An Upper Bound on Capacity of 5G mmWave Cellular with Multi-Connectivity Capabilities

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Outage events caused by dynamic blockage of a radio signal propagation path are one of the key challenges in 5G millimetre-wave (mmWave) cellular networks. To mitigate them, 3GPP standardization has recently ratified multi-connectivity techniques aiming to enable user connectivity to several base stations simultaneously, while switching between them whenever the currently active connection becomes blocked. In this work, we obtain a closed-form upper bound on the probability density function of the respective system capacity in a random field of moving blockers.

Background and Motivation: The 3rd Generation Partnership Project (3GPP) has recently introduced *multi-connectivity* (MC) capabilities that target to prepare the emerging 5G millimetre-wave (mmWave) cellular systems for serving reliability-demanding applications. Accordingly, a user equipment (UE) maintains multiple spatially-diverse connections with different base stations (BSs) [1]. Hence, MC is expected to mitigate the consequences of the line-of-sight (LoS) blockage by various objects in the radio channel owing to dynamic re-association of the UE with the currently non-blocked BSs [2].

The authors of [3] consider the capacity and outage probability as the two metrics that can be improved by enabling MC operation. The capacity gains of MC have been demonstrated in [4] as the result of a simulation campaign. In [5], a framework to model the signal-to-interference-plus-noise ratio (SINR) distribution in MC-ready scenarios has been developed. A comparison of various BS switching strategies for MC-capable mmWave networks has been performed in [6].

Despite a number of past studies that assessed the capacity gains with MC, none of those reported an upper bound on the capacity achievable in dense mmWave deployments. Such a bound is a valuable contribution to determine the extent, to which a particular MC scheme performs far from the theoretical limit. To this aim, we derive a theoretical upper bound on the probability density function (pdf) of capacity in the 5G mmWave cellular system supporting MC capabilities.

System Model: We assume that the locations of mmWave BSs follow a Poisson point process (PPP) in $\Re^2$ with the intensity of λ. This assumption is aligned with the recent studies of dense BS deployments [7]. The height of the BS is fixed and equals h_A.

Human bodies act as potential blockers to the mmWave connections of the UEs. We represent these obstacles by cylinders with the constant radius and height, r_B and h_B, respectively. The spatial density of blockers is maintained constant, μ. Human blockers are mobile and move around the area according to the random direction model (RDM) [8].

The path loss is modelled by using $L(r) = Ar^{-\gamma}$, where γ is the loss exponent, A is the factor accounting for the transmit power and antenna gains, and r is the distance between the UE and its serving BS. One can determine A and γ by utilizing a propagation model for the frequency band of interest e.g., [9] for the spectrum range of $0.5 - 100$ GHz.

We concentrate on the tagged cell-edge UE located farther away than R_B from its nearest BS, where a blockage event leads to outage. The distance R_B is calculated based on the propagation model and signal-to-noise (SNR) threshold at the UE. The height of the UE is fixed at h_U.

According to the considered MC scheme, the UE in question always associates with its nearest BS that experiences non-outage conditions. Re-association time is assumed to be negligible. The metric of interest is the time-averaged UE capacity over the bandwidth of B, which is defined as

$$C = \lim_{t\to\infty} \frac{1}{t} \int_0^t B \log_2[1 + S(t)], \tag{1}$$

where $S(t)$ is the SNR at time t.

Upper Bound on Capacity: Consider a UE located farther away than R_B from its nearest BS. Observe that the subject UE faces outage conditions whenever there is at least one blocker, which occludes its LoS path to the BS. The conventional approach to derive an upper bound on the UE capacity is to (i) determine the capacity offered by the i-th nearest BS, $i = 1, 2, \ldots$, (ii) estimate a fraction of time that the UE remains associated with the i-th nearest BS [10], and (iii) use these factors to weight the associated capacities. However, this procedure does not lead to a closed-form formulation, as it involves an infinite weighted sum of pdfs.

Instead of tackling this problem directly, it could be noticed that the blockers moving around according to the RDM model constitute a PPP at all times, since a random displacement of the PPP is again PPP [11]. Hence, the fraction of time that the i-th nearest BS resides in the outage conditions with the tagged UE coincides with the probability that this BS is blocked under a static PPP of blockers. Given that according to the considered MC scheme the UE always associates with its nearest BS, which experiences non-outage conditions, the problem at hand is reduced to (i) determining the blockage with the i-th BS located at the distance of r, $r > R_B$, (ii) deriving the pdf of the distance to the nearest BS in non-outage conditions within a PPP field of blockers, and (iii) transforming the resulting pdf by utilizing the capacity function.

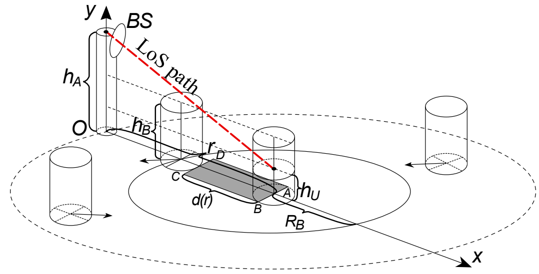

Fig. 1. Configuration of mmWave LoS blockage zone.

Let $p_B(r)$ be the probability that the LoS path between the UE and the mmWave BS, which are separated by the planar distance of r, is blocked. For the proposed model, one can identify the so-called "LoS blockage zone" around the UE, denoted ABCD in Fig. 1. At realistic UE–BS distances, the area of the LoS blockage zone may be approximated by a rectangle with the sides of $2r_B$ and $d(r) = r(h_B - h_U)(h_A - h_U) + r_B$. The probability that the LoS path is not blocked thus corresponds to the void probability of the Poisson process of blockers, which leads to

$$p_B(r) = 1 - e^{-2r\mu r_B \frac{h_B - h_U}{h_A - h_U}}. \tag{2}$$

Define a new process that only inherits those BSs, which are currently not blocked. It is produced as a probabilistic thinning [11] of the original formulation with the probability of $1 - p_B(r)$. The resulting process is a non-homogeneous PPP of BSs with the instantaneous intensity of $\lambda[1 - p_B(r)]$, $r > 0$, which decreases along the radial lines.

Let X be the planar distance to the nearest non-blocked BS. We obtain the pdf $f_X(r)$ by exploiting the probability that all the BSs within a circle of radius r remain non-blocked, $Pr\{B_0\}$, and the probability that there is at least one non-blocked BS in the annulus $(r, r + \Delta r)$, $Pr\{B_1\}$. The probability that there are no BSs in $B(0, r)$ is

$$Pr\{B_0\} = e^{-\Lambda(0,r)\pi r^2}, \tag{3}$$

where $\Lambda(0, r)$ is the mean intensity of the process given by

$$\Lambda(0, r) = \frac{\lambda(h_A - h_U)\left(e^{\frac{2\mu r r_B(h_U - h_B)}{h_A - h_U}} - 1\right)}{2\mu r r_B(h_U - h_B)}. \tag{4}$$

The probability that there is at least one BS in $B(r, r + \Delta r)$ is thus

$$Pr\{B_1\} = 1 - e^{-\pi r^2[\Lambda(0,r+\Delta r) - \Lambda(0,r)]}. \tag{5}$$

Substituting (4) into (5) and letting $\Delta r \to 0$, we arrive at

$$Pr\{B_1\} = \frac{\pi\lambda e^{\frac{2\mu r r_B(h_U - h_B)}{h_A - h_U}}(h_U + 2\mu r r_B(h_B - h_U) - h_A)}{2\mu r_B(h_B - h_U)} + \\ + \frac{\pi\lambda\,(h_A - h_U)}{2\mu r_B(h_B - h_U)}. \tag{6}$$

Combining (3) and (6), we establish $f_X(r)$ in the closed-form as

$$f_X(r) = \frac{e^{\frac{2\mu r r_B(h_U - h_B)}{h_A - h_U}}(h_A + 2\mu r r_B(h_U - h_B) - h_U) - h_A + h_U}{2\mu r_B(h_U - h_B)(\pi\lambda)^{-1}} \times \\ \times e^{\frac{\pi\lambda r(h_U - h_A)\left(e^{\frac{2\mu r r_B(h_U - h_B)}{h_A - h_U}} - 1\right)}{2\mu r_B(h_U - h_B)}}, \; r > 0. \tag{7}$$

Since we consider the UE that is located farther away than R_B from its nearest BS, the conditional pdf of distance to the i-th BS, given that it is greater than R_B, is $f_X(r|R_B) = f_X(r)/[1-\int_0^{R_B} f_X(r)]$, which yields

$$f_X(r|R_B) = f_X(r)e^{\frac{-\pi\lambda r_B(h_U-h_A)\left[e^{\frac{2\mu R_B r_B(h_U-h_B)}{h_A-h_U}}-1\right]}{2\mu r_B(h_U-h_B)}}. \qquad (8)$$

Therefore, the derivation of the capacity pdf reduces to establishing the pdf of the non-linear transformation of X in the form

$$\phi(r) = B\log_2\left(1+\frac{A[r^2+(h_A-h_U)]^{-\gamma/2}}{BN_0}\right), \qquad (9)$$

where r is the planar distance to the nearest non-blocked BS, B is the bandwidth of interest, and $N_0 = -174$ dBm is the noise per one Hz.

Recall that the pdf of a random variable C, $f_C(y)$, which is expressed as a function $y = \phi(r)$ of another random variable X having the pdf $f_X(r)$, is [12]

$$f_C(y) = \sum_{\forall i} f_X(\phi_i^{-1}(y))\left|[\phi_i^{-1}(y)]'\right|, \qquad (10)$$

where $r = \phi_i^{-1}(y)$ is the i-th branch of the inverse function.

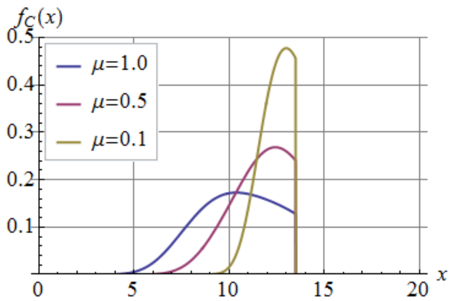

Fig. 2. Capacity pdf as a function of μ, $\lambda = 10^{-4}$

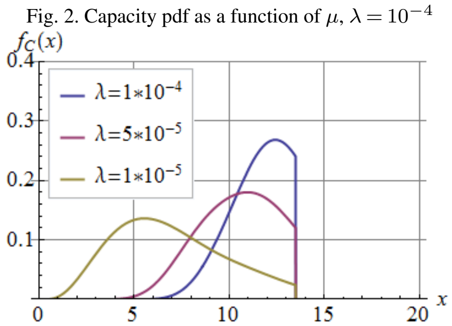

Fig. 3. Capacity pdf as a function of λ, $\mu = 0.5$

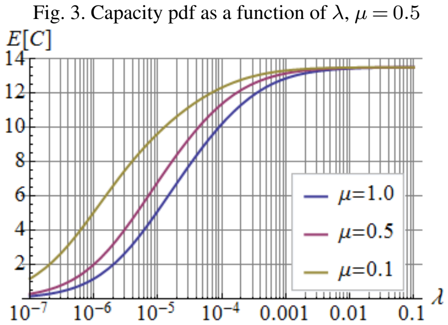

Fig. 4. Mean capacity as a function of λ, μ

The inverse and modulo for the inverse derivative of the capacity function in (9) over the region of interest are given by

$$\phi^{-1}(y) = -\sqrt{\left(-\frac{BN_0\left(1-2^{\frac{y}{B}}\right)}{A}\right)^{-2/\gamma} - h_A + h_U},$$

$$[\phi^{-1}(y)]' = \frac{N_0\log(2)2^{y/B}\left(-\frac{BN_0\left(1-2^{y/B}\right)}{A}\right)^{-\frac{2}{\gamma}-1}}{A\gamma\sqrt{\left(-\frac{BN_0\left(1-2^{y/B}\right)}{A}\right)^{-2/\gamma} - h_A + h_U}}. \qquad (11)$$

Substituting (7) into (8) and then $f_X(r|R_B)$ and (11) into (10), the sought closed-form result for $f_C(y)$ follows readily.

Numerical Examples: We proceed by illustrating the obtained results. The capacity pdf and the mean capacity for several densities of blockers and BSs, as well as $h_A = 4$, $h_B = 1.7$, $h_U = 1.5$, $r_B = 0.3$, $N_0 = -174$ dBm, and $B = 1$ Hz. The path loss parameters were set to $\gamma = 2.1$ and $A = 28935037$, which corresponds to 28 GHz carrier frequency [9], while the SNR threshold was equal to 0 dB, which translates to $R_B = 53$ m.

As one may observe in Fig. 2, for constant density of BSs, $\lambda = 10^{-4}$, (corresponds to the mean inter-BS distance (IBD) of 50 m), changing the density of blockers from 0.1 to 1.0 results in degraded system performance as the pdf shifts to the region of its smaller values. Similar effects are observed by decreasing the density of the BSs in Fig. 3 from 10^{-4} (IBD= 50 m) down to 5×10^{-5} (IBD= 70 m) and then to 10^{-5} (IBD= 160 m).

Finally, by analysing the mean capacity presented in Fig. 4, one may learn that the highest gains of the MC operation are achieved for the BS density range of 10^{-6} (IBD= 500 m) to 10^{-3} (IBD= 15 m). A further increase in the density of BSs brings no additional capacity gains.

Conclusions: In this letter, we offered a closed-form upper bound on the pdf of capacity for the 5G mmWave cellular system supporting MC capabilities. The proposed bound can be utilized as a benchmark result for the performance evaluation of practical MC schemes.

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