Solar Irradiation Independent Expression for Photovoltaic Generator Maximum Power Line

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Abstract—In order to enhance maximum power point tracking (MPPT) speed of photovoltaic generators (PVG) upon fast irradiation changes, maximum power line (MPL) - based control is often used. MPL is a curve, linking all possible MPP coordinates for a given temperature. In the literature so far, PVG MPL was either assumed linear, which is inaccurate for all irradiation levels, or possessed photocurrent dependence, requiring real time estimation of the latter. In this letter, an irradiation-independent explicit expression for PVG MPL is derived, valid for all practical irradiation levels thus allowing real-time implementation without the need of photocurrent estimation.

Index Terms—Photovoltaic generator, MPPT, maximum power line, single diode model.

I. INTRODUCTION

PHOTOVOLTAIC generators possess the so-called *soft source* characteristics, featured by a single maximum power point (on a basic unit level), whose coordinates are environmental conditions dependent, thus requiring MPPT to allow optimal utilization in terms of cost and reliability [1]. Generalized photovoltaic energy conversion system is shown in Fig. 1, consisting of PVG, interfacing power converter (IPC) and load.
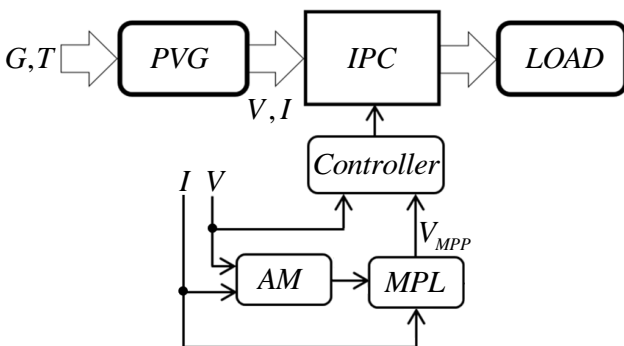

Fig. 1: Photovoltaic energy conversion system with model based MPPT.

Environmental variables, affecting PVG operation, are solar irradiation G and temperature T. To optimally utilize the harvested energy, PVG voltage and current are typically sampled and fed back into an MPPT algorithm, which generates IPC control signal. In order to increase the response time of conventional MPP trackers (dictated by combined system dynamics [2]) under fast-varying solar irradiation, voltage-oriented MPPT is preferred since irradiation changes have relatively weak influence on MPP voltage while greatly affecting MPP current [3] - [5]. The authors of [6] and [7] have shown that, for a given temperature, MPPs for different irradiation levels approximately lie on a straight line, as depicted in Fig. 2. Then, model-based MPPT was proposed as follows: PVG output current is measured first; then, corresponding MPP voltage is calculated using the MPL expression and set as reference voltage V_{MPP} (cf. Fig. 1). Voltage controller processes the difference between reference and actual PVG voltage, and the IPC is operated to reduce tracking error ($V_{MPP}–V$) to zero. Since the MPL is temperature dependent, some adaptation mechanism (*AM*, cf. Fig. 1) is required both to provide the information regarding the temperature and account for parameter inaccuracies, which inevitably occur either during model-based MPL calculations or due to PVG ageing process. The AM may be realized by temperature measurement [8], estimation [9], perturb-and-observe MPPT [6] or indirect extraction of temperature-related information [7]. Linear MPL (LMPL, cf. Fig. 2) approximation is very simple and easy to implement yet inaccurate for low solar irradiation levels.
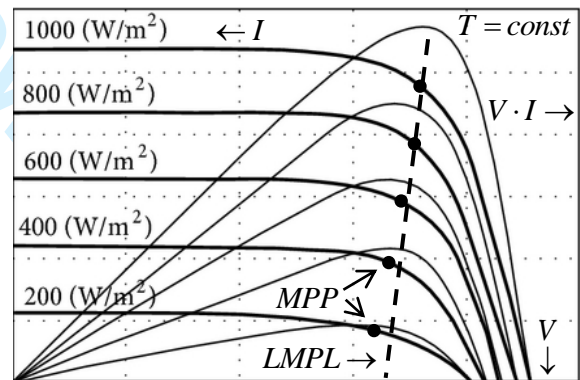

Fig. 2: I-V and P-V curves of a typical PVG for a given temperature. MPPs are indicated by black dots; LMPL is denoted by dashed line.

More accurate MPL expressions, were implicitly obtained in e.g. [10] – [15], requiring numerical solution by suitable iterative procedure. Moreover, some of the expressions contain photocurrent as a variable, which is irradiation dependent. Consequently, instead of describing the MPL, they actually determine single MPP coordinates for given irradiation and temperature. The first drawback was recently addressed in [8], [16], [17], where explicit analytical MPL expression was obtained by means of the Lambert-W function. However, the derived expression is still irradiation dependent. In this letter, an explicit, irradiation independent MPL expression is obtained based on the single-diode PVG equivalent circuit. The derived expression is successfully utilized to obtain the MPL of an off-the-shelf commercial photovoltaic panel.

II. MPL EXPRESSION DERIVATION

A. Single-diode PVG equivalent circuit

Consider the well-known single diode PVG equivalent circuit, shown in Fig. 3. Electrical characteristics of the PVG are described by the following equation

$$I = I_{ph} - I_D - \frac{V + R_s I}{R_p} \tag{1}$$

with

$$I_D = I_s(e^{\frac{V + R_s I}{NaV_t}} - 1), \tag{2}$$

where I_{ph} is the photocurrent, I_s denotes reverse saturation current, a stands for the ideality factor, V_t symbolizes the thermal voltage, N is the number of series-connected cells forming the PVG and R_s and R_p are equivalent series and shunt resistances, respectively.
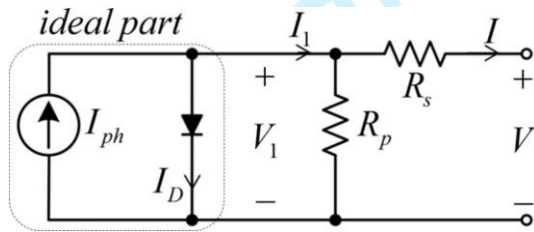

Fig. 3: Single-diode equivalent circuit of a PVG.

The thermal voltage is given by

$$V_t = \frac{k_b T}{q} = V_t^* \frac{T}{T^*}, \tag{3}$$

where $k_b = 1.380 \times 10{-}23$ [J·K^{-1}] is Boltzmann's constant; $q = 1.602 \times 10\text{-}19$ [C] is the elementary charge and T [K] is the PVG temperature. The asterisk '*' here and thereafter denotes that the parameter value corresponds to "known" conditions, (usually STC of 1000W·m^{-2} irradiation and 25°C temperature). Reverse saturation current is obtained for mono-crystalline silicon as

$$I_s = I_s^* \left(\frac{T}{T^*}\right)^3 \exp\left\{\frac{E_g}{k_b T^* a}\left(1 - \frac{T^*}{T}\right)\right\} \tag{4}$$

with E_g denoting energy band gap of the material. Photocurrent dependence on ambient conditions is given by

$$I_{ph} = k_{ph} \frac{G}{G^*} I_{ph}^* \left(1 + \varepsilon\left(\frac{T}{T^*} - 1\right)\right), \tag{5}$$

where ε is the short-circuit current temperature coefficient, G [W·m^{-2}] the solar irradiation and k_{ph} the spectral factor ($k_{ph} = 1$ for air mass ratio of 1.5). Apparently, while thermal voltage and reverse saturation current are influenced only by temperature, photocurrent is both temperature and irradiation dependent.

B. Existing nonlinear MPL expressions

Note that the following holds at MPP:

$$\frac{d(VI)}{dV} = 0 \Rightarrow \frac{dI}{dV}\bigg|_{MPP} = -\frac{I_{MPP}}{V_{MPP}}. \tag{6}$$

Hence, substituting (1) and (2) to (6) yields

$$\frac{I_{MPP}}{V_{MPP}} = \left(\frac{I_s}{NaV_t} e^{\frac{V_{MPP} + R_s I_{MPP}}{NaV_t}} + \frac{1}{R_p}\right)\left(1 + \frac{R_s I_s}{NaV_t} e^{\frac{V_{MPP} + R_s I_{MPP}}{NaV_t}} + \frac{R_s}{R_p}\right)^{-1}, \tag{7}$$

implicitly linking MPP coordinates for a given temperature based on four out of five equivalent circuit parameters without possessing irradiation dependence.

On the other hand, in order to derive an explicit MPL expression, the procedure proposed in [8], [17] has been applied as follows. First, MPP voltage coordinate of the PVG equivalent circuit ideal part (cf. Fig. 3) was obtained analytically from (1) and (2) as

$$V_{1,MPP} = NaV_t W\left\{\frac{I_{ph} + I_s}{I_s} e^1 - 1\right\}. \tag{8}$$

Then, corresponding MPP current was obtained from (8) as

$$I_{1,MPP} = \frac{I_s}{NaV_t} e^{\frac{V_{1,MPP}}{NaV_t}} V_{1,MPP}. \tag{9}$$

Eventually, MPP coordinates of the PVG were approximated by

$$V_{MPP} = V_{1,MPP} - \left(I_{1,MPP} - \frac{V_{1,MPP}}{R_p}\right)R_s \tag{10}$$

and

$$I_{MPP} = I_{1,MPP} - \frac{V_{1,MPP}}{R_p}, \tag{11}$$

respectively, assuming that the MPP of the ideal part coincides with the PVG MPP (which is true for $R_s \ll R_p$, as proved in [16],[17]). Corresponding MPL may then be explicitly obtained by combining (8) - (11) as

$$V_{MPP} = NaV_t\left(W\left\{\frac{I_{ph} + I_s}{I_s} e^1 - 1\right\} - 1\right) - R_s I_{MPP}. \tag{12}$$

Still, (8) and thus (12) remain dependent on solar irradiation (cf. (5)). Thus, in order to apply (12) in real time, G must be known. However, direct high-bandwidth irradiation measurement is noisy and unreliable while estimation of I_{ph} requires relatively complex calculations [8]. On the other hand, combining (10) and (11) into (9) yields an irradiation independent, yet implicit expression

$$\frac{1}{R_p}V_{MPP} + \left(1 + \frac{R_s}{R_p}\right)I_{MPP} = \frac{I_s}{NaV_t} e^{\frac{V_{MPP} + I_{MPP}R_s}{NaV_t}} (V_{MPP} + I_{MPP}R_s), \tag{13}$$

which is actually a reformulation of (7). This may be used to derive two boundary MPL points, given by

$$V_{MPP}\big|_{I_{MPP}=0} = NaV_t \ln\frac{NaV_t}{I_s R_p} \tag{14}$$

and

$$I_{MPP}\big|_{V_{MPP}=0} = \frac{NaV_t}{R_s}\ln\left(\frac{R_p + R_s}{R_p R_s}\frac{NaV_t}{I_s}\right), \tag{15}$$

respectively.

C. Proposed MPL expression

Note that (9) defines MPL expression for PVG equivalent circuit ideal part and may be rewritten as

$$V_{1,MPP} = NaV_t W\left\{\frac{I_{1,MPP}}{I_s}\right\}. \tag{16}$$

Apparently, (16) is irradiation independent but deviates from PVG MPL, as shown in [8, Fig. 2]. However, recall the MPL purpose: for a given PVG current, it should yield the corresponding MPP voltage. Consequently, we are not looking for MPP coordinates at given environmental conditions but for V_{MPP} corresponding to measured current for a given temperature. Thus, substituting (16) to (10)

$$V_{MPP} \cong \left(1+\frac{R_s}{R_p}\right)NaV_tW\left\{\frac{I_{1,MPP}}{I_s}\right\} - R_sI_{1,MPP}. \quad (17)$$

Relating to $I_{1,MPP}$ in (17) as the measured current, the expression explicitly approximates the desired MPL without any dependence on photocurrent. Corresponding power-voltage plane MPL is then given by

$$P_{MPP} \cong \left(1+\frac{R_s}{R_p}\right)NaV_tW\left\{\frac{I_{1,MPP}}{I_s}\right\}I_{1,MPP} - R_sI_{1,MPP}^2. \quad (18)$$

III. Validation

Consider REC-AE220 multi-crystalline solar module with STC specifications summarized in Table I. The five corresponding equivalent circuit parameters were derived in [9] and are given in Table II.

TABLE I
MAIN STC SPECIFICATIONS OF REC-AE220

Peak power	220 [W]
MPP voltage, V_{MPP}	28.7 [V]
MPP current, I_{MPP}	7.7 [A]
Open circuit voltage, V_{OC}	36.6 [V]
Short circuit current, I_{SC}	8.2 [A]
Temperature coefficient of I_{SC}, ε	0.074 [%/°C]

TABLE II
REC-AE220 EQUIVALENT CIRCUIT PARAMETERS AT STC

Reverse saturation current, I_s	$1.6 \cdot 10^{-10}$ [A]
Photocurrent, I_{ph}	8.21 [A]
Series resistance, R_s	0.47 [Ω]
Parallel resistance, R_p	608 [Ω]
Normalized thermal voltage, NaV_t	1.48 [V]

Fig. 4 depicts I-V and P-V curves families of REC-AE220 for $T = 25$°C and irradiation levels of $100W \cdot m^{-2} - 1000W \cdot m^{-2}$ (with $100W \cdot m^{-2}$ resolution). First, MPL is derived by the procedure described by (8) - (11) [8], [17], requiring the knowledge of I_{ph} (referred to as "existing MPL"). Then, (15) is used to derive the MPL (referred to as "proposed MPL") without any information regarding I_{ph}. Apparently, both MPLs coincide, verifying the validity of the proposed method. Furthermore, in order to confirm (17) for non-STC conditions, I-V and P-V curves families of REC-AE220 for $T = 50$°C and different irradiation levels were derived by modifying photocurrent, reverse saturation current and nominalized thermal voltage according to (3) – (5). Existing and proposed MPLs were then obtained as above. The results are shown in Fig. 5, again indicating overlapping of both curves, supporting the validity of (17).
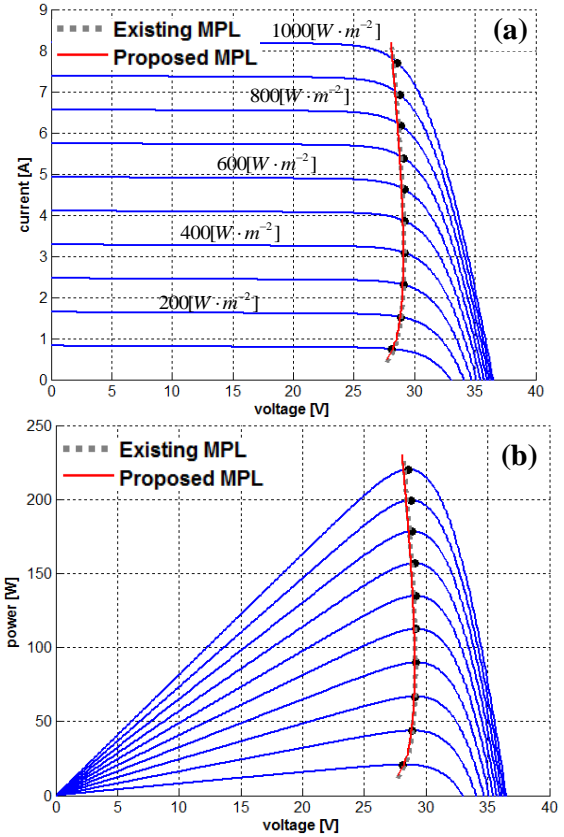

Fig. 4: Families of I-V curves (a) and P-V curves (b) of REC-AE220 for $T = 25$°C with corresponding MPLs. Actual MPPs are indicated by black dots.
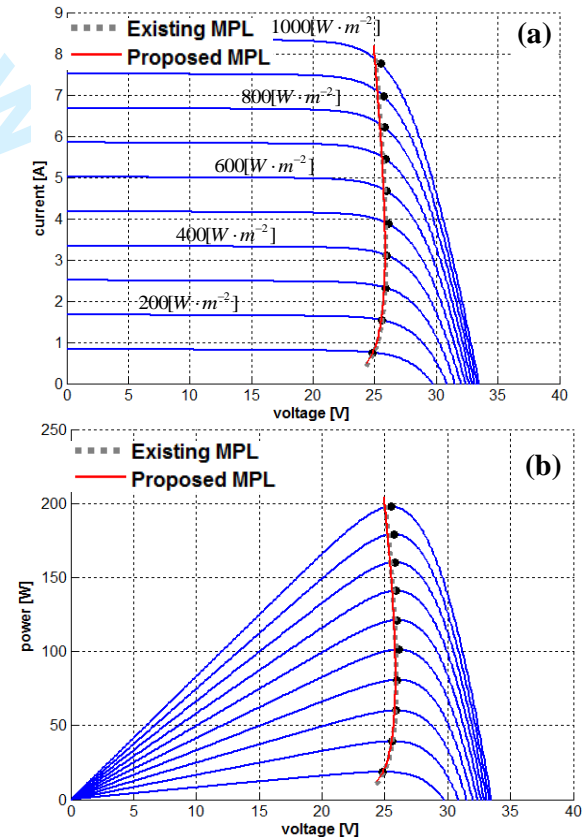

Fig. 5: Families of I-V curves (a) and P-V curves (b) of REC-AE220 for $T = 50$°C with corresponding MPLs. Actual MPPs are indicated by black dots.

IV. CONCLUSION

In this letter, an explicit, yet irradiation independent expression for photovoltaic generator maximum power line was derived, allowing efficient real-time implementation without the need for photocurrent estimation. The results indicate that the derived curve overlaps with the one obtained in earlier works, requiring photocurrent related information, validating the accuracy of the proposed approach.

V. REFERENCES

[1] S. Gadelovits, A. Kuperman, M. Sitbon, I. Aharon and S. Singer, "Interfacing renewable energy sources for maximum power transfer – Part 1: Statics," *Renew. Sust. Energy Rev.*, vol. 31, pp. 501-508, 2014.

[2] S. Kolesnik, M. Sitbon, S. Gadelovits, T. Suntio and A. Kuperman, "Interfacing renewable energy sources for maximum power transfer - Part II: Dynamics," *Renew. Sust. Energy Rev.*, vol. 51, pp. 1771 – 1783, 2015.

[3] T. K. Soon and S. Mekhlief, "A fast-converging MPPT technique for photovoltaic system under fast-varying solar irradiation and load resistance," *IEEE Trans. Ind. Inform.*, vol. 11, no. 1, pp. 176 – 186, Feb. 2015.

[4] A. Kuperman, M. Averbukh and S. Lineykin, "Maximum power point matching versus maximum power point tracking for solar generators," *Renew. Sust. Energy Rev.*, vol. 19, pp. 11 – 17, 2013.

[5] S. Gadelovits, M. Sitbon, T. Suntio and A. Kuperman, "Single-source multi-battery solar charger: case study and implementation issues," *Prog. Photovolt. Res. Appl.*, vol. 23, no. 12, pp. 1916 – 1928, Dec. 2015.

[6] M. Sokolov and D. Shmilovitz, "A modified MPPT scheme for accelerated convergence," *IEEE Trans. Energy Convers.*, vol. 23, no. 4, pp. 1105-1107, Dec. 2008.

[7] V. Scarpa, S. Buso and G. Spiazzi, "Low-complexity MPPT technique exploiting the PV module MPP locus characterization," *IEEE Trans. Ind. Electron.*, vol. 56, no. 5, pp. 1531-1538, May 2009.

[8] G. Farivar, B. Asaei and S. Mahrnami, "An analytical solution for tracking photovoltaic module MPP," *IEEE J. Photovolt.*, vol. 3, no. 3, pp. 1053-1061, Jul. 2013.

[9] G. Farivar and B. Asaei, "A new approach for solar module temperature estimation using the simple diode model," *IEEE Trans. Energy Convers.*, vol. 26, no. 4, pp. 1118-1126, Dec. 2011.

[10] T. Ikegami, T. Maezono, F. Nakanishi, Y. Yamagata, and K. Ebihara, "Estimation of equivalent circuit parameters of PV module and its application to optimal operation of PV system," *Sol. Energy Mat. Sol. Cells*, vol. 67, no. 1–4, pp. 389–395, Mar. 2001.

[11] J. Ding and R. Radhakrishnan, "A new method to determine the optimum load of real solar cells using the Lambert W-function," *Sol. Energy Mat. Sol. Cells*, vol. 92, no. 12, pp. 1566–1569, Dec. 2008.

[12] M. Zagrouba, A. Sellami, M. Boua¨ıcha, and M. Ksouri, "Identification of PV solar cells and modules parameters using the genetic algorithms: Application to maximum power extraction," *Sol. Energy*, vol. 84, no. 5, pp. 860–866, May 2010.

[13] N. S. Singh, A. Jain, and A. Kapoor, "A new method to determine the optimum load of a real solar cell using special trans function theory (STFT)," *Int. J. Renew. Energy Res*., vol. 3, no. 2, pp. 378–382, 2013.

[14] N. S. Singh, A. Jain, and A. Kapoor, "An exact analytical method for calculating the parameters of a real solar cell using special trans function theory (STFT)," *Int. J. Renew. Energy Res*., vol. 3, no. 1, pp. 201–206, 2013.

[15] A. Askarzadeh and A. Rezazadeh, "Extraction of maximum power point in solar cells using bird-mating optimizer-based parameters identification approach," *Sol. Energy*, vol. 90, pp. 123–133, Apr. 2013.

[16] A. Kuperman, "Comments on '–An analytical solution for tracking photovoltaic module MPP'," *IEEE J. Photovolt.*, vol. 4, no. 2, pp. 734-735, Mar. 2014.

[17] E. Batzelis, G. Kampitsis, S. Papathanassiou and S. Manias, "Direct MPP calculation in terms of single-diode PV model parameters," *IEEE Trans. Energy Convers.*, vol. 30, no. 1, pp. 226-236, Mar. 2015.