a result of discrete convolution. Note that  $q=l+j$ . These numbers follow each other with sampling interval  $\Delta t = T / (N_{samp} / L)$ , where  $N_{samp}$  is the number of samples at pulse duration  $T_s$ .



**Figure 1.** Block diagram of the digital formation of optimal FTN signals with PAM.

The digital non-recursive filter (figure 1) consists of code converter (CC), digital multiplier (DM) of  $l$ -bit and  $j$ -bit numbers, impulse response shaper IRS (sequences  $K^{[l]}[i]$  of  $l$ -bit numbers), random access memory RAM and accumulator ACC. Digital filtration includes two stages. During the first one, each  $j$ -bit  $c^{[j]}$  is multiplied by all  $P$  values of  $l$ -bit numbers  $K^{[l]}[i]$  and obtained numbers are written to RAM. The second stage represents reading information from RAM into the accumulator. Such principle of digital non-recursive filter design allows getting a stack structure of the device. It is especially important when signals with long duration are applied (e.g.,  $T_s=8T$ ). The impulse response is formed with the help of Fourier coefficients  $a_0, \dots, a_{m-1}$  which are fed into IRS before transmission starts.

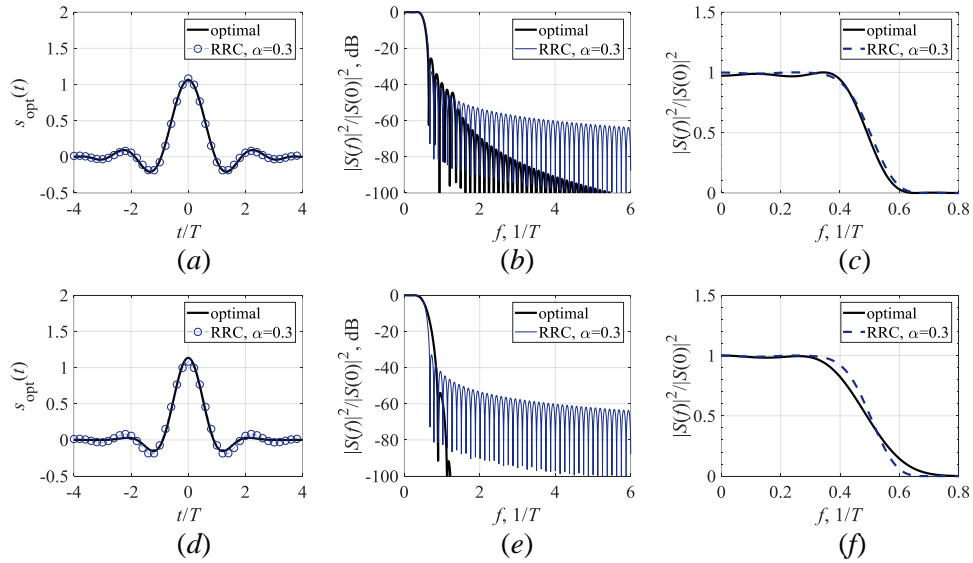
On the output of the digital non-recursive filter, the sequences of numbers go to the digital to analog converter (DAC) which forms step-changing voltage. After low-pass filtration which eliminates the influence of spectrum replicas on the spectrum shape in occupied frequency bandwidth  $\Delta F$  step-changing signal shape transforms into analog shape  $s_{opt}(t)$ .

### 3. Results of simulation modeling

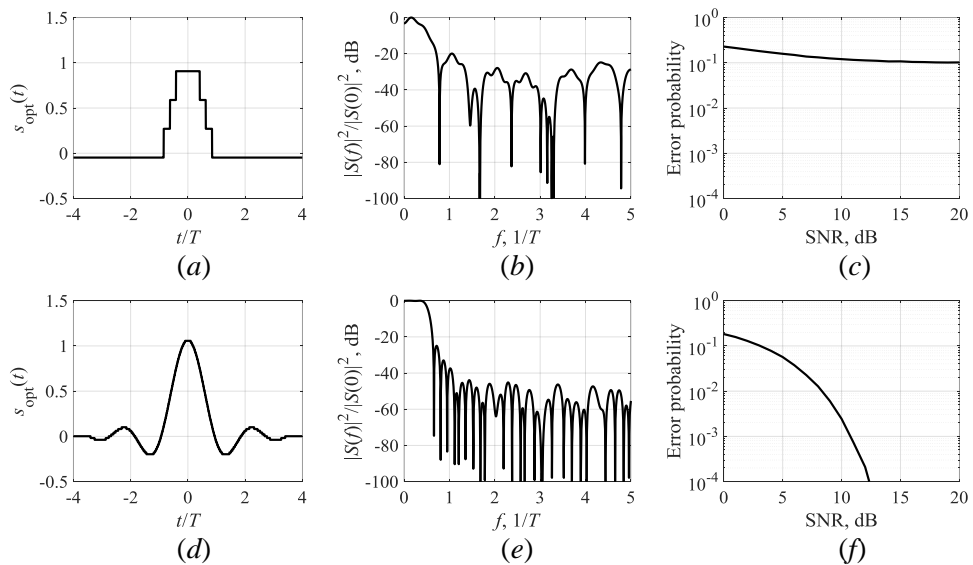
The simulation modeling was aimed at estimating time and spectral parameters of the proposed digital formation method for optimal FTN signals with PAM. During modeling the volume of channel alphabet was equal to  $M=4$  and optimal FTN pulse duration  $T_s=8T$  was chosen. The optimization problem was solved with constraints on the reduction rate of out-of-band emissions outside  $\Delta F$  and on the coefficient of mutual correlation  $K_0$ .

In figure 2 the calculated shapes  $s_{opt}(t)$  and their energy spectra for different values of the reduction rate of out-of-band emissions are illustrated ( $K_0=0.01, R=1/T$ ). The shapes of FTN signals on the basis of root-raised cosine (RRC) pulses [1] (depicted by circles in figure 2,  $a, d$ ) with roll-off factor  $\alpha=0.3$  are given for comparison. In figure 2,  $b, c, e, f$ , the shapes of energy spectra of such signals in the area of out-of-band emissions (figure 2,  $b, e$ , blue thin line) and in the area near to zero frequency (figure 2,  $c, f$ , dashed line) are shown. It can be seen that an increase in the rate of out-of-band emissions changes the shape of optimal pulse  $s_{opt}(t)$ . The amplitude of side lobes reduces as well as the width of the main petal of  $s_{opt}(t)$ . The energy spectrum of optimal FTN signal has a higher reduction rate of out-of-band emissions than known FTN signals based on RRC pulses.

Let us investigate the influence of the number of quantization levels for the sequence  $K^{[l]}[i]$  containing  $l$ -bit numbers in IRS on spectral characteristics of the signal. The results of simulation modeling of optimal FTN pulses presented in figure 2, $a$  for  $M=4$  may be found below (figure 3).



**Figure 2.** Shapes of  $s_{opt}(t)$  and normalized energy spectra for reduction rate of out-of-band emissions  $1/f^6$  (a, b, c);  $1/f^{22}$  (d, e, f).

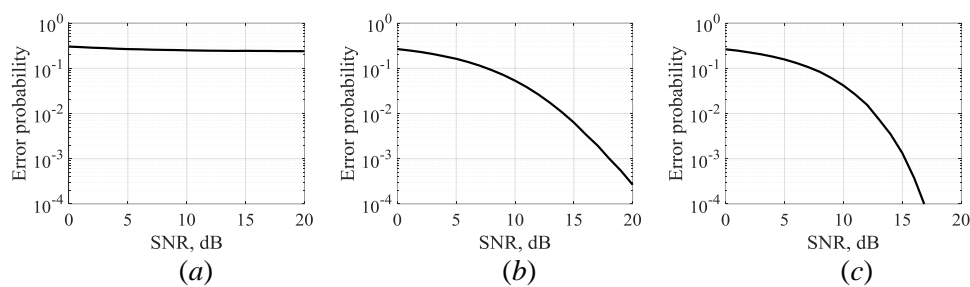


**Figure 3.** Shapes of quantized optimal pulses (a, d), normalized energy spectra (b, e) and BER performance (c, f) for  $M=4$ .

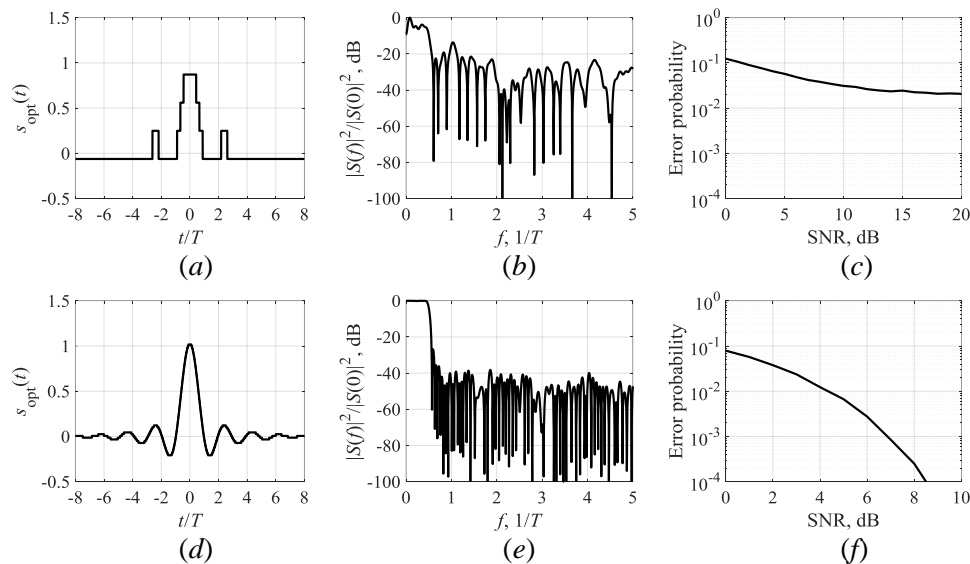
During modeling the number of quantization levels for samples  $K^{[l]}[i]$  was chosen to 4, 16 and 64 correspondingly while DAC (figure 1) resolution was equal to 2, 4, 6 relatively. For illustration, we placed here the results for 4 (figure 3,a-c) and 64 (figure 3,d-f) quantization levels and DAC resolution equal to 2 and 6 correspondingly. Analyzing energy spectra of such quantized signals, we can notice an additional spectral level of quantization noise in the area of out-of-band emissions which reaches about  $-30$  dB and  $-50$  dB. Error probabilities in the simulation model were calculated using coherent bit-by-bit detection algorithm for the channel with additive white Gaussian noise (AWGN) with average power spectral density  $N_0$  and symbol rate  $R=1/T$ . In the graphs of figure 3, c, f, the x-axis represents values of signal-to-noise ratio (SNR). When the number of quantization levels is equal to

64, BER performance is close to the theoretical one. However, if 16 quantization levels are used, energy losses do not exceed 0.5 dB for error probability  $p=10^{-4}$ .

Let us look at the results of quantization for optimal FTN pulses with PAM when the volume of channel alphabet is increased to the value  $M=8$ . Degradation of optimal pulse shapes and their energy spectra is similar to the case of  $M=4$ . But the probabilities of error detection for quantized pulses with  $M=8$  differ from those obtained for  $M=4$  significantly. In figure 4 the probabilities of error detection depending on SNR are given for pulses illustrated in figure 2,a. Comparing curves in figure 4, we can conclude that only using 64 quantization levels gives BER performance close enough to the theoretical one. For 16 levels of quantization, energy losses reach about 3 dB for error probability  $p=10^{-3}$ . Thus, if the volume of channel alphabet increases, the requirements for an acceptable number of quantization levels for samples of optimal FTN signals with PAM become more stringent.



**Figure 4.** BER performance of quantized optimal FTN pulses for  $M=8$  and the number of quantization levels 4 (a), 16 (b), 64 (c).



**Figure 5.** Shapes of quantized optimal pulses (a, d), normalized energy spectra (b, e) and BER performance (c, f) for  $M=2$  and  $T_s=16T$ .

Let us review the influence of quantization on time and spectral parameters of optimal FTN signals with longer duration ( $T_s=16T$ ). The shapes of quantized optimal pulses (a, d), normalized energy spectra (b, e) and probabilities of error detection (c, f) may be found in figure 5. Here optimal FTN pulse duration  $T_s=16T$  and the rate of transmitting the symbols of binary channel alphabet  $R=1/T$  were used. The reduction rate of out-of-band emissions is equal to  $1/f^6$  and coefficient of mutual correlation is  $K_0=0.01$ . If we compare the shapes of quantized optimal FTN pulses with different duration (figure 3 and figure 5), we will see that for a longer duration the distortions of pulse shape become greater

[12]. It leads to raising the level of out-of-band emissions up to  $-22$  dB. Increasing the number of quantization levels up to 16 results in the level of out-of-band emissions caused by quantization about  $-30$  dB. It is 10 dB higher than for signals with duration  $T_s=8T$ . Spectral noise achieves  $-40$  dB for 64 levels of quantization. Now we should pay attention to BER performance of optimal coherent bit-by-bit detection algorithm for optimal FTN signals and  $M=2$  (figure 5, *c, f*). Error probabilities almost match each other for the cases of 16 and 64 levels of quantization. Therefore, decreasing the volume of channel alphabet to  $M=2$  allows reducing the requirements for the number of quantization levels.

#### 4. Conclusion

Formation of optimal FTN signals with PAM via digital non-recursive filters gives an opportunity to obtain arbitrary signal shapes. It makes this method different from the formation method via classic linear filtration. Such filtration is used in order to obtain FTN signals on the basis of RRC pulses which provide transmission at the rate higher than the Nyquist limit, for example [1]. The performance of digital non-recursive filters depends on the time needed for operation execution, the way of organizing the calculations and the bitness of impulse response numbers. In order to improve performance, it is necessary to reduce the number of quantization levels of the impulse response. As a result of quantization of optimal FTN pulses with PAM, their spectral characteristics degrade as well as correlation properties and probabilities of error detection. For example, when FTN signal with duration  $8T$  and PAM is formed with the use of 4, 16, 64 quantization levels of impulse response samples, additional spectral level of quantization noise appears. It is equal to  $-30$  dB,  $-40$  dB and  $-50$  dB correspondingly. If pulse duration is increased by two times, the spectral level of quantization increases too and reaches  $-22$  dB,  $-30$  dB and  $-40$  dB for the same quantization levels. Quantization of samples of optimal FTN pulses with PAM causes degradation of correlation properties of a random signal sequence. Thus, for channel volume  $M=8$  using 16 quantization levels leads to energy losses about 3 dB for error probability  $p=10^{-3}$ . Increasing the number of levels up to 64 allows reaching theoretical BER performance. Summarizing the research on the possibility of application of digital non-recursive filters with a reduced number of quantization levels for the formation of optimal FTN signals with PAM, we can state that it is rational to choose at least 64 quantization levels. It provides energy losses no more than 0.2 dB and additional spectral quantization noise at the level  $-40$  dB for volumes of channel alphabet up to  $M=8$ .

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