

# Optimization issues of the broke management system in papermaking

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## Abstract

This paper presents an optimization strategy for the design and operation of a broke management system in a papermaking process. A stochastic model based on a two-state Markov process is presented for the broke system and a multiobjective and bi-level stochastic optimization model is developed featuring (i) a multiobjective operational subproblem for the optimization of the broke dosage, and (ii) a multiobjective design problem formulation. An efficient optimization strategy is proposed for the operational subproblem along with a simulation based Pareto optimal solution for the design problem, and illustrated with a detailed case study.

**Keywords:** process design and control, multiobjective optimization, papermaking process, broke management, stochastic process

## Nomenclature

Symbol	Variable name
$b$	Binary break variable
$c_f$	Filler content deviation
$d$	Design variable at the upper level
$F_Z$	Cumulative distribution of the number of breaks
$h$	Vector of the filler response coefficients
$K_H$	Optimization horizon
$p_0$	Parameter of the accepted overflow risk
$P_0(k)$	Function of the accepted overflow risk $k$ time steps from the present time
$p(b(n))$	Probability of break on or off at time $n$
$q_1$	Transition probability from 0 to 1, i.e. the break probability
$q_2$	Transition probability from 1 to 0
$q_{\max}$	Parameter for the transition probability $q_1$ - the upper limit at high broke dosage
$q_{\min}$	Parameter for the transition probability $q_1$ - the lower limit at low broke dosage
$T_{\text{of}}$	Broke tower overflow time
$u$	Control variable at the lower level, i.e. the broke dosage from the tower to the system
$u_{\text{eff}}$	Effective broke dosage
$u_{\text{hist}}$	Broke dosage history, i.e. previous values

$u_{th}$	Parameter for the transition probability $q_1$ - defines the dosage threshold which considerably increases the break risk
$v_0$	Amount of broke generated per time step during normal run (no break)
$v_1$	Amount of broke generated during a break
$V(n)$	Amount of broke in the tower at time $n$
$V_{max}$	Volume of the broke tower
$VU$	Volume unit
$Z$	Number of breaks
$\alpha$	Scalarization parameter
$\beta$	Scalarization parameter
$\gamma(k)$	Time-wise weighting factor for the objectives
$\Psi$	Collections of the stochastic state variables
$\sigma_w$	Parameter for the transition probability $q_1$ - describes how rapidly the transition from regular break risk to increased break risk occurs

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## 1. Introduction

Papermaking is a complex process in which paper is produced from wood, water and chemicals. The process starts from producing chemical and/or mechanical pulp from wood. The produced pulp is mixed with recycled production and fed to the paper machine together with water, chemicals, and filler. In the paper machine, paper web is formed and water is removed from the web in several subprocesses, the paper web is dried with hot metal cylinders, and finally the surface is finished and end product produced by cutting the web into rolls or sheets. The water

removed is collected and used again. If the paper web breaks during the run, all the production is discarded. The discarded production, called broke, is stored and reused in papermaking. Figure 1 presents a simplified example of the papermaking process, involving broke production.

<Figure 1 >

The papermaking process is clearly affected by web breaks, as breaks cause delays, upset production and typically result in financial penalties. The occurrence of breaks is random; nevertheless, empirical correlation exists relating the occurrence of breaks to the amount of the recycled broke, the broke dosage. Recycling the broke is economically justified and the broke storage towers are designed to overcome such abnormal situations and to enable the process to run smoothly. As the size of the broke tower increases, the operation of the broke volume becomes easier, albeit at the expense of capital costs and sometimes at the expense of problems related to microbial contamination, not considered here.

The focus of this paper is to study such broke management issues as discussed above. At the design level, the objective in broke management is to minimize the capital costs associated with the broke tower, whereas at the operational level, the objective is to optimize the performance of the process by selecting an appropriate level of broke dosage. Given that, due to the randomness of breaks, the broke system is a highly stochastic process, we address the overall broke system problem through a multiobjective stochastic optimization model (see e.g. Clark and Westerberg, 1983; Miettinen, 1999; Chinchuluun, and Pardalos, 2007) which involves two levels – an operational/inner optimization subproblem featuring a stochastic broke tower model, and a design/outer optimization. The problem is formulated in the following form

$$d^* = \arg \min_d \begin{cases} E_{\Psi} (f_1(d, \Psi(0))) \\ \vdots \\ E_{\Psi} (f_N(d, \Psi(0))) \end{cases}$$

s.t.

$$d \in D$$

$$\Psi(n+1) = M(d, \Psi(n), u^*(n))$$

$$u^*(n) = \arg \min_{\{u(n+k)\}_{k=0}^{K_H-1}} \begin{cases} E_{\Psi'} \left( \sum_{k=0}^{K_H-1} \gamma(k) g_1(d, \Psi'(n+k), u(n+k)) \right) \\ \vdots \\ E_{\Psi'} \left( \sum_{k=0}^{K_H-1} \gamma(k) g_N(d, \Psi'(n+k), u(n+k)) \right) \end{cases}$$

s.t.

$$\Psi'(n+k) = M'(d, \Psi'(n+k-1), u(n+k-1))$$

$$\Psi'(n) = \Psi(n)$$

$$u \in U$$

$$\Psi' \in \Psi_{range}$$

(P)

where  $d$  is a design variable,  $u$  is a control variable at the operational level  $K_H$  time steps ahead,  $M$  is a nonlinear, stochastic state model,  $\Psi$  and  $\Psi'$  are collections of the stochastic state variables,  $\gamma(k)$  is a time-wise weighting factor for the objectives, and  $f$  and  $g$  are the objective functions for design and operational levels, respectively. In the broke management case, tower volume is an example of the design variable  $d$  and broke dosage an example of the control variable  $u$ ,  $M$  describes the volume dynamics, and break status and current broke volume in the tower are examples of the stochastic state variable  $\Psi$ . Capital cost of the broke tower is an example of the design objective  $g$  and squared deviation from the filler content target an example of the operational objective  $f$ .  $E_{\Psi}$  and  $E_{\Psi'}$  represent the expected values of the objective functions with respect to  $\Psi$  and  $\Psi'$ . Note that (P) corresponds to a multiobjective stochastic simultaneous design and operational optimization problem.

Integrated design and operational problems have been widely studied in the open literature, for example see the works of Bansal (2000) and Sakizlis (2003) and the literature provided within - see also Table 1 where general methods and algorithms for the solution of different classes of (P) are discussed in detail.

<Table 1>

In papermaking, integrated design and operation has not been widely studied, although some studies in optimization of the decision making have been done over the years - see Table 2 for an overview of some of the representative works done in the area.

<Table 2>

One of the key challenges of the design/operational problem in the broke management is the presence of stochastic/random events in the model, i.e. the web breaks, having a huge effect on the operation of the system through the broke accumulation and dosage. This, as will be discussed later in this paper, makes the solution of problem (P) more complex than the existing methods (as outlined in Table 1). The objective of this paper is to develop and present an efficient solution strategy for problem (P) in the broke management.

The paper is organized as follows. In section 2, the broke management models are described, whereas an optimization solution strategy for both the operational and design problems are presented in sections 3 and 4. Examples are used throughout to demonstrate some key features of the proposed approach. In section 5, the robustness of the design solution with respect to the highly uncertain web break model is studied. Finally, section 6 provides a summary of the key points and highlights future/ongoing research directions.

## 2. Broke system

Broke management is a process in which the discarded production is stored into a broke tower and dosed for reuse back to the process (Paulapuro, 2008). Economically it is important to reuse the broke, although the reuse may cause disturbances to the process. Disturbances are due to the material composition of the broke, which can be different from the composition of the fresh pulp. Furthermore, the pulp in the broke has been dried at least once and the properties are somewhat inferior to the virgin pulps. Thus, the higher the broke dosage is the higher the probability for a new break is. Another challenge in broke management is that the dosage of the broke affects the end product quality. Recycled broke contains chemicals and filler: thus broke increases the filler and chemical content of the mixed pulp. Quality variations can be controlled with feedback actions, but fast changes of the broke dosage cause transient deteriorations in the quality of the paper produced. A third challenge in broke management is to prevent the broke tower overflow.

### 2.1. Broke tower model

A broke tower is a tank in which the broke is stored. Discarded production from the process is fed to the tower, and stored broke from the tower is dosed back to the process (Figure 2).

<Figure 2>

The discrete-time dynamics of the tower can be described as

$$V(n+1) = V(n) - u(n) + (1 - b(n))v_0 + b(n)v_1 \quad (1)$$

where  $V(n)$  is the amount of broke in the tower at time  $n$ ,  $u(n)$  is the dosage from the tower to the system,  $v_0$  is the amount of broke generated per time step when there is no break, and  $v_1$  is the amount of broke generated during a break ( $v_1 \gg v_0$ ),  $b(n)$  being a binary break variable as

$$b(n) = \begin{cases} 1 & \text{if a break is on at time } n \\ 0 & \text{else} \end{cases} \quad (2)$$

Table 3 presents an example of typical process values with respect to the nominal values used in this case study.

<Table 3>

As breaks are random events, they cannot be predicted and therefore break instants are not known in advance. However, the probabilities of breaks can be described as a two-state Markov chain

$$\begin{bmatrix} p(b(n+1)=0) \\ p(b(n+1)=1) \end{bmatrix} = \begin{bmatrix} 1 - q_1(u_{eff}(n)) & q_2 \\ q_1(u_{eff}(n)) & 1 - q_2 \end{bmatrix} \begin{bmatrix} p(b(n)=0) \\ p(b(n)=1) \end{bmatrix} \quad (3)$$

where  $p(b(n)=0)$  and  $p(b(n)=1)$  are the probabilities of break on and off at time  $n$ ,  $q_1(u_{eff}(n))$  is a transition probability per time step from 0 to 1, i.e. the break probability, and  $q_2$  is a constant transition probability from 1 to 0.

Many attempts have been made to describe the reasons of web breaks (see e.g. Roisum,1990; Orcotoma et al., 1997; Dabros et al., 2005; Ahola, 2005). Here we apply a simple mechanistic idea that broke weakens the web and increases the risk of breaks. That is firstly because the fibers in broke are weaker than the corresponding virgin fibers, and secondly because broke has chemical and microbiological disturbances increasing the risk of breaks. The model is constructed such that it leads to the “vicious circle of breaks”: as breaks occur and broke tower starts to fill up, higher amount of broke must be dosed to the system which then increases the risk of further breaks. Thus, the break probability model  $q_1$  assumed to be known in operational and design optimization transition probability is expressed as



$$q_1(u_{eff}(n)) = q_{min} + \frac{q_{max} - q_{min}}{1 + \exp\left(-\frac{u_{eff}(n) - u_{th}}{\sigma_w}\right)} \quad (4)$$

where  $u_{th}$  defines the threshold which the dosage considerably increases the break risk,  $\sigma_w$  describes how rapidly the transition from regular break risk to increased break risk occurs as a function of dosage,  $q_{min}$  is the break probability at low broke dosage,  $q_{max}$  is the upper limit at very high dosage, and  $u_{eff}$  is an effective dosage defined as

$$u_{eff}(n) = \sum_{i=1}^{\infty} s(i)u(n-i) \quad \sum_{i=1}^{\infty} s(i) = 1 \quad (5)$$

where  $s$  is a vector of coefficients defining the dynamics between the dosage and the break probability. Figure 3 shows an example of the break probability curve as a function of the effective dosage. In reality the break probability model (Eq. 4) is based on rather crude empirical knowledge and thus the parameters of the model are significantly uncertain. Thus, robustness of the solution with respect to break model parameters must be analyzed.

<Figure 3>

Based on Eq. (1), the volume of broke in the tower at time  $n$  can be calculated as

$$V(n) = V(0) + nv_0 - \sum_{n'=0}^{n-1} u(n') + Z(n, b(0))(v_1 - v_0) \quad (6)$$

where  $Z(n, b(0))$  is the number of breaks after  $n$  time steps when the initial break state has been  $b(0)$ . As breaks are random,  $Z$  is not known in advance. However, by knowing  $q_1$ ,  $q_2$ , and  $b(0)$ , the distribution of  $Z$  can be calculated by repeated iteration of Eq. (3). Figure 4 shows the effect of the initial state  $b(0)$  to the distribution of  $Z(n, b(0))$  for four alternative time horizons. It can

be seen that for short horizons the initial state has a remarkable effect, but as the horizon increases the effect becomes less important.

<Figure 4>

## 2.2. Paper quality considerations

The dosage of the broke affects the end product quality. Recycled broke contains chemicals and filler, thus broke increases the filler and chemical content of the mixed pulp. The dosage of the new chemicals and filler can be controlled, but as the quality of the paper is measured from the end product, there is a delay in such a feedback. The recycled broke is mixed with fresh pulp and the mixed mass flows through several tanks and subprocesses. Therefore the changes in the broke dosage cause transient disturbances to the end product as the control cannot be adjusted quickly enough to the new conditions.

In this study, filler content variation of the end product is used as an indicator of the paper quality. The filler content deviation is defined as a transient/impulse with coefficients that sum up to zero as

$$c_f(n) = \sum_{i=1}^{\infty} h(i)u(n-i) \quad \sum_{i=1}^{\infty} h(i) = 0 \quad (7)$$

where  $h(n)$  is the vector of the filler response coefficients chosen to correspond to typical closed loop filler dynamics on paper machines.

## 3. Broke management optimization - operational level

In broke management optimization, the target is to optimize the schedule of the broke dosage back to the papermaking system. The objectives in broke management are to minimize the

probability of the broke tower overflow (i), and to maximize the effective production time (ii), the uniformity of quality of the end product (iii), and the smoothness of the broke dosage (iv). To overcome the delay between the control and its effect, and to avoid the broke tower overflow, decisions should be made further down in the time horizon taking into account the future behavior. The operational problem is to optimize the schedule of the recycled broke dosage  $K_H$  time steps ahead and can be formulated as follows.

$$\begin{aligned}
 & \min_{\{u(n+k)\}_{k=0}^{K_H-1}} \left\{ \begin{array}{l}
 \max_{k=1, \dots, K_H} \left[ \frac{p(V(n+k) > V_{\max})}{P_0(k)} \right] \quad \text{(i)} \\
 \sum_{k=1}^{K_H} \gamma(k) q_1(u_{\text{eff}}(n+k)) \quad \text{(ii)} \\
 \sum_{k=1}^{K_H} \gamma(k) c_f(n+k)^2 \quad \text{(iii)} \\
 \sum_{k=1}^{K_H} \gamma(k) (u(n+k-1) - u(n+k))^2 \quad \text{(iv)}
 \end{array} \right. \\
 & \text{s.t. } P(V(n+k) < 0) = 0 \quad k = 1, \dots, K_H \\
 & \quad 0 \leq u(n+k) \leq u_{\max} \quad k = 1, \dots, K_H \\
 & \text{given } V(n), b(n), \{u(k)\}_{k=-\infty}^{n-1}
 \end{aligned} \tag{8}$$

where  $V_{\max}$  is the volume of the broke tower, i.e. the maximum amount of broke in the tower,  $P_0(k)$  is a function of accepted risk of an overflow  $k$  time steps from the present time  $n$ , and parameter  $\gamma(k)$  is a time-wise weighting factor for the objectives treated as a degree of freedom. For rest of the notations see section 2. In Eq. (8), the objective of the broke tower overflow (i) is considered as a min-max optimization of the probabilities over the optimization horizon. The objective of the effective production time (ii) can be seen as a minimization of the time within breaks and is thereby considered as a minimization of the break probability  $q_1$ . The objective of the uniformity of quality (iii) is considered as a minimization of the filler content variation, and

the smoothness of the dosage (iv) as a minimization of the difference of two consecutive dosages.

The operational optimization problem (Eq. 8) can be viewed as an example of a stochastic receding-horizon model predictive control (see e.g. Rawlings and Mayne, 2009; Maciejowski 2002; Findeisen et al., 2007) with multiple objectives, in which the control is optimized at each time step for a certain horizon and the first value is applied. Multiobjective model predictive control problems has been studied e.g. by Bemporad and Muñoz de la Peña (2009) and De Vito and Scattolini (2007).

In this paper, as the operational problem – being a subproblem of the design – must be solved online without a decision maker, the multiobjective problem has been chosen to be reformulated into a single objective optimization. That has been done by introducing parameters indicating the relative importance of the objectives (ii), (iii) and (iv), and considering the probability of broke tower overflow (i), as a parameterized constraint. As high dosage of broke increases the probability of a new break ( $q_1$ ), the objective of the maximization of the effective production time (ii), is considered as a penalty term for high dosages. This leads to the following single objective optimization problem

$$\begin{aligned} \{u^*(n+k)\}_{k=0}^{K_H-1} &= \arg \min_{\{u(n+k)\}_{k=0}^{K_H-1}} \sum_{k=1}^{K_H} \gamma(k) \left( c_f(n+k)^2 + \alpha(u(n+k) - u(n+k-1))^2 + \beta u(n+k-1)^2 \right) \\ \text{s.t. } P(V(n+k) > V_{\max}) &< 1 - (1 - p_0)^k \equiv P_0(k) \quad k = 1, \dots, K_H \\ P(V(n+k) < 0) &= 0 \quad k = 1, \dots, K_H \\ 0 \leq u(n+k) &\leq u_{\max} \quad k = 1, \dots, K_H \end{aligned} \quad (9)$$

where  $\alpha$ , and  $\beta$  are scalarization parameters (see e.g. Miettinen, 1999) required to transform the multiobjective problem into a single objective problem, and  $p_0$  is a parameter for the accepted

overflow risk. These parameters are considered as degrees of freedom at the design level. The objective function can now be reformulated in a quadratic form e.g. as

$$\{u^*(n+k)\}_{k=0}^{K_H-1} = \arg \min_{\{u(n+k)\}_{k=0}^{K_H-1}} u^T G u + c^T u \quad (10)$$

The constraint of the broke tower overflow in Eq. (9) can be rewritten as

$$-\sum_{k=0}^{K_H-1} u(n+k) \leq V_{\max} - V(n) - k v_0 - (v_1 - v_0) F_{Z(k,b(n))}^{-1} (1 - P_0(k)) \quad (11)$$

where  $F_Z$  is the cumulative distribution of  $Z$ . Thus, for known  $F_Z$  the operational optimization turns into quadratic programming (QP) form as the objective function is quadratic and the constraints are linear. However, as shown earlier in section 2, the distribution of  $Z$  depends on the transition probability  $q_1$ , which depends on the broke dosage  $u$  which is the optimization variable. To overcome this, the distribution of  $Z$  is approximated using the optimized broke dosage from the previous time as an initial guess. By using this approximation,  $Z$  and thus  $F_Z$  can be calculated and the problem solved as a quadratic optimization problem. To obtain more exact result, the procedure is repeated by calculating the distribution of  $Z$  for optimized  $u^*$  and optimizing the dosage again. That procedure is repeated until the optimized dosage is stable i.e.  $\max |u_j^* - u_{j-1}^*| < \varepsilon$ . The algorithm for the operational optimization can be summarized as follows.

**Step 1.** Define an initial guess of  $u_0(n) \dots u_0(n+K_H-1)$ , and set  $j = 1$

**Step 2.** Calculate  $F_Z$

**Step 3.** Solve the linearly constrained QP problem (Eq. 10-11) and obtain  $u_j^*(n) \dots u_j^*(n+K_H-1)$

**Step 4.** If  $\max |u_j^* - u_{j-1}^*| < \varepsilon$  go to step 5, otherwise  $j = j + 1$  and go to step 2

**Step 5.** Apply  $u_j^*(1)$  to the process

The algorithm seems to converge well. If it does not, and the solution keeps oscillating between two solutions, either one of the solutions is chosen. The effect of a single non-optimal solution is not significant as the optimization is repeated at the next time step.

Figure 5 shows an example of the operational optimization. In this example, the effect of the initial break state was studied by solving the same optimization problem for initial break states  $b(n) = 0$  (top) and  $b(n) = 1$  (bottom). In both cases, the current amount of broke in the tower was 275 volume units the maximum tower volume being 400 volume units. In the case  $b(n) = 0$ , the optimized broke dosage schedule is lower for the entire horizon than in the case  $b(n) = 1$ . That is because during a break it is reasonable to raise the dosage to prevent the tower overflow in the future, as the break might last for several time steps. If there is not a break, it is reasonable to keep the dosage lower to ensure lower break probability.

<Figure 5>

As the operational problem reformulates into a quadratic problem, it can be solved online and model predictive control procedure can be applied to optimize the dosage of the recycled broke. To test the controller, the system can be simulated by running the optimization together with a simulator as shown in Figure 6. The current states of the process ( $\Psi_n = [V(n), u_{hist}, b(n)]$ ) are sent to the optimizer and the QP problem is solved and the optimized dosage  $u^*$  for the horizon obtained. The first value  $u^*(1)$  is applied to the simulator and the states of the simulator are updated based on that.

<Figure 6>

In Figures 7 and 8, two examples of the broke tower model simulation are presented. Both simulations were started from the same initial states  $V(0)=0$ ,  $b(0)=0$ , and dosage history  $u_{hist}$ , and ran until an overflow occurred. Due to the randomness of breaks, the simulation profiles differ significantly from each other. That shows the key challenge of the broke management: the number of breaks, overflow time and filler content variation all vary from one run to another, even when the process is operated from the very same initial state.

<Figure 7>

<Figure 8>

#### 4. Broke system optimization - design level

The main target in the broke system design is to optimize the size of the broke tower. That is to find a trade-off between the investment cost of the tower and the performance of the process (objectives i-iv). As the process is stochastic, the performance is described as an expected behavior and the design optimization takes a form

$$\min_{V_{\max}} \begin{bmatrix} H(V_{\max}) \\ -E_{\Psi}\{T_{of}\} \\ E_{\Psi}\{q_1\} \\ E_{\Psi}\{c_f^2\} \\ E_{\Psi}\{(u(n+1) - u(n))^2\} \end{bmatrix} \quad (12)$$

*s.t.*  $V_{\max} \geq 0$

where  $H(V_{\max})$  is the investment cost of a tower volume  $V_{\max}$ , and  $T_{of}$  is the broke tower overflow time as  $T_{of} = \min_n \{n | V(n) > V_{\max}\}$ .  $E_{\Psi}\{\}$  denotes the expectation value of the system performance

as  $\Psi$  is the stochastic process with applied dosage policy. For the optimization of the performance, also the scalarization parameters  $\alpha$  and  $\beta$ , and the accepted overflow risk  $p_0$  from the operational problem (Eq. 9) are degrees of freedom. Therefore the design provides the relative importance of the operational objectives, to be applied uniformly in all operational situations. Thus, design determines the operational policy concurrently with the process structure, i.e. broke tower volume.

Although the wide range of publications in the area of integrated design and control (see section 1), the proposed methods cannot be directly applied to our study. That is due to the remarkable uncertainty of the system studied. As the uncertainty in broke management is not constant, but the probability of a break depends on the previous dosages of the recycled broke, the broke system case differs significantly from the cases presented in the literature. The key challenge in the design optimization is that as the uncertainty results from the dynamics at the operational level, the probability distributions are not known in advance. Thus, the problem is, how to optimize the expected performance of the process, i.e. the overflow time, the filler content variation, and the time within breaks, when the operation and the uncertainties are not known.

In this study, a simulation based algorithm is used to obtain the probability distributions of the design objectives. The expected behavior of each design option is obtained by running the tower model simulation together with the operational optimization several times from the same initial conditions. The overflow time of the tower, the filler content variation of the end product per time unit, and the time within breaks per time unit during the simulations are collected, and the expected behavior is calculated as a mean of these simulations. The procedure to calculate the expected performance for a design can be seen in Figure 9.



<Figure 9>

In Figure 10, histograms of the tower overflow time are presented for tower volumes 400 and 600 volume units. Histograms were calculated running the simulator 100 times from the same initial conditions. The mean values of the two cases were 1000 and 1100 time units, with standard deviations as 732 and 716 units, respectively. It can be seen that the histograms resemble exponential distribution, though Weibull distribution might be conceivable.

<Figure 10>

To choose the optimal design, the procedure presented in Figure 9 must be repeated for each of the design alternatives. The design solutions with respect to the objectives can then be presented to the decision maker and the most preferred design selected from the Pareto optimal (see e.g. Miettinen, 1999) set of designs according to the decision maker's assessment.

As multiobjective solutions with dimension higher than three are difficult to present to the decision maker, the result is chosen to be presented with a subsystem approach and the most preferred design solution selected using a similar method presented by Engau and Wiecek (2007; 2008). The selection is based on defining the order of importance of the objectives. The simulated set of design solutions are first presented to the decision maker with respect to the two most important objectives. The decision maker chooses the most preferable solution amongst these primary objectives, and the chosen solution is shown in the second plot with respect to the secondary objectives. After the initial choice, a second round brute force optimization step is carried out by generating a large set of new designs in a neighborhood of the design chosen in the first round. This time the decision maker chooses a set of most preferable solutions amongst the most important objectives. The chosen set of solutions are then shown in respect of the less

important objectives. The final solution can be then chosen from the Pareto optimal set amongst these secondary objectives. The proposed solution algorithm is summarized as follows.

**Step 1.** For a finite set of design options

a) Simulate the process operation (with online optimization) several times for each design starting from the same initial state until an overflow occurs

b) Calculate the means of the overflow time, the filler content variation, and the time within breaks

**Step 2.** Present the solutions with respect to the two primary objectives. Choose an initial design.

**Step 3.** Generate a large set of new design solutions in the neighborhood of the initial choice.

**Step 4.** Choose the acceptable solution range, and plot with respect to the secondary objectives.

**Step 5.** Choose the final design from the Pareto curve or generate new set design solutions.

Figure 11 shows an example of a set of 126 designs ( $V_{\max}, \beta, p_0$ ) in which 20 simulations were carried out for each design and the expectation value was calculated as the mean of the 20 observations. In this example, 7 discrete values were set for the maximum tower volume  $V_{\max}$ , 6 discrete values for the parameter  $\beta$ , and 3 discrete values for the parameter  $p_0$ . The goal of smooth dosage was removed from design considerations, and the corresponding parameter  $\alpha$  was determined intuitively. The design solutions are first presented with respect to investment cost and the overflow time (Figure 11 top). The decision maker's initial choice is marked by a circle. The same solution set is also presented with respect to the filler content variation and time within breaks and the same design marked with a circle. Then a new set of 144 design solutions were

generated in the neighborhood of the initial design, and an acceptable range of solutions chosen in respect of the investment cost and overflow time (Figure 12). The final solution can be chosen amongst the Pareto optimal curve with respect to the filler content variation and time within breaks (Figure 13).

<Figure 11>

<Figure 12>

<Figure 13>

## 5. Robustness of the solution

The results in sections 3 and 4 rely on the assumption that the break probability model (see Eq. 4 and Figure 3), i.e. how the dosage affects the break probability, is known to the decision maker. In reality that is not the case, as usually there is significant uncertainty about such a model. To examine the robustness of the solution, design simulations were carried out using a different break probability function for the simulator than for the optimizer (Figure 14). The break model in the simulator is denoted by  $S_r$  (real) whereas that assumed by the optimizer is denoted by  $S_o$  (optimizer).

<Figure 14>

Three break probability models were cross-studied, thus 9 simulation sets for each design option were carried out. The parameters of the cross-studied break probability model are presented in Table 4.

<Table 4>

In Table 5, the results of the cross-studied robustness simulation for one design are collected. It can be seen that if a probability model ‘Low’ is used for the optimizer but the real (simulator) probability is ‘High’, the mean overflow time is more than 50 time units lower and the mean filler variation more than 0.04 units higher compared to the case where the probability model ‘High’ is used for the optimizer. That is result of a tighter broke dosage policy in the case ‘High/High’ ( $S_r/S_o$ ) compared to the case ‘High/Low’ ( $S_r/S_o$ ). If the break probability is high it is reasonable to keep the dosage higher to prevent the tower overflow. On the other hand, the time within breaks is a bit higher in that case as the tighter dosage policy increases the risk of a break. The same occurrence can be seen in the other cases.

<Table 5>

To examine the effect of the break probability model, robustness was studied for a wide range of designs. Figure 15 presents the change in the performance if the optimization is done based on the break probability ‘Low’, but the true probability is ‘High’. The Pareto optimal set of designs is presented in a three-dimensional space of investment cost, mean overflow time, and mean filler content variation. The black triangles in Figure 15 represent the Pareto optimal solutions for the case ‘Low/Low’ ( $S_r/S_o$ ). The white circles represent the performance of the same designs for the case ‘High/Low’ ( $S_r/S_o$ ). It can be seen how the performance a Pareto optimal design falls if the true break probability is ‘High’ compared to the assumed case ‘Low’. Figure 16 presents a similar study for the cases ‘High/Low’ and ‘High/High’. In that case the difference between the solutions shows how the performance changes if the model is changed. That can be seen as an example of a model error.

<Figure 15>

<Figure 16>

Based on the results in Table 5 and Figures 15 and 16, it can be seen that it is reasonable to overestimate the break probability, rather than underestimate. If a too low break probability model is chosen, the performance can be remarkably lower, as the overflow time decreases and filler content variation increases. On the other hand, if a too high break probability model is chosen the number of breaks increases.

## **6. Conclusion and discussion**

In this paper, we have formulated the broke system design problem as a stochastic multiobjective bilevel optimization with design and operational levels. For the operational level we have presented an efficient strategy to solve the problem online without a decision maker by formulating the problem in a quadratic programming form and using scalarization of the multiple objectives. For the design problem we have presented a straight forward method to generate Pareto optimal solutions through simulations such that the scalarization parameters of the operational problem are additional degrees of freedom of the design problem. The proposed strategy is based on an assumption that the break probability is known to the decision maker. As that is not the case in practice, we have studied the robustness of the solution by cross-studying different break probability cases.

At the design level the tower volume and the scalarization parameters of operational problem are optimized. However, as only the tower volume needs to be fixed at the design stage, the scalarization parameters can be changed afterwards according to the current process state and targets. That can be done based on the operational decision maker's opinion. One option is to

define a Pareto optimal subset such that within the subset the tower volume is fixed but the scalarization parameters can be varied. That gives freedom to the decision maker to choose the most preferred operational solution according to the current process state and needs, and the operation is Pareto optimal from the design perspective. In our future work we shall explore this opportunity to delegate decision making to the operational level rather than fixing the operational objective in design.

In our future studies, the strategies presented in this paper are going to be expanded to cover the entire material flow in papermaking, especially the optimization of the four pulp/water tower operation and designs. This will be based on the operational and design optimization strategies presented in this paper.

## **7. Acknowledgements**

The main part of this study was carried out during the visit of Aino Ropponen to Imperial College, Centre for Process Systems Engineering (CPSE). CPSE is gratefully acknowledged for hosting the visit. The study has been financed by Forestcluster Ltd and the Finnish Funding Agency for Technology and Innovation (TEKES), which is gratefully acknowledged.

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## Figures

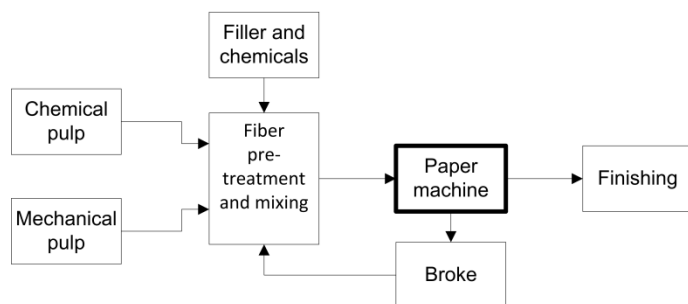


Figure 1. A simplified example of the papermaking process.

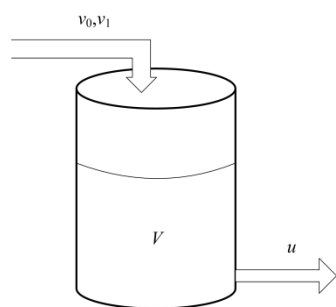
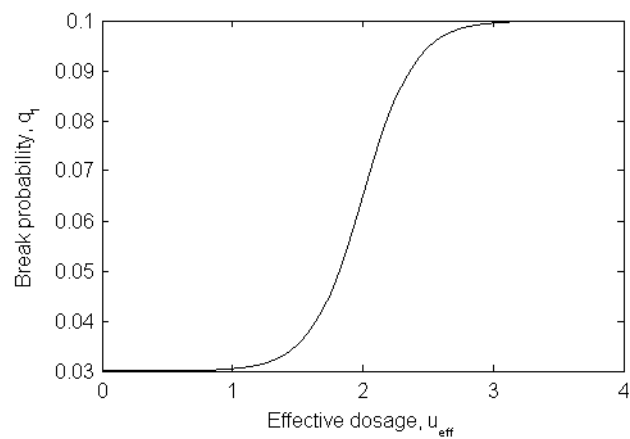


Figure 2. Broke tower.

Figure 3. Break probability as a function of the effective dosage ( $u_{th}=2$ ,  $\sigma_w=0.2$ ,  $q_{min}=0.03$ ,  $q_{max}=0.1$ ).

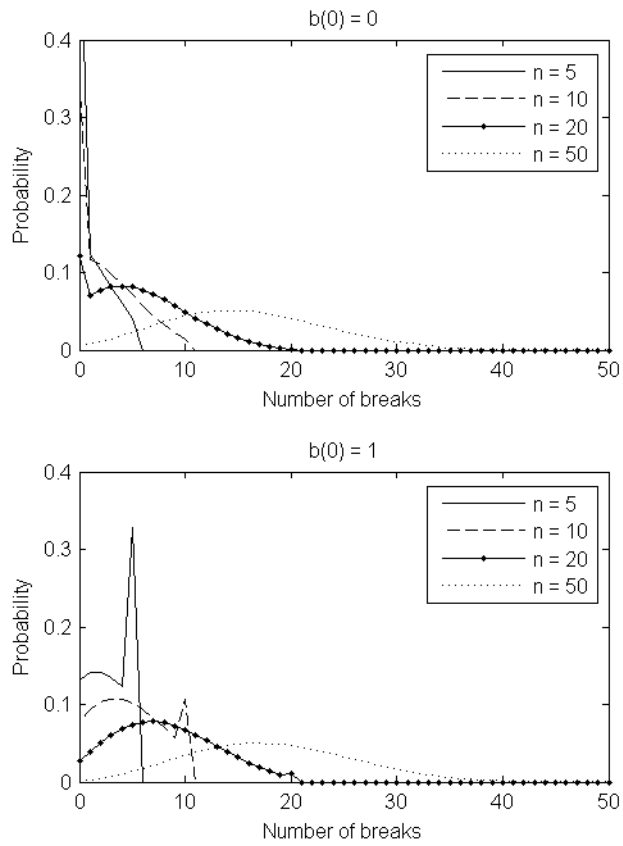


Figure 4. The distribution of the number of break instants for different time horizons ( $n$ ) using constant break probability  $q_1=0.1$ . The initial states are  $b(0)=0$  (top) and  $b(0)=1$  (bottom), and the probability that break ends is  $q_2=0.2$ .

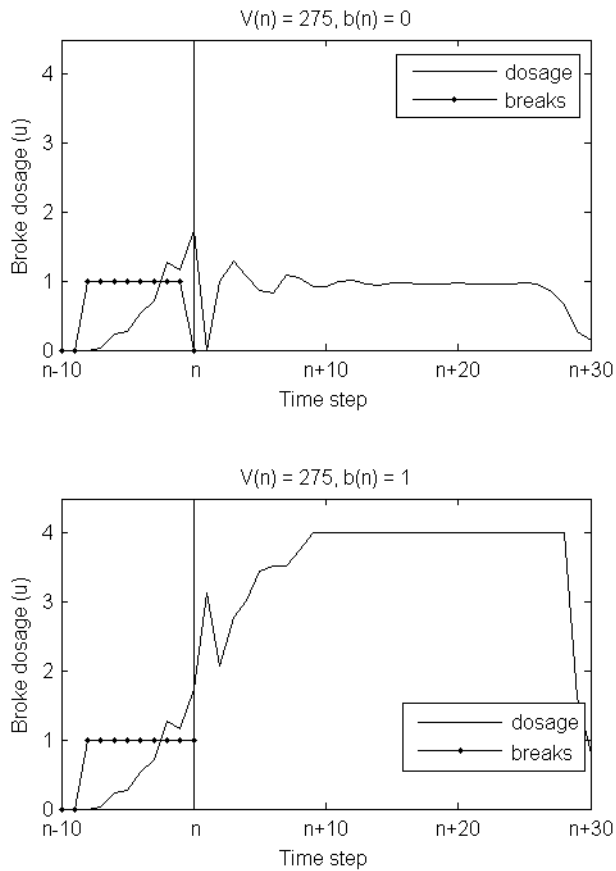


Figure 5. Two different optimization results with the same dosage history until  $u(n)$  and volume  $V(n) = 275$  volume units ( $V_{\max} = 400$  VU,  $K_H = 30$ ,  $\alpha = 0.01$ ,  $\beta = 0.01$ ,  $p_0 = 0.001$ ,  $u_{th} = 2$ ,  $\sigma_w = 0.2$ ,  $q_{min} = 0.03$ ,  $q_{max} = 0.1$ ,  $q_2 = 0.2$ ,  $u_{max} = 4$  VU,  $\gamma(k) = 0.99^k$ ). The upper figure is representing the optimized dosage schedule at the time  $n$  for the initial state  $b(n) = 0$  (no break) and the lower figure for the initial state  $b(n) = 1$  (break).

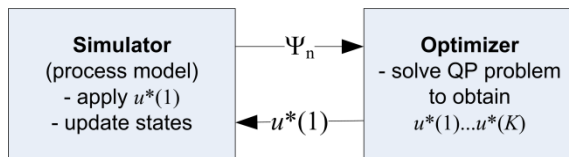


Figure 6. System can be simulated by running the optimization in parallel with the simulation

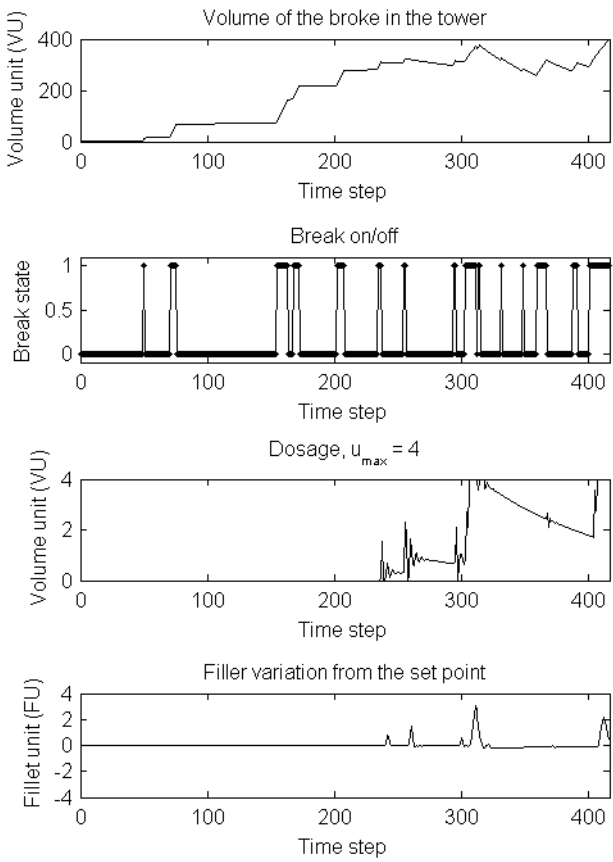


Figure 7. An example of a broke tower simulation until an overflow occurs ( $V(0) = 0$ ,  $V_{\max} = 400$  VU,  $K_H = 30$ ,  $\alpha = 0.1$ ,  $\beta = 0.01$ ,  $p_0 = 0.01$ ,  $u_{th} = 2$ ,  $\sigma_w = 0.2$ ,  $q_{min} = 0.03$ ,  $q_{max} = 0.1$ ,  $q_2 = 0.2$ ,  $u_{max} = 4$  VU,  $\gamma(k) = 0.99^k$ ).

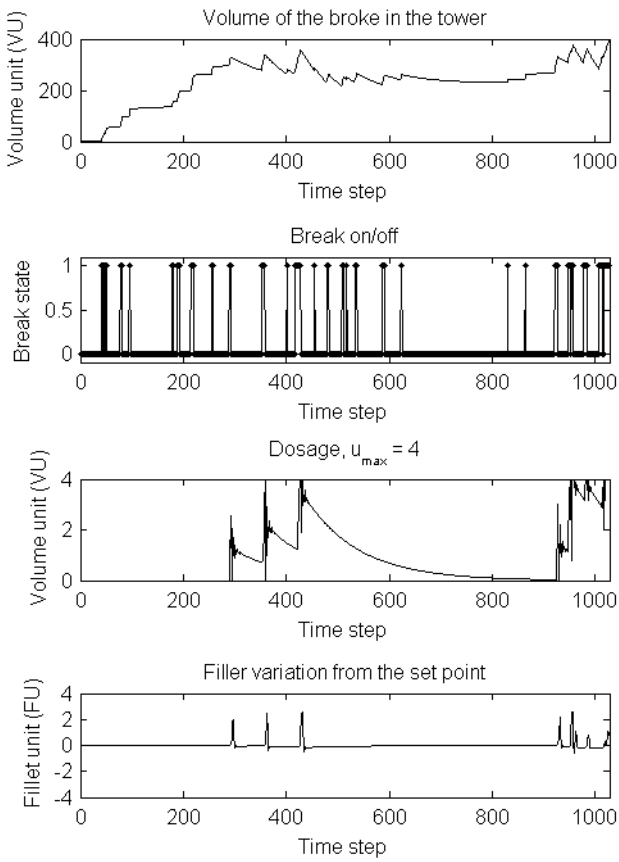


Figure 8. An example of a broke tower simulation starting from the same initial state as in Figure 7 ( $V(0) = 0$ ,  $V_{\max} = 400$  VU,  $K_H = 30$ ,  $\alpha = 0.1$ ,  $\beta = 0.01$ ,  $p_0 = 0.01$ ,  $u_{th} = 2$ ,  $\sigma_w = 0.2$ ,  $q_{min} = 0.03$ ,  $q_{max} = 0.1$ ,  $q_2 = 0.2$ ,  $u_{max} = 4$  VU,  $\gamma(k) = 0.99^k$ ), but ending up in a remarkable different solution.

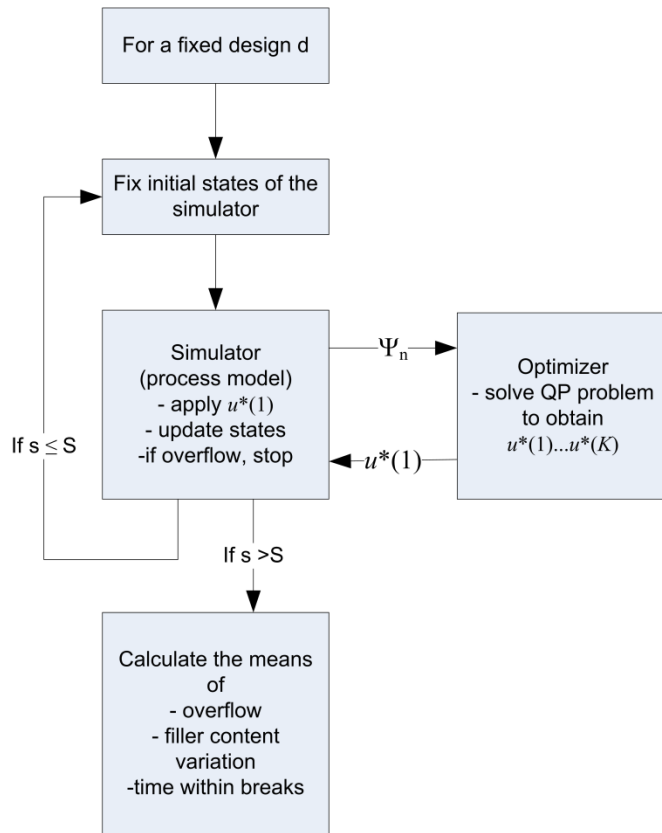


Figure 9. The design procedure to be repeated for the each design option  $S$  being the number of repetitions to calculate the means.



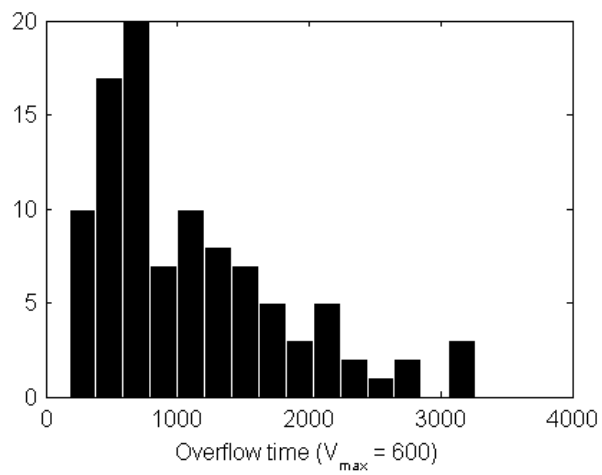
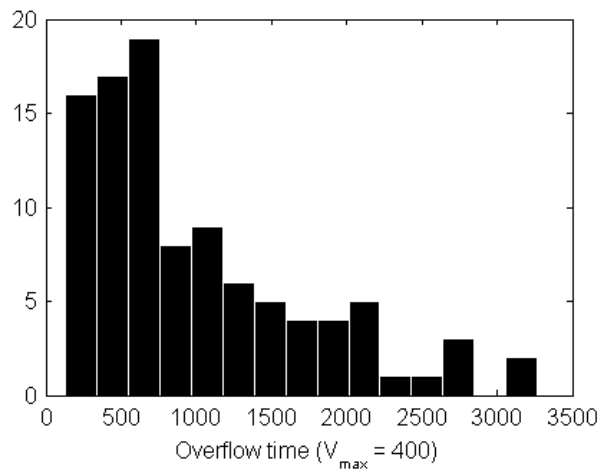


Figure 10. Histograms of the tower overflow time for the tower volumes 400 (top) and 600 (bottom) volume units.

Histograms calculated running simulation 100 times from the same initial conditions.

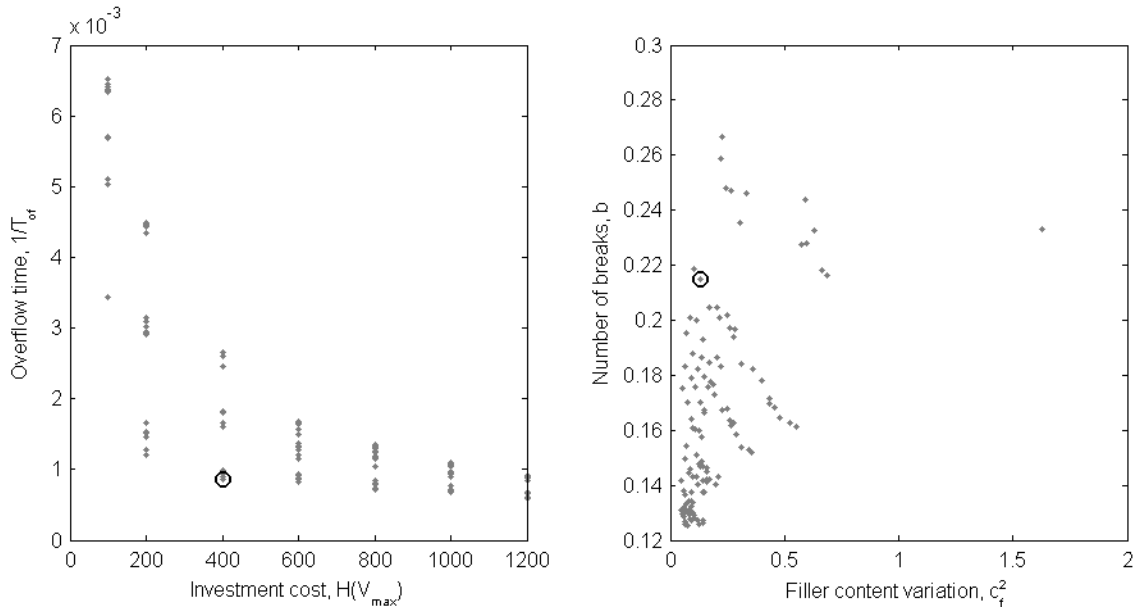


Figure 11. An example of the simulation results of 126 design options ( $V_{\max}=[100\ 200\ 400\ 600\ 800\ 1000\ 1200]$ ,  $\beta=[0.01\ 0.02\ 0.03\ 0.05\ 0.08\ 0.1]$ ,  $p_0=[0.1\ 0.01\ 0.001]$ ,  $\alpha=0.1$ ) shown in two figures. The set of designs in respect to the investment cost and the tower overflow (top), and the filler content variation and the number of breaks (bottom). The decision maker's choice is marked with a circle.

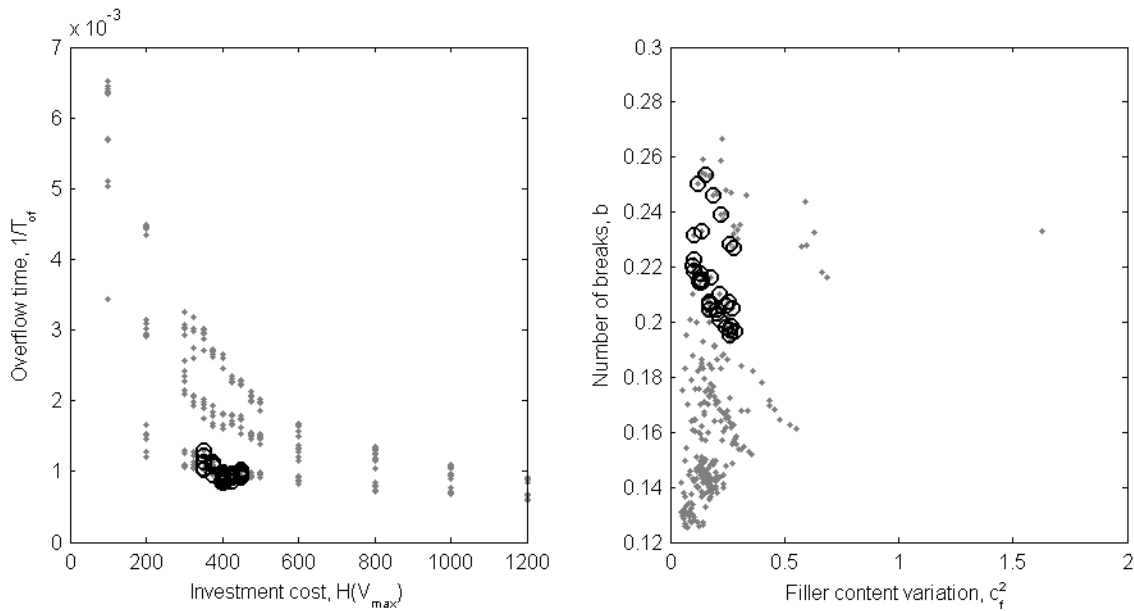


Figure 12. A new set of 144 design solutions generated in the neighborhood of the chosen design ( $V_{\max}=[300\ 325\ 350\ 375\ 425\ 450\ 475\ 500]$ ,  $\beta$ ,  $p_0$ , and  $\alpha$  as before). The decision maker's choices are marked with a circle.

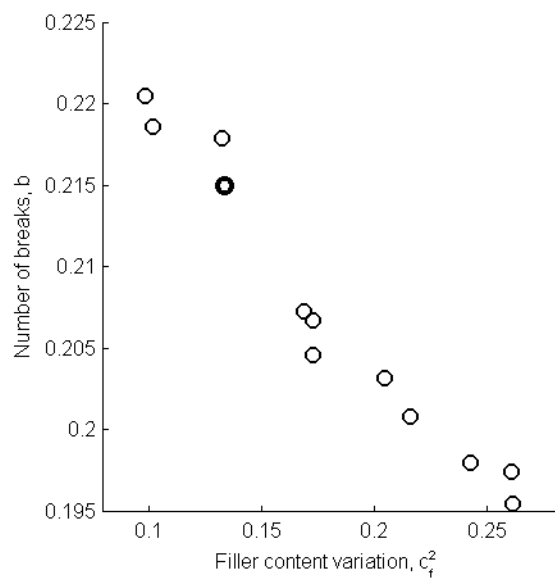


Figure 13. The final design should be chosen amongst these.

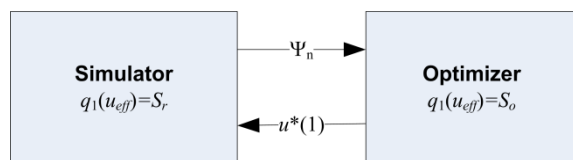


Figure 14. Robustness of the solution was studied using distinct break probability models for the simulator and the optimizer.

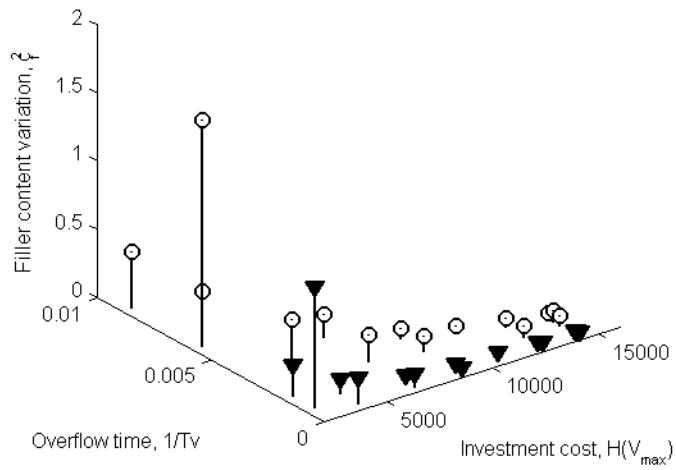


Figure 15. Black triangles represent the Pareto optimal solutions for the case ‘Low/Low’. White circles represent the performance in the same designs for the case ‘High/Low’. Lines are the projections to the filler plane.

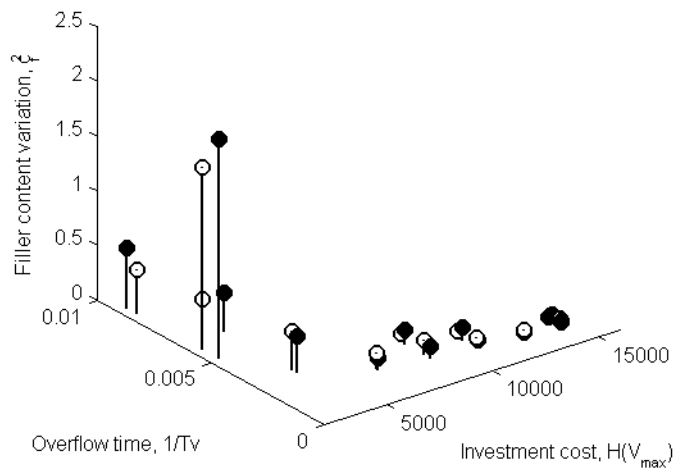


Figure 16. White circles represent the Pareto optimal solutions for the case ‘High/Low’. Black circles represent the performance in the same designs for the case ‘High/High’.

## Tables

Table 1. Integrated design and operational optimization

Authors	Work outline
Bregel and Seider (1992)	Strategy for coordinated design and control optimization of nonlinear processes by simulating MPC with disturbance scenarios.
Luyben and Floudas (1994)	Multiobjective mixed-integer nonlinear optimization formulation of process design alternatives solved with $\varepsilon$ -constraint method and Generalized Benders decomposition.
Pistikopoulos and Ierapetritou (1995)	Formulation of a two-stage approach for design optimization with uncertain parameters.
Perkins and Walsh (1996)	Process design and optimal control structure selection based on worst-case scenarios.
Mohideen et al. (1996)	Simultaneous process and control design algorithm formulated as a mixed-integer stochastic optimization and solved using multiperiod decomposition approach.
Acevedo and Pistikopoulos (1998)	Two-stage framework for stochastic MINLP process synthesis problems.
Bansal (2000)	Parametric programming framework for flexibility analysis and design. Generalized Benders' decomposition algorithm for MIDO problems.
Bansal et al. (2000)	Simultaneous design and control MIDO problem solved with GBD algorithm.
Kookos and Perkins (2001)	Dynamic optimization solution for simultaneous design of process and control based on lower and upper bounds of the best achievable solution
Sakizlis (2003)	Simultaneous process and control design of nonlinear dynamic systems using multi-parametric programming approach.
Sakizlis et al. (2004)	Overview of the methods in simultaneous process control and design under uncertainty.
Seferlis and Georgiadis (2004)	Collection of the recent studies in the field of integrated design and control.
Ricardez-Sandoval et al. (2009)	Overview of the studies on integrated design and control. Robust approach based on formulating the problem as a nonlinear constraint optimization problem.

Table 2. Optimization in papermaking

Authors	Work outline
Bonhivers et al. (2002)	Model predictive control for a broke dosage with given break duration. Broke dosage considered not affecting the break risk.
Sundqvist et al. (2003)	Multiobjective design optimization of mechanical pulping (TMP) process based on worst-case scenario.
Dabros et al. (2005)	Direct-search based optimization of the profile of the broke dosage changes. Broke dosage not considered affecting the break risk.
Madetoja et al. (2006)	Multiobjective optimization architecture using interactive solution procedure applied in papermaking.
Madetoja et al. (2008)	Decision support system approach for paper making including simulation and optimization of a virtual papermaking line.
Pulkkinen and Ritala (2008)	Operational optimization and scheduling of a stochastic problem in mechanical pulping (TMP) process.
Ropponen et al. (2010)	Operational optimization of the multiobjective, stochastic problem in broke management. Increase of break risk due to the increased broke dosage taken into account.

Table 3. Example of typical process values with respect to the nominal case study values. VU refers to a volume unit.

Variable	Nominal value	Process (e.g.)
Time step	1	5 min
Broke generated during normal run ( $v_0$ )	0.1 VU/time step	0.14 m <sup>3</sup> /min
Broke generated during a break ( $v_1$ )	10 VU/time step	14.4 m <sup>3</sup> /min

Typical broke dosage ( $u$ )	2 VU/time step	2.88 m <sup>3</sup> /min
Tower volume ( $V_{max}$ )	400 VU	2880 m <sup>3</sup>

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Table 4. Additional break probability functions (see Eq.4). In sections 3 and 4, break probability model medium (Med) was applied.

Break probability	$q_{min}$	$q_{max}$
High	0.05	0.12
Med	0.03	0.1
Low	0.01	0.08

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Table 5. Robustness analysis of a selected design for three break probability functions.

Break probability ( $S_r/S_o$ )	Overflow time	Filler variation	Time in breaks
High/ High	491.64	0.1335	0.2981
High/ Med	467.78	0.1547	0.2931
High/ Low	427.94	0.1800	0.2866
Med/ High	895.44	0.1175	0.2323
Med/ Med	900.94	0.1359	0.2250
Med/ Low	896.40	0.1518	0.2199
Low/ High	3327.9	0.0737	0.1056
Low/ Med	3225.1	0.0840	0.0997
Low/ Low	2650.2	0.0962	0.0989

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