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Frequency-Response Masking Based Design of Nearly Perfect-Reconstruction Two-Channel FIR Filterbanks with Rational Sampling Factors

Robert Bregović, *Member, IEEE*, Yong Ching Lim, *Fellow, IEEE*, and Tapio Saramäki, *Fellow, IEEE*

***Abstract*—In order to ensure a good filterbank (FB) performance in cases, where there are significant changes in the subband signals, the filters in such FBs must have very narrow transition bandwidths. When using conventional finite-impulse response (FIR) filters as building blocks for generating these FBs, this implies that their orders become very high, thereby resulting in a high overall arithmetic complexity. For considerably reducing the overall complexity, this contribution exploits the frequency-response masking (FRM) technique for synthesizing FIR filters for those above-mentioned FBs, where rational sampling factors are used. Comparisons between various optional methods of utilizing the FRM technique for designing FBs under consideration shows that the most efficient one, from both the design and the implementation viewpoints, are FBs that are constructed such that the bandedge-shaping or periodic filters are evaluated at the input sampling rate and the masking filters at the output sampling rate. This is shown by means of illustrative examples.**

***Index terms*—Frequency-response masking technique, two-channel filterbanks, nonuniform filterbanks, rational sampling rate conversion.**

I. INTRODUCTION

Multirate filterbanks (FBs) are very popular in digital signal processing due to their ability of dividing an input signal into two or more subband signals (two- or *M*-channel FBs) such that each subband signal contains only one part of the input signal. Processing these subband signals gives in most applications better overall performances than the corresponding techniques that concentrate on directly processing the input signal. Moreover, these subband signals are usually decimated before processing, thereby resulting in a smaller amount of data to be processed [1]–[3].

In most FBs, the same decimation factor is used in all channels, thereby resulting in subbands having equal bandwidths. Such FBs are known as uniform FBs. However, in many applications, these FBs do not provide the best achievable performance. For example, in some speech processing applications, it is more beneficial to use a FB with channel bandwidths following the Bark scale [4], that is, a psychoacoustic scale with bands corresponding to the human hearing system. An adequate approximation to the Bark scale can be achieved by using a FB having the decimation factors $6/1$, $6^2/5$, $6^3/5^2$, …, $6^{M-1}/5^{M-2}$, $6^{M-1}/5^{M-1}$ with M being the number of channels and the decimation factor $6/1$ belonging to the highpass channel. Here, the decimation factor denoted by q/p means that the signal is first up-sampled by a factor of p and then down-sampled by a factor of q. In the above FB, each channel has a different decimation factor, and correspondingly, a different channel bandwidth. Such FBs are known as non-uniform FBs.

The most straightforward approach for synthesizing the above FB is to use a tree structure [2] with non-uniform two-channel FBs as building blocks. At the first level of such a structure, the input signal is separated into two channels with the decimation factors 6/5 and 6/1. At the second level, the lowpass channel (channel with the decimation factor 6/5) is further separated by using the same non-uniform two-channel FB. This is repeated until the desired number of channels is achieved. Such non-uniform FBs with the decimation factors being q/p, where q and p are positive integers, are known as FBs with rational sampling factors or, shorter, as rational FBs.

The basic principles of rational FBs have been discussed in [5], [6]. In the case of two-channel rational FBs, there exist two alternative design approaches. In the first approach, as described in [7]–[10], two sets of filters are used (see, e.g. Figure 2 in [7]). The first set of filters separates the input signal into two parts, whereas the second one eliminates the aliasing (imaging) effects that are results of the sampling rate alternations. The first set of filters has been designed in [7] and [8] by applying a least-squares design method and in [9] by using a minimax design technique. In [10], these filters have been synthesized by exploiting the frequency-response masking (FRM) technique. The second set of filters in [7]–[10] has been designed by using the Remez multiple exchange algorithm. In the second approach, as described in [5], [11]–[16], only one set of filters is used for taking care of the signal separation and aliasing effects. Among these papers, it is worth pointing out [5], where an iterative method has been proposed for designing such FBs, and [13], where a method for designing orthogonal PR FBs has been discussed. The second approach will be utilized in this paper and will be described in more detail in Section II.

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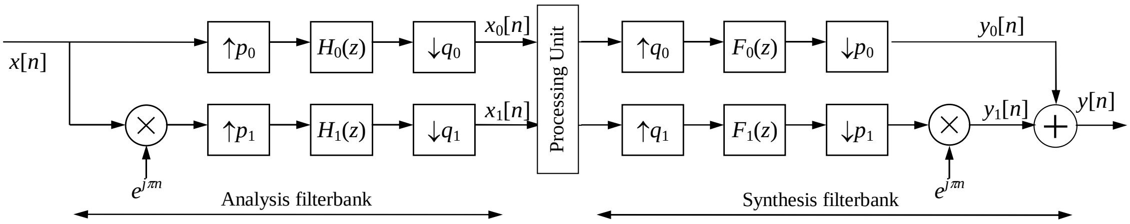

Figure 1. Two-channel FB with rational sampling factors.

Ideally, a FB should not introduce any distortion to the signal passing through the bank, that is, the output signal should be a delayed version of the input signal. FBs satisfying this property are known as perfect-reconstruction (PR) FBs. However, in most applications, it is enough to require that the errors introduced by the FB are smaller than the changes in the signal taking place in the processing unit due to the signal processing algorithms (applications). Such FBs that change themselves the signal (introduce distortions) are known as nearly PR (NPR) FBs. By properly releasing the PR property, other FB properties can be improved, for example, a better channel separation can be achieved with the same FB delay and implementation complexity [3], [17]. The rest of this paper concentrates only on NPR FBs.

In both PR and NPR FBs, due to the multirate operations taking place inside the FB, namely, up- and down-samplings, subband signals suffer from alias errors, which can also be interpreted as energy leakage between the channels (subbands). These errors are directly proportional to the transition bandwidths of the filters as illustrated in [7] (see also Example 1 in Section VI). These alias errors can be compensated completely (PR FBs) or partially (NPR FBs) in the synthesis bank assuming that the subband signals are not modified. However, in most systems, these subband signals are changed in the processing unit. In this case, as a direct consequence of the alias errors, there is a two-fold effect on the overall FB performance. The first effect is that when signals are modified significantly enough in the subbands, the synthesis bank is no longer capable of properly reconstructing the original signal even in the PR case. The second effect is that the signal processing algorithms applied to one of the channels is forced to process also some parts of the signals aliased from the adjacent channels, thereby making the signal processing less efficient.

In order to reduce the alias errors, and consequently, to reduce the above-mentioned two effects on the FB performance, it is required that filters used for generating the FB have narrow transition bands and high stopband attenuations. When utilizing the attractive properties of finite-impulse response (FIR) filters for this purpose, the above requirements imply the use of very high-order FIR filters that are quite difficult to design and are numerically expensive to implement. From the filter design theory, it is well-known that in such cases (high filter orders, narrow transition bands), it is beneficially to use, for both the filter design and implementation, the FRM technique [18]–[20]. Furthermore, it has been recently shown that this is also true for two-channel uniform FBs [21]. Therefore, it is to expect that the use of the FRM technique is also beneficial when synthesizing two-channel rational FBs.

The use of the FRM technique for designing two-channel rational FBs has been discussed in [10], [14]–[16]. It has been shown, on one hand, that when using FRM FIR filters as building blocks in the FBs, the design and/or implementation complexity can be reduced when compared with those FBs, where direct-form FIR filters are utilized. On the other hand, it has been observed that the use of the FRM approach for synthesizing such building blocks for two-channel rational FBs is not very straightforward because the properties of the resulting FBs depend strongly on how this approach has been utilized. In order to show what existing design approach (structure) is the most efficient one for synthesizing the two-channel rational FBs under consideration, this contribution concentrates on the technique presented by the authors in [15] with the following three-fold extension. First, a more detailed description of the overall synthesis scheme is provided including both the practical implementation and optimization issues. Second, a special emphasis is laid on comparing four different two-channel rational FBs generated by using the above-mentioned synthesis techniques.

Third, measures are involved for evaluating both the design and implementation complexities. When evaluating the implementation complexities of all FBs under consideration, the implementation technique proposed by the authors in [24] for implementing systems with rational sampling rate is utilized. This technique enables one to efficiently exploit the coefficient symmetries of linear-phase FIR filters in such systems.

The outline of this paper is as follows: Section II gives the structure and the basic relations of the two-channel rational FBs under consideration. The FRM technique is briefly reviewed in Section III and the various FRM-based techniques for designing two-channel rational FBs are discussed in Section IV. Section V introduces the proposed synthesis scheme for designing two-channel rational FBs under consideration, whereas Section VI provides some examples illustrating the properties of the resulting FBs as well as a detailed comparison with some other existing techniques for synthesizing such FBs. Finally, in Section VII some concluding remarks are made.

II. TWO-CHANNEL FILTERBANKS WITH RATIONAL SAMPLING FACTORS

This section considers the structure and basic relations for rational two-channel FBs under consideration together with the basic requirements to be met by the filters building the FB in order to achieve good filter and FB properties [5]–[16].

A. Filterbank Structure

The rational two-channel FB considered in this paper is shown in Figure 1. This FB consist of an analysis FB and a synthesis FB separated by a processing unit. In the sequel, it is assumed that the processing unit does not alter the subband signals. Moreover, the channel containing the filter transfer functions $H_0(z)$ and $F_0(z)$ $\{H_1(z)$ and $F_1(z)\}$ is referred to as the lowpass {highpass} channel. The p_k's and q_k's for $k = 0, 1$ can be any positive integers as long as $p_k < q_k$. Additionally, for a maximally decimated (critically sampled) FB, in order to cover the overall frequency range of the signal from 0 to π, the following relation has to be satisfied [7]:

$$\frac{p_0}{q_0} + \frac{p_1}{q_1} = 1 . \tag{1a}$$

In this case, the lowpass {highpass} channel covers the frequency range from 0 to $(p_0/q_0)\pi$ {from $(p_0/q_0)\pi$ to π}, as illustrated in Figure 2. Moreover, (1a) can be rewritten as:

$$\frac{p_1}{q_1} = 1 - \frac{p_0}{q_0} = \frac{q_0 - p_0}{q_0} , \tag{1b}$$

which implies that $q_1 = q_0$ and $p_1 = q_0 - p_0$.

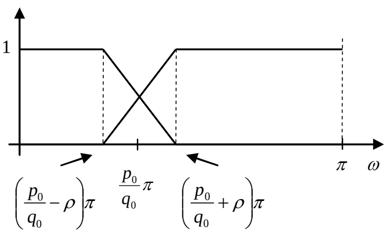

Figure 2. The band division of the analysis FB in Figure 1.

Furthermore, the signal in the highpass channel of the FB shown in Figure 1 is modulated by $e^{j\pi}$ in both the analysis and the synthesis banks. This corresponds to highpass-to-lowpass and lowpass-to-highpass transforms in the analysis and synthesis banks, respectively. The purpose of this modulation is twofold. First, it makes all the filters in the FB lowpass filters. This fact simplifies the notations required when stating the optimization problem in Section V.A for the proposed FB and when describing the algorithm for solving this problem in Section V.B. Second, most importantly, this modulation enables one to implement a FB for all combinations of p_0 and q_0 with the aid of FIR filters having real-valued coefficients. For example, when omitting this modulation, a FB with $q_0 / p_0 = 3/1$ cannot be realized without having bandpass filters with complex-valued coefficients in the highpass channel [6], [7]. In this case, if the filters with real-valued coefficients are only used, then aliasing occurring after the down-sampling cannot be cancelled in the synthesis bank.

B. Basic Relations

The relation between the output and input sequences for the system shown in Figure 1 is expressible in the z-domain as

$$Y(z) = Y_0(z) + Y_1(z) , \tag{2a}$$

where

$$Y_k(z) = \frac{1}{p_k q_k} \sum_{q=0}^{q_k-1} \sum_{p=0}^{p_k-1} \Big\{ F_k(z^{1/p_k} W_{p_k}^{p}) H_k(z^{1/p_k} W_{p_k}^{p} W_{q_k}^{q}) X(z W_{q_k}^{p_k q}) \Big\} \tag{2b}$$

for $k = 0, 1$ and

$$W_p^q = e^{-2\pi j q/p} . \tag{2c}$$

The above equation can also be written in the following form that is more commonly used when analyzing FBs [10] [1]:

$$Y(z) = T_0(z) X(z) + \sum_{l=1}^{q_0-1} T_l(z) X(z W_{q_0}^{l p_0}) . \tag{3}$$

Here, $T_0(z)$ is the distortion transfer function determining the amount of the distortion caused by the overall system for the un-aliased component $X(z)$ of the input signal and the $T_l(z)$'s for $l = 1, 2, \ldots, q_0-1$ are the alias transfer functions determining how well the aliased components $X(zW_{q_0}^{lp_0})$ of the input signal are attenuated.

For an NPR FB with reasonable good properties, $T_0(z)$ and $T_l(z)$'s should approximate a pure delay z^{-D} and zero, respectively. In this case, the output signal is an approximately delayed version of the input signal, that is, $y[n] \approx x[n-D]$. Here, D is the FB delay in samples. In the z-domain, this corresponds to $Y(z) \approx z^{-D} X(z)$.

C. Filter and Filterbank Requirements

In order to synthesize a FB having good channel selectivity and a small FB distortion (alias errors), the filters building the FB should satisfy the following requirements [9], [10]:

First, the FB delay D evaluated as

$$D = \frac{\left(N_{h_0} + N_{f_0}\right)}{2p_0} + D_0 = \frac{\left(N_{h_1} + N_{f_1}\right)}{2p_1} + D_1 , \tag{4}$$

has to be odd. Here, N_{h0}, N_{h1}, N_{f0}, and N_{f1} are the orders of the linear-phase FIR filters $H_0(z)$, $H_1(z)$, $F_0(z)$, and $F_1(z)$, respectively. When properly selecting the filter orders, the values D_0 and D_1 become equal to zero. Otherwise, one of these values is zero, whereas the remaining one is a positive integer.

Second, this contribution assumes that the relations between the synthesis filters and the analysis filters are given by

$$F_0(z) = H_0(z) \tag{5a}$$

[1] Due to the restriction given by (1a) and the periodicity of the W_p^q term, (3) can also be written as $Y(z) = T_0(z)X(z) + \sum_{l_1=1}^{q_1-1} \hat{T}_{l_1}(z) X(zW_{q_1}^{l_1 p_1})$. In this case, $\hat{T}_{l_1}(z) = T_l(z)$ for $l = 1, 2, ..., q_0-1$ and $l_1 = q_0 - l$.

and

$$F_1(z) = -H_1(z)\ . \tag{5b}$$

This reduces the number of unknowns, thereby simplifying the design complexity. In some cases, by relaxing this constraint, a FB having slightly better properties can be designed. However, in this case, the number of unknowns increases approximately by a factor of two.

Third, the passband and stopband edges of the filters building the FB are given by

$$\omega_p^{(k)} = (\frac{1}{q_k} - \frac{1}{p_k}\rho)\pi \tag{6a}$$

and

$$\omega_s^{(k)} = (\frac{1}{q_k} + \frac{1}{p_k}\rho)\pi \tag{6b}$$

for $k = 0, 1$, respectively. The parameter ρ should be selected according to the above equations such that the edge frequencies given by the design requirements are achieved. For all filters in the FB, the centers of the transition bands are located at π/q_k for $k = 0, 1$. These stopband and passband edge frequencies are defined in such a way that the transition bands of both filters are equal after mapping these filters to the input sampling rate. This means that the input signal sees filters with the same transition bandwidth of $2\rho\pi$ as shown in Figure 2, but the transition bandwidths of the individual filters are different due to different sampling rates in the two channels.

The following two sections will state some additional requirements that should be imposed to the FRM technique when designing rational FBs.

III. Frequency-Response Masking (FRM) Technique

When using the FRM technique, the transfer function $H(z)$ of an FIR filter is typically expressed as [18]–[23]

$$H(z) = G(z^L)E_0(z) + G_c(z^L)E_1(z)\ , \tag{7a}$$

where

$$G_c(z) = z^{-N_g/2} - G(z) \tag{7b}$$

with N_g, the order of $G(z)$, being even. Here, $G(z)$ and $G_c(z)$ are referred to as the model and the complementary model filters, respectively, whereas $E_0(z)$ and $E_1(z)$ are referred to as the masking filters. Moreover, all these filters are linear-phase filters. Figure 3 illustrates the roles of these filters for generating the desired overall response. It should be pointed out that in Figure 3 the so-called Case A design is considered. In this design, one of the transition bands of $G(z^L)$ is used as that of $H(z)$. In the corresponding Case B design, one of the transition bands of $G_c(z^L)$ is used as that of $H(z)$. More detail about Case B designs can be found in [22], [23]. When using the FRM technique for designing FBs under considerations, both Case A and Case B designs can be used.

Due to the up-sampling by L, the periodic transfer functions $G(z^L)$ or $G_c(z^L)$ have narrow transition bands and sparse impulse responses with only every Lth value being non-zero, whereas $E_0(z)$ and $E_1(z)$ are low-order filters due to the wide transition bands provided that L is properly selected (for more detail, see, e.g., [22], [23]). An efficient structure for implementing $H(z)$, as given by (7a) and (7b), is shown in Figure 4.

When designing an FIR filter by using the FRM technique, various parameters, including L, l, the orders of $G(z)$, $E_0(z)$, and $E_1(z)$, as well as whether the filter design is a Case A or a Case B design must be determined based on the given filter design criteria and the resulting implementation complexity [22], [23].

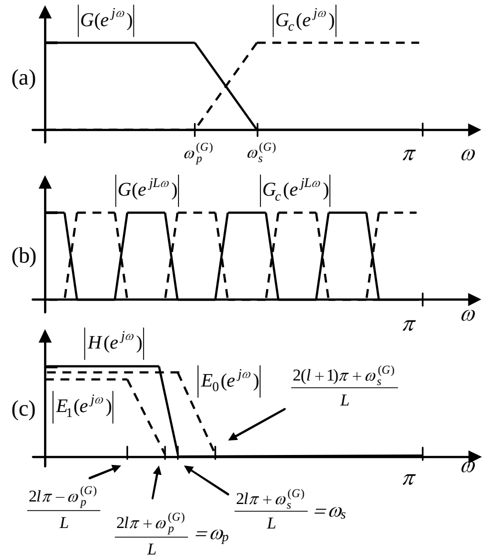

Figure 3. Design of a lowpass filter by using the FRM technique (Case A). (a) Model and complementary model filters. (b) Periodic model filters. (c) Overall filter (solid-line) and masking filters (dashed-lines).

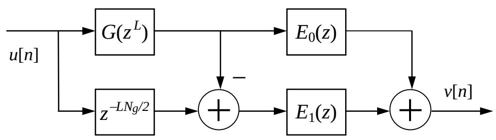

Figure 4. Block diagram for a filter designed by using the FRM technique.

Moreover, from the FB design point of view, when selecting parameter L, the following two conditions must be satisfied [18]–[20]:

Condition 1: Neither $L\omega_p/\pi$ nor $L\omega_s/\pi$ is an integer.

Condition 2: $\lfloor L\omega_p/\pi \rfloor = \lfloor L\omega_s/\pi \rfloor$, where $\lfloor x \rfloor$ stands for the integer part of x.

In the case of a conventional FRM FIR filter, it is always possible to find the parameter L by which the above two conditions are satisfied. However, it is not trivial to meet these two conditions when exploiting the FRM technique in FB applications, where the goal is at the same time to change the sampling rate and to generate systems having good properties (e.g., a good channel selectivity and a low distortion) and an efficient implementation form. The next section will consider various methods that utilize the FRM technique for designing rational FBs.

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IV. Methods for Designing Two-Channel Filterbanks with Rational Sampling Factors based on the Frequency-Response Masking Technique

This section briefly describes four methods (structures) for designing rational FBs. All these methods are based on utilizing the FRM technique for building the filters in the FB. Moreover, the differences between these methods are highlighted in order to clarify the fact that methods based on FRM structures have different design and implementation complexities.

Method 1: In the first method, as proposed in [10] and illustrated in Figure 5, two filters are used in each channel of the analysis and synthesis FB. In this case, the filter $A_0(z)$ is designed by using the FRM technique and is used for shaping the transition band, whereas the filter $B_0(z)$ is designed by using the Remez multiple exchange algorithm and is utilized for eliminating imaging that is the result of the up-sampling by the factor p_0. The filter $B_0(z)$ has typically a wide transition band and as such it would not benefit from the FRM technique.

This method is efficient from the design point of view (small number of unknowns) because the filter $B_0(z)$ is designed separately, but results in FBs with longer delays than FBs generated by other existing design methods. Moreover, the implementation complexity is quite high due to the fact that in each channel, two separate filters have to be implemented. Better results could be obtained if $B_0(z)$ was designed at the same time as $A_0(z)$, but this would considerably increase the design complexity.

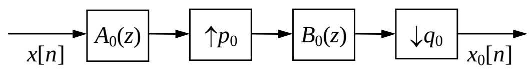

Figure 5. Block diagram of the lowpass channel of the analysis FB designed by using Method 1 [10].

Method 2: In the second method under consideration, the FB is implemented as shown in Figure 1 with all filters designed utilizing the FRM technique. The implementation of one out of the two channels in the analysis FB is illustrated in Figure 6.[2] The disadvantage of this method is in the fact that, due to the cascade configuration of the model and masking filters, even though only every q_kth output sample of the masking filters is used, there is a need to evaluate all output samples of the model filter. Moreover, the model filter is implemented at the highest sampling rate in the overall system. This considerably increases the implementation complexity.

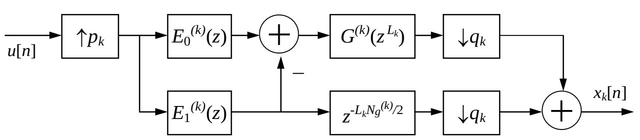

Figure 6. Block diagram of an FRM filter implemented as a part of an analysis FB designed by using Method 2.

[2] Due to the modulation in the lower branch in Figure 1, $u[n]$ is $x[n]$ and $(-1)^n x[n]$ for $k=0$ and $k=1$, respectively.

Nevertheless, based on Figure 6, it can be observed that the construction of very computationally-efficient FRM-based linear-phase structures to be used as the building blocks involved in the structure of Figure 1 implies that the following two ultimate goals are achieved:

1) When implementing the filters by using the FRM approach, the coefficient symmetries should be exploited.
2) The implementation structure for the FRM approach must utilize the facts that in the analysis bank of Figure 1 only every p_kth input sample is nonzero for the transfer functions $H_k(z)$ for $k=0,1$, and that there is a need to generate only every q_kth sample after the transfer functions $H_k(z)$ for $k=0,1$. Similar facts should be utilized when implementing the $F_k(z)$'s for $k=0,1$.

Recently, the authors of this paper have shown in [24] that these goals can be easily achieved when using direct-form linear-phase FIR filters. However, it is not so straightforward to generate an efficient implementation structure when these filters are designed by using the FRM approach, as was already discussed above. Two alternative solutions (implementations) are discussed next and these methods are referred to as Method 3 and Method 4.

Method 3: In the third method, as proposed in [14], the FB is constructed in such a way that all sub-filters of the FRM filter are evaluated at the subband sampling rate, that is, at the lowest available FB sampling rate. It is somehow intuitive that this should give best results from the implementation viewpoint. Although, as the results to be given later on will indicate, the implementation complexity is one of the lowest one among the methods considered in this paper, the design of such systems becomes difficult due to the fact that the structure in Figure 4 cannot be directly used for building filters in the FB shown in Figure 1. In this case the parameters L_k have to be selected as $L_k = l_k \cdot q_k$, where the l_k's for $k=0, 1$ are both integers, in order to locate the centers of the transition bands for filters building the FB under consideration at π / q_k for $k=0, 1$ [6]. The above selections violate one of the two FRM conditions stated in Section III and, therefore, the structure in Figure 4 is not directly usable.

In order to alleviate this problem, in this method, the filters building the FB are designed by using the modified FRM technique proposed by Lim and Yang [20]. The corresponding structure used for implementing the lowpass channel of the analysis FB is shown in Figure 7. Here, the transfer functions $E_s(z)$, $E_d(z)$, and $G_1(z)$ are generated by properly modifying the transfer functions $E_0(z)$, $E_1(z)$, and $G(z)$ of the basic FRM structure (for more detail, see [14], [20]). Due to these modifications, it is not possible to utilize anymore the relations (5a) and (5b). Therefore, all analysis and synthesis filters have to be designed separately. Moreover, there is an additional unfavorable condition on the selection of the FRM parameters L_0 and L_1 as discussed in [14], [20]. Satisfying the above-mentioned two constraints in Method 3 results in a higher number of design unknowns, that is, in an increased design complexity compared with other methods under consideration.

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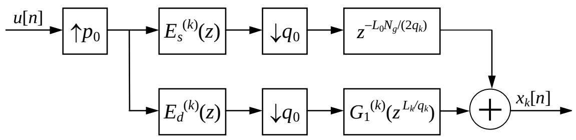


Figure 7. Block diagram of an FRM filter implemented as a part of an analysis FB designed by using Method 3 [14].

Method 4: In the fourth method, referred later on also to as the proposed method, all filters in Figure 1 are synthesized by using the FRM technique in such a way that, first, the periodic filters are evaluated at the input sampling rate and, second, the masking filters are evaluated at the subband sampling rate. For this purpose, the transfer functions of the analysis filters $H_k(z)$ for $k = 0, 1$ are defined, according to (7a) and (7b), as

$$H_k(z) = G^{(k)}(z^{L_k})E_0^{(k)}(z) + G_c^{(k)}(z^{L_k})E_1^{(k)}(z), \qquad (8a)$$

where

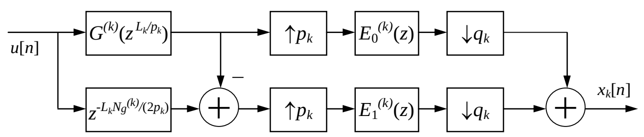
$$G_c^{(k)}(z) = z^{-N_{gk}/2} - G^{(k)}(z) \qquad (8b)$$

and are implemented as shown in Figure 8(a). By using such an implementation, both ultimate goals stated when describing Method 2 can be easily satisfied, thereby significantly reducing the implementation complexity. When implementing the filters in the synthesis bank, as given by (5a) and (5b), the duality with the corresponding filters in the analysis part guarantees the same reduction. The resulting implementation structure for the synthesis bank is shown in Figure 8(b).

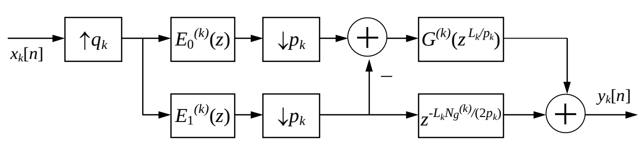
In order to implement the FB in the above manner, in addition to the two conditions stated in Section III, the parameters L_k for $k = 0, 1$ are restricted to have following values:

$$L_k = l_k \cdot p_k, \qquad (9)$$

where l_k is an integer and $k = 0, 1$. As in the case of original FRM filters, it is always possible to find good values for the L_k's. Moreover, when comparing (6a), (6b), and (9), it can be observed that $l_0 = l_1 = L_{frm}$ is a good selection. Here, L_{frm} is the parameter that is used for designing filters with the transition bandwidths equal to $2\rho\pi$ [for more detail, see the paragraph after (6b)].



(a)



(b)

Figure 8. Block diagram for an FRM filter implemented as a part of a FB designed by using Method 3. (a) Analysis FB. (b) Synthesis FB.

It should be pointed out that Method 4 can be interpreted as a hybrid version between Method 1 and Method 2. Only one filter per channel is designed as in Method 2, but the filters are implemented before and after the up-sampler as in Method 1, as is desired. Moreover, when comparing the proposed method with Method 1, it can be observed that in this method the masking filters are merged with the anti-aliasing filter $B(z)$ in Figure 5, which decreases the number of filter coefficients in the overall FB implementation as well as the FB delay.

As it will be shown, by means of examples, in Section VI, Method 4 generates rational two-channel FBs with best performances. Therefore, an efficient design method for synthesizing the above-defined FBs is discussed in the next section.

V. FILTERBANK DESIGN

This section states the design criteria for the rational FBs synthesized by using Method 4, proposes an efficient synthesis scheme, and estimates the resulting FB design and implementation complexities.

A. *Filterbank Design Problem*

In order to emphasize the dependence between the unknowns and each design criterion under consideration in the FB design problem, the coefficients required for building filters $H_k(z)$ for $k = 0, 1$ are denoted in the vector form by $\boldsymbol{h}_k = \{\boldsymbol{g}^{(k)}, \boldsymbol{e}_0^{(k)}, \boldsymbol{e}_1^{(k)}\}$ with the vectors $\boldsymbol{g}^{(k)}$, $\boldsymbol{e}_0^{(k)}$, and $\boldsymbol{e}_1^{(k)}$ containing the unknown FRM filter coefficients. When forming these vectors, the coefficient symmetries are exploited. By using the above notations, the following optimization problem is stated:

Given the desired passband and stopband ripples δ_p and δ_s, the amplitude and alias distortions δ_d and δ_a, the decimation factor q_0 / p_0, and ρ, the factor defining the passband and the stopband edge frequencies, find filter coefficients of all FRM filters to minimize

$$\delta = \max\left(\left|H_0(\boldsymbol{h}_0, e^{j\omega})\right|, \left|H_1(\boldsymbol{h}_1, e^{j\omega})\right|\right) \quad \text{for} \quad \omega \in [\omega_s^{(k)}, \pi] \qquad (10a)$$

subject to:

$$\left|H_k(\boldsymbol{h}_k, e^{j\omega})\right| - 1 \le \delta_p \text{ for } \omega \in [0, \omega_s^{(k)}] \text{ and } k = 0,1 \qquad (10b)$$

$$\left|\left|T_0(\boldsymbol{h}_0, \boldsymbol{h}_1, e^{j\omega})\right| - 1\right| \le \delta_d \quad \text{for} \quad \omega \in [0, \pi] \qquad (10c)$$

$$\left|T_l(\boldsymbol{h}_0, \boldsymbol{h}_1, e^{j\omega})\right| \le \delta_a \text{ for } \omega \in [0, \pi] \text{ and } l = 1, 2, ..., q_0 - 1 \qquad (10d)$$

with the stopband edge frequencies being defined by (6b). Here, $H_k(\boldsymbol{h}_k, e^{j\omega})$ for $k = 0, 1$ are frequency responses of the corresponding FB filters, whereas $T_0(z)$ and the $T_l(z)$'s, as defined by (3), are the distortion and alias error transfer functions, respectively. In addition, it is assumed that the filters in the synthesis bank are related to those of the analysis bank trough (5a) and (5b).

The above criteria are selected in such a way that, first, the larger one of the maximum values of the filter amplitude responses in the stopbands is minimized. Second, the maximum value of the filter amplitude responses is limited to be less than or equal to $1+\delta_p$. As will be seen in connection with examples, the minimum value of the filter amplitude response is indirectly limited through the constraints of (10a) and (10c). Third, the distortion error has to be less than or equal to δ_d and the worst-case alias error has to be less than or equal to δ_a. This gives the overall control over the NPR property of the FB. Before starting the design procedure, the parameters of the FRM filters have to be chosen as discussed in [22], [23], [25] such that, after optimization, the minimum value of δ becomes less than or equal to δ_s.

B. Filterbank Design Algorithm

There are various ways to solve the above design problem. One alternative is to use the following procedure:

Step 1. Select the filter orders $N_g^{(k)}$, $N_{e0}^{(k)}$, and $N_{e1}^{(k)}$; the L_k's for $k \in 0, 1$; and check for both $k=0$ and $k=1$ whether the corresponding design is a Case A or Case B design[3] according to the discussion in [22], [23], [25].

Step 2. Discretize the interval $[0, \pi]$ into points $\omega_m \in [0, \pi]$ for $m = 0, 1, \ldots, M$. It has turned out that in order to arrive at an accurate enough overall solution, a good choice for M is $M = 8 \cdot \max\{N_{h0}, N_{h1}\}$.

Step 3. Solve the design problem defined by (10a)–(10d) on the above-defined discrete grid of frequencies. This can be done by using standard non-linear optimization routines, for example, the function `fminimax` included in the Optimization Toolbox provided by MathWorks, Inc. [25]. If all the filter orders are properly selected, then solving the above problem results in a solution satisfying the design criteria and having the minimum implementation complexity. Otherwise, the filter orders have to be changed until achieving the desired goal.

As initial filters for Step 3 of the above procedure, lowpass FRM filters satisfying the given passband and stopband ripples are used.

Various combinations of the filter orders and the values of the L_k's give rise to solutions meeting the given criteria. Therefore, the above algorithm has to be carried with different combinations of the above parameters and, finally, the combination of these parameters that satisfies the given criteria with the lowest implementation cost is selected.

C. Filterbank Design and Implementation Complexity Estimation

In general, the complexity of an optimization (design) problem increases with the number of unknowns. In the problem at hand, namely, the synthesis of rational FBs, FIR filters are used for building the FB and the filter coefficients are determined with the aid of a minimax design criterion. In this case, the number of grid points required by the objective function to be optimized as well as those needed for the constraints are directly proportional to the number of unknowns. In many cases, the arrival at the optimized solution depends on the optimization algorithm and its practical implementation itself, but the convergence at a roughly optimized solution heavily depends on the number of grid points in use. For instance, when applying the Remez multiple exchange algorithm implemented by McClellan, Parks, and Rabiner [26] to designing an arbitrary-magnitude response linear-phase filters, a proper number of grid points guaranteeing the arrival at a satisfactory result has been observed to be roughly 8 times the filter order, or alternatively, 16 times the number of unknowns in the case, where the coefficient symmetries are exploited. On the other hand, the amount of calculations required by the optimization algorithm depends heavily on the number of grid points that, in turn, is proportional to the number of unknowns involved in the optimization in the case, where FIR filters are used together with a minimax criterion. Based on the above-mentioned facts, a good measure of the complexity of the optimization problem at hand is simply the number of unknowns used in the optimization.

For a FB designed by Method 4, the number of design parameters is equal to the number of unknown filter coefficients, that is,

$$Y = Y_0 + Y_1, \tag{11a}$$

where

$$Y_k = \frac{N_g^{(k)} + 2}{2} + \left\lceil \frac{N_{e0}^{(k)} + 1}{2} \right\rceil + \left\lceil \frac{N_{e1}^{(k)} + 1}{2} \right\rceil \tag{11b}$$

for $k = 0, 1$. Here, $\lceil x \rceil$ stands for the smallest integer being greater than or equal to x and it is assumed that the order of the periodic filter is even.

The implementation cost C, defined as the number of multiplications per input sample[4] for a FB designed by Method 4, is given by

$$C = 2(C_0 + C_1), \tag{12a}$$

where C_0 and C_1 are the number of multiplications per input sample in the analysis lowpass and highpass channels, respectively. A simple estimation is obtained as follows:

---

[3] Determining proper values for $N_g^{(k)}$, $N_{e0}^{(k)}$, $N_{e1}^{(k)}$, and L_k's for $k \in 0, 1$ is complicated due to the fact that when applying the FRM approach for designing filters for constructing rational two-channel FB's, there exist several above-mentioned constraints that have to be taken into account. Nevertheless, as the initial selection for those parameters, in the proposed design method, parameters for the conventional FRM filters can be used. How to properly determine various parameters, including L, l, the orders of $G(z)$, $E_0(z)$, and $E_1(z)$, as well as whether the filter design is a Case A or a Case B design [cf. Figure 3] in order to arrive at the best solution, in terms of the minimizes number of coefficients required in the original FRM approach [18] to meet the given criteria has been shown, in terms of illustrative examples, in [22], [23]. It is worth to point out that finding the best solution can be performed very fast by first determining the start-up orders of the three filters by properly using the estimation formula [27]. These estimated values are in the very close vicinities of the actual filter orders, thereby significantly simplifying for finding the best solution for the given criteria. The best solutions are obtained at those values of L, for which the transition bandwidths of $E_0(z)$ and $E_1(z)$ are practically the same.

[4] In most designs, when implementing a digital system, the multiplications are the most time-consuming part of the system. Other parameters, such as additions and memory consumptions, although very important, are not so relevant. Moreover, the last two parameters depend heavily on the implementation platform and, as such, are more difficult to estimate.

$$C_k = \frac{N_g^{(k)}+2}{2} + \left\lceil \frac{N_{e0}^{(k)}+1}{2q_k} \right\rceil + \left\lceil \frac{N_{e1}^{(k)}+1}{2q_k} \right\rceil \tag{12b}$$

for $k = 0, 1$.

It should be pointed out that (12a) and (12b) give a rough estimation for the implementation complexity. More precisely, this estimation assumes that when implementing the FB, all parameters of the system (linear-phase, down-sampling, up-sampling) are utilized as well as possible, that is, in the ideal case. Therefore, this estimation is always smaller than the one achievable in practice, but as it will be seen in the next section, the difference between this estimation and the practically achievable one is in most cases small. For a particular combination of the filter orders and the rational sampling factors, a more precise estimation for implementing the masking filters can be derived by using the discussion given in [24]. In [22], [23], it has been shown, in terms of illustrative examples, how to properly determine various parameters, including L, k, the orders of $G(z)$, $E_0(z)$, and $E_1(z)$, as well as whether the filter design is a Case A or a Case B design [cf. Figure 3] in the original FRM approach

VI. Examples and Comparisons

This section shows, by means of examples, the efficiency of the proposed technique (Method 4 in Section IV) for designing and implementing two-channel rational FBs of Figure 1. Moreover, the FBs resulting when using the proposed method are compared with the equivalent FBs that are synthesized using some other existing design techniques.

It should be pointed out that the figures in this section will concentrate on showing the characteristics of the filter transfer function $H_1(-z)$, instead of $H_1(z)$, in order to make them more distinguishable from those of $H_0(z)$. Moreover, in all figures, the amplitude responses of both filters are normalized such that the passband average takes on the value of unity.

Example 1: It is desired to synthesize a two-channel rational FB to satisfy the following requirements: $p_0 = 5$, $q_0 = 6$, $\rho = 0.02$, $\delta_s = \delta_p = 0.01$, and $\delta_d = \delta_a = 0.01$. Such a FB divides the input signal into two nonuniform subbands as illustrated in Figure 9.

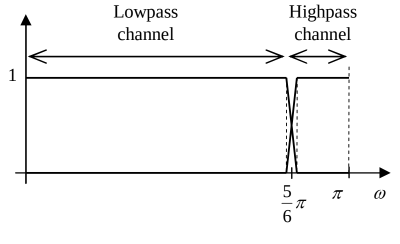


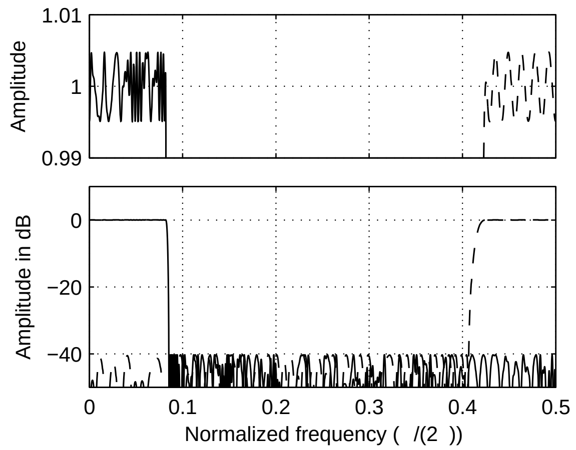
Figure 9. The band division of the analysis FB in Example 1.

By using the proposed method, the above design specifications can be achieved by the proposed FB with the following parameters:

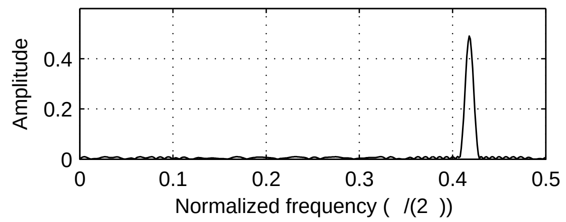
$$H_0(z) \Rightarrow \begin{cases} N_g^{(h0)} = 30,\ L_0 = 20 \\ N_{e0}^{(h0)} = N_{e1}^{(h0)} = 85 \end{cases}$$
$$H_1(z) \Rightarrow \begin{cases} N_g^{(h1)} = 22,\ L_1 = 4 \\ N_{e0}^{(h1)} = N_{e1}^{(h1)} = 49. \end{cases} \tag{13}$$

As already mentioned in Section V, there exist many solutions meeting the given criteria with various combinations of the orders of the FIR filter synthesized using the FRM approach as well as the L_k's. The FB designed with the above parameters provides a proper tradeoff between the design complexity, the implementation complexity, and the FB delay. These parameters have been found by a trial-and-error method that roughly follows the guidelines given in [23] for designing conventional FRM filters.

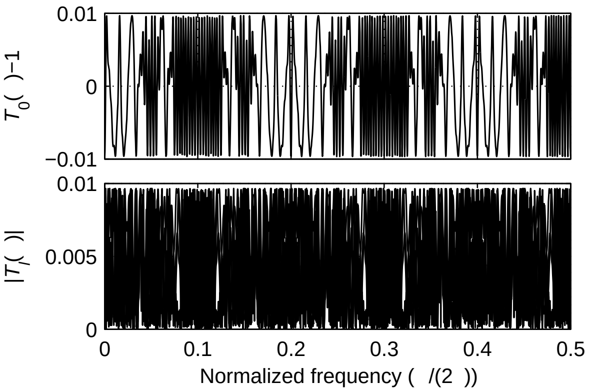
The amplitude responses for the optimized analysis filters together with their passband details as well as the alias and amplitude distortions of the overall FB are shown in Figure 10. As seen in Figure 10(a), the passband ripples are approximately half the specified ones. This is because the synthesis filters are related to the analysis filters through (5a) and (5b). Therefore, the passband ripples of the filters $H_0(z)$ and $F_0(z)$ {$H_1(z)$ and $F_1(z)$} are accumulated to contribute to the overall FB distortion.



(a)



(b)



(c)

Figure 10. Various responses for the optimized FB in Example 1. (a) Analysis filters. (b) Alias errors in channels. (c) Amplitude and alias distortions.

Figure 10(a) also illustrates the relations between the filter transfer functions $H_0(z)$ and $H_1(z)$, the down-sampling factors, and the transition bandwidths. These filters work at higher (different) sampling rates because the input signals are up-sampled before filtering by the factor of p_k for $k = 0, 1$. Moreover, due to this up-sampling and (6a) and (6b), the transition bandwidths of filters $H_0(z)$ and $H_1(z)$ are also different. Nevertheless, as discussed before as well as illustrated in Figure 9, the input signal sees the same transition bandwidth in both channels. The corresponding alias errors in the sub-channels are shown in Figure 10(b). As seen in this figure, although the filters have quite narrow transition bands, the aliasing errors in the subbands are still considerable. These errors would be much larger for filters with wider transition bands. Due to the compensation of alias errors in the synthesis FB, as shown in Figure 10(c), the overall alias errors are negligible. It should be emphasized that the alias cancellation mechanism in the synthesis bank is efficient only as long as the signal changes in the subbands are small enough, that is, smaller than the errors caused by the overall NPR FB.

In order to show the efficiency of the proposed technique, Table I compares the proposed FB with those designed and implemented using other existing techniques, both in terms of the implementation complexity, measured as the number of multiplications per input sample, and in terms of the design complexity, measured as the number of unknowns required in the actual optimization. This table shows that the proposed FB, when compared with the equivalent direct-form implementation (design) [12], has a lower implementation complexity (157.9 vs. 103.3) and a smaller number of design unknowns (164 vs. 400) at the expense of a slightly higher FB delay (137 vs. 133). Furthermore, when considering in Table I the FRM-based designs described in [10] and [14], it is observed that the proposed method has a significantly lower design complexity compared with the design in [14] and a considerably lower implementation cost compared with the design in [10]. Overall, the proposed method combines the best properties of both these designs in addition of keeping the FB delay low. Therefore, in most cases, the proposed method is superior to the other existing ones. Only if the design complexity has to be further reduced, the technique proposed in [10] should be considered. However, as seen in Table I, the price to be paid for this reduction is a higher FB delay and a considerably higher implementation complexity.

Furthermore, when comparing the proposed method with the minimax design described in [9], it is seen that the number of multiplications per input sample required by the proposed FB is only roughly one third. This is mainly due to the fact that in [9] two sets of filters are used in every channel, as was briefly discussed in Section I. An orthogonal design generated by using the method presented in [13] is also included in Table I. Because the least-squared criterion is used in this synthesis scheme, its comparison with other FBs in Table I is not very straightforward. For providing a rough comparison, the orders of the filters in this technique were chosen in such a way that the average stopband energy of the two filters in the analysis FB became practically the same as the energy of the corresponding filters generated by the proposed method. It should be noted that the filters designed by method proposed in [13] have lower stopband attenuations at the stopband edges. The high number of unknowns in this approach is not a problem due to the efficiency of the proposed design method in [13]. The resulting FB has a lower FB delay, but a considerably higher implementation complexity. This is mainly due to the fact that in this case nonlinear-phase filters are used to build the FB. Hence, from the implementation viewpoint, nonlinear-phase filters are not very efficient building blocks for rational sampling rate FBs. In most cases, they might result in FBs having slightly shorter delays, but the implementation complexity will be considerably higher.

TABLE I DESIGNED FB PROPERTIES:
D - DELAY, *Y* - NUMBER OF DESIGN UNKNOWNS, AND *C* - NUMBER OF MULTIPLICATIONS PER INPUT SAMPLE EVALUATED BY USING [24]. [5]

Design	*D*	*Y*	*C*
Proposed (Method 4)	137	164	103.3
[12]	133	400	157.9
[10] (Method 1)	159	124	270.3
[14] (Method 3)	141	578	113.7
[9]	163	144	307.0
[13]	103	596	854.7

Example 2: In this example it is desired to synthesize two-channel rational FBs to satisfy the following requirements: $p_0 = 1$, $q_0 = 6$, $\rho = 0.02$, $\delta_s = \delta_p = 0.01$, and $\delta_d = \delta_a = 0.01$. Such a FB divides the input signal into two nonuniform subbands as illustrated in Figure 11.

It should be noted that the present design specifications are similar to the ones in Example 1. The only difference is in the rational sampling factors, which are exchanged between the lowpass and the highpass channel as illustrated in Table II. As discussed in Section II.A, due to the modulation in the highpass channel by $e^{j\pi}$, all filters in the FB are lowpass filters. Consequently, when exchanging the sampling rate factors in the lowpass and highpass channels, it is enough to exchange

[5] By using the estimation formulas for implementation complexity given by (12a) and (12b), the estimated complexity for the proposed design is 101.3, that is very close to the one given in table (103.3).

the roles of filter transfer functions $H_0(z)$ and $H_1(z)$ as well as those of $F_0(z)$ and $F_1(z)$. In this case, the same filters as in Example 1 can be used. The corresponding amplitude and aliasing distortions are shown in Figure 12. By closely comparing Figure 10(b) with Figure 12, it can be observed that the errors are just the frequency-domain-reversed versions of each other.

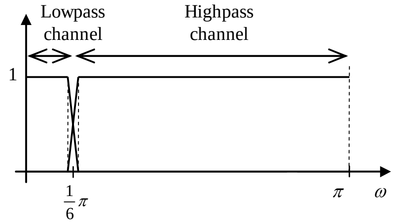


Figure 11. The band division of the analysis FB in Example 2.

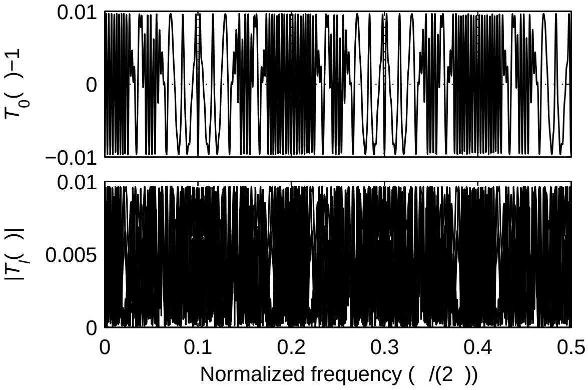


Figure 12. Amplitude and alias distortions for the FB in Example 2..

TABLE II RATIONAL SAMPLING RATE FACTORS IN EXAMPLE 1 AND EXAMPLE 2

Example	q_0/p_0	q_1/p_1
1	6/5	6/1
2	6/1	6/5

Example 3: In this example it is desired to generate a family of two-channel rational FBs designed by the proposed method such that they satisfy the following requirements: $p_0 = 2$, $q_0 = 5$, $\rho = 0.02$. For such a family, the filter orders are fixed as follows:

$$H_0(z) \Rightarrow \begin{cases} N_g^{(h0)} = 30, \;\; L_0 = 8 \\ N_{e0}^{(h0)} = N_{e1}^{(h0)} = 94 \end{cases}$$
$$H_1(z) \Rightarrow \begin{cases} N_g^{(h1)} = 34, \;\; L_1 = 12 \\ N_{e0}^{(h1)} = N_{e1}^{(h1)} = 93. \end{cases} \tag{14}$$

For generating the desired family, FBs were designed for various values of the distortion (δ_d) and aliasing error (δ_a). In all cases, the identity $\delta_d \equiv \delta_a$ was assumed. The results are summarized in Figure 13 that shows the stopband attenuation in the decibel scale as a function of $\delta_d \equiv \delta_a.$ As seen in this figure, the FB selectivity is much better for FBs having a worse reconstruction property and vice versa, as can be expected. In addition, this figure indicates that there is chance to generate a trade off between the stopband attenuation (FB selectivity) and the FB reconstruction property.

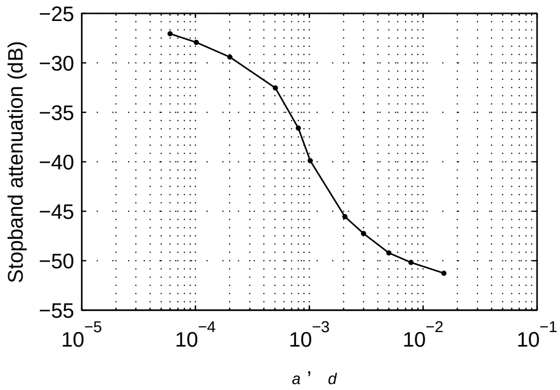


Figure 13. Stopband attenuation as a function reconstruction errors $\delta_d \equiv \delta_a$ for FBs in Example 3.

VII. CONCLUDING REMARKS

In this paper an efficient approach for utilizing the FRM technique for designing rational sampling rate FBs has been proposed. Based on the observations made in this paper, the following facts should be emphasized.

First, due to the modulation in the highpass channel by $e^{j\pi}$, all filters in the FB are lowpass filters. Consequently, filters designed for a FB with rational sampling factor q_0/p_0, can also be used in a FB with rational sampling rate factor $q_0/(q_0-p_0)$ by simply exchanging the roles of the lowpass and highpass filters, as shown in Example 2.

Second, linear-phase filters outperform in most cases their nonlinear-phase counterparts due to the existence of a more efficient implementation. As shown in this paper, the FRM technique can be efficiently utilized for designing and implementing such filters.

Third, by using NPR filter banks, better channel selectivity can be achieved for a given filter orders, FB delay, and consequently, also for a given implementation complexity.

Fourth, FRM filters are not appropriate for building PR FBs.

Fifth, by using filters with narrow transition bands, the aliasing errors in the channels are reduced. However, narrow transition bands mean longer filters, that is, a higher FB delay. The proposed method is efficient in designing such long filters. On the other hand, the application itself defines the appropriate level for aliasing errors in the subbands, and as such, the required filter orders.

Sixth, the differences in the design and implementation complexities of methods under consideration arise from the relations between the required properties and up-sampling and down-sampling blocks.

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