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C/N_0-based criterion for selecting BOC-modulated GNSS signals in cognitive positioning

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***Abstract*—A Carrier-to-Noise spectral density-based criterion for selecting Binary Offset Carrier signals (both sine and cosine) for the purpose of a cognitive tracking unit is derived in this letter. The paper presents the C/N_0 gap between two signals in terms of positioning capability expressed via the Cramer Rao lower bounds on time-delay estimation accuracy.**

***Index Terms*—Binary Offset Carrier (BOC), Carrier to Noise spectral density (C/N_0), cognitive positioning, Global Navigation Satellite Systems (GNSS), Cramer Rao Lower Bound (CRLB).**

I. Problem formulation

CURRENTLY two GNSS systems are fully operational (GPS and Glonass) and two more are emerging and promise to be fully functional in the next $5-7$ years (Galileo and Compass). Three out of these four systems are employing direct sequence spread spectrum and Binary Phase Shift Keying (BPSK) with rectangular (RECT) pulse shaping or BOC modulations [4], and the fourth one (Glonass) may also have a future spread spectrum component compatible with the other three systems. It is envisioned that the future sky will shelter more than 110 navigation satellites, each transmitting various forms of BPSK/BOC-modulated signals, in various frequency bands. While a higher number of satellite signals may mean better availability of the location estimates worldwide, the problem of selecting the most relevant signals (e.g., in terms of positioning accuracy) from the wide pool of available signals is also becoming important. Cognitive positioning architectures have already emerged [1], [2], [3], focusing on signal identification, medium awareness and efficient combination of existing localization sources. The problem addressed in this paper is the problem of 'relevant' signal selection, where a 'relevant' signal is defined as the signal with the highest accuracy capability among a pool of available signals. The focus is on BOC-modulated spread spectrum systems (BPSK being a particular case of sine-BOC waveforms [4], [5]). The accuracy is defined in terms of the code tracking performance bounds. It is known that the wider the available bandwidth and the higher the BOC modulation order, the smaller tracking variance and the better multipath robustness we have [6], [9]. Alternatively, by increasing the Carrier-to-Noise spectral density ratio C/N_0, we can also decrease the tracking error variance (and sometimes also the multipath error) [10]. Thus, there are three important parameters that affect the tracking accuracy: the BOC modulation type (sine or cosine), the BOC modulation order (i.e., twice the ratio between sub-carrier rate and chip rate) and the C/N_0. This paper investigates how these three parameters influence the achievable accuracy and what would be the best signal to employ among several possible signals, with different values of the aforementioned parameters. It is to be noticed that we talk here about the C/N_0 at the receiver side, which is influenced of course by the satellite transmit power, but also by the wireless channel characteristics and the satellite elevation. Our statement is that stronger signals have better positioning capabilities only if their power difference is larger than a certain threshold that is dependent on the BOC modulation type and BOC modulation order of each signal, and that this threshold is C/N_0 dependent. In cognitive positioning, when several signals are available for positioning purposes, it is tremendously important to be able to select the signal with the best positioning capabilities among the available signals, and our paper gives an answer to this problem of signal selection. We derive here the exact shape of this threshold. Our analysis is valid for any BOC-modulated waveform. Section II presents the signal model. Section III computes the signal tracking accuracy in terms of Cramer Rao Lower Bounds (CRLB) performance metric and gives the C/N_0 rule. Section IV discusses the conclusions and the further open issues.

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II. Signal model

The received BOC-modulated signal through a channel with the impulse response $h(t)$ can be modeled as [4]

$$r(t) = c(t) \otimes s_{BOC}(t) \otimes h(t) + \eta(t) \tag{1}$$

where $\otimes$ is the convolution operator, $\eta(t)$ is a white noise Gaussian term with double-sided power spectral density N_0, $c(t) = \sqrt{E_b} \sum_{n=-\infty}^{\infty} b_n \sum_{k=1}^{S_F} c_{k,n} \delta(t - nS_F T_c - kT_c)$ is the spreading code part, including data bits b_n, $\delta(\cdot)$ is the Dirac pulse, $c_{k,n}$ are the chip values for k-th chip and n-th data bit, S_F is the spreading factor (e.g., 4092 chips for Galileo E1 Open service signal), T_c is the chip interval, $s_{BOC}(t)$ is the BOC-modulation waveform including the pulse shaping part (and its detailed expression is shown in Section III, (3)), E_b is the bit energy, and $h(t)$ is the channel impulse response. The relationship between the typically used C/N_0 and the bit-energy to noise ratio is related to the signal bandwidth B_W after dispreading operation [12]:

$$C/N_0 = \frac{E_b}{N_0} + 10log_{10} B_W \tag{2}$$

Here, $B_W = 1$ kHz, coming from the 1 ms reference code epoch rate taken from GPS C/A code. Additional coherent integration will appear as a gain factor of $10log_{10}N_c$, where N_c is the coherent integration length.

III. Tracking accuracy analysis

A. BOC modulation

The signal structure is a sequence of chips at the chip rate $f_c = \frac{1}{T_c}$, and the shape of each chip can be thought as a binary sub-carrier of frequency f_{sc} higher or equal to f_c. There are two main implementations of BOC, namely sine and cosine BOC [7], [8] which are the building blocks of many other BOC classes. The generic model of a sine/cosine BOC-modulation waveform, derived by the authors in [4],[5] is:

$$\begin{aligned} s_{BOC}(t) &= p_{T_{B_1}}(t) \otimes \sum_{i=0}^{N_B-1} \sum_{k=0}^{N_{cos}-1} (-1)^{i+k} \delta\Big(t \\ &- iT_B - kT_{B_1}\Big), T_B = \frac{T_c}{N_B}, \\ & T_{B_1} = \frac{T_c}{N_B N_{cos}}, \end{aligned} \tag{3}$$

with $N_{cos} = 1$ for SinBOC and $N_{cos} = 2$ for CosBOC. The typical notation is $BOC(m, n)$, where m and n indexes are defined as $m = \frac{f_{sc}}{1.023MHz}$ and $n = \frac{f_c}{1.023MHz}$, $p_{T_{B_1}}(t)$ is the rectangular pulse of support T_{B_1}, and $N_B = \frac{2f_{sc}}{f_c}$ is the BOC modulation order. Thus, the main two parameters differentiating BOC waveforms are the BOC modulation order N_B and the modulation type N_{cos}.

The signal is shaped by the BOC modulation, whose Fourier transform derived from (1) is:

$$\begin{aligned} S_{BOC}(f) &= T_B\Big(\frac{1-(-1)^{\frac{N_B}{N_{cos}}} e^{-j2\pi f T_B}}{1+e^{-j2\pi f T_{B_1}}}\Big) \\ & sinc(\pi f T_B)\Big(\frac{1-(-1)^{N_B} e^{-j2\pi f T_c}}{1+e^{-j2\pi f T_B}}\Big) \\ & e^{-j2\pi f T_{B_1}} \end{aligned} \tag{4}$$

The square absolute value of $S_{BOC}(f)$ shapes the transmitted signal spectrum and it has two main lobes at frequencies $\pm f_{lobe}$. BPSK is a particular case of above, with $N_B = N_{cos} = 1$ and $f_{lobe} = 0$ MHz. The exact values of f_{lobe} can be easily obtained from numerical implementation of (3) and are not reproduced here for lack of space.

B. CRLB-based tracking variance

At the same C/N_0 level, the higher modulation order we have in a BOC-modulated signal, the better positioning accuracy a BOC signal can provide because higher modulation orders mean higher receiver bandwidth. For example a SinBOC$(4,1)$ would exhibit better tracking accuracy than a SinBOC$(1,1)$ signal, at the same C/N_0, a fact that can also be easily checked from the formulas in this section. However, when a higher-order modulation signal has a lower C/N_0, the choice is not obvious. The analysis is based on the Cramer Rao Lower Bound (CRLB) in order to study the maximum achievable performance. The CRLB of a signal is given by [13]:

$$\sigma^2_{CRLB} = \frac{B_L}{(2\pi)^2 (C/N_0)_{lin} \int_{-B_W/2}^{B_W/2} f^2 \overline{G_{BOC}}(f) df} \tag{5}$$

where $C/N_0)_{lin} = 10^{\frac{C/N_0}{10}}$ is the C/N_0 in linear scale, and C/N_0 is shown in (2), B_L is the bandwidth of the delay tracking loop and $\overline{G_{BOC}}(f)$ is the normalized power spectral density of the noise filtered via the BOC modulation:

$$\overline{G_{BOC}}(f) = \frac{\Big|S_{BOC}(f)G(f)\Big|^2}{\int_{-\infty}^{\infty} \Big|S_{BOC}(f)G(f)\Big|^2 df}, \tag{6}$$

$G(f)$ is the front-end receiver transfer function,$S_{BOC}(f)$ is the transfer function of BOC modulation from (4). We remark that similar results can be obtained with narrow non-coherent correlator, coherent early-minus correlator and dot-product discriminator, whose formulas for BOC modulations can be found for example in [6].

C. C/N_0 gap

The problem addressed in this section is how to find an approximation of the C/N_0 gap between a lower-order BOC modulation and a higher-order BOC modulation, such that the lower-order BOC modulation would exhibit the same tracking variance as the higher-order BOC modulation. Assuming that two signals i, j are present, each characterized by a certain $\Big(C/N_0, S_{BOC}(f)\Big)_{i,j}$ pair, the question is which signal has greater potential for positioning, or equivalently a lower tracking variance. Basically, this means finding the gap:

$$\Delta(C/N_0)_{i,j} = (C/N_0)_i - (C/N_0)_j \tag{7}$$

(in dB scale) between any two modulation pairs i, j such that: $\sigma^2_{CRLB_i} = \sigma^2_{CRLB_j}$. From (5), we have:

$$\sigma^2_{CRLB_i} = \frac{B_L}{(2\pi)^2 10^{\frac{(C/N_0)_i}{10}} \int_{-B_W/2}^{B_W/2} f^2 \overline{G_{BOC_i}}(f) df} \tag{8}$$

After straighforward manipulations, from (7)and (5) we obtain:

$$\frac{10^{\frac{(C/N_0)_i}{10}} 10^{\frac{\Delta(C/N_0)_{i,j}}{10}}}{10^{\frac{(C/N_0)_i}{10}}} = \frac{\int_{-B_W/2}^{B_W/2} f^2 \overline{G_{BOC_j}}(f) df}{\int_{-B_W/2}^{B_W/2} f^2 \overline{G_{BOC_i}}(f) df} \tag{9}$$

which implies that the C/N_0 gap is independent on the nominal C/N_0 and depends only on the ratio of RMS bandwidths of the considered signals. The further simplification of (9) gives a closed-form expression for the C/N_0 gap, where the integrals from the right-hand side are in fact directly proportional with the signal RMS bandwidth:

$$\Delta(C/N_0)_{i,j} = 10log_{10} \frac{\int_{-B_W/2}^{B_W/2} f^2 \overline{G_{BOC_j}}(f) df}{\int_{-B_W/2}^{B_W/2} f^2 \overline{G_{BOC_i}}(f) df} \tag{10}$$

This translates into: modulation j is better than modulation i as long as $(C/N_0)_i$ is less than $(C/N_0)_j + \Delta(C/N_0)_{i,j}$.

D. Numerical results

1) Single path channels: If we assume brick-wall filters $H(f)$ and that the reference modulation i is the BPSK(1) modulation and the other modulations $j = 2, \ldots, 30$ are all sine-BOC($j/2, 1$) modulations (i.e., $N_B = j$), then Figure 1 shows the C/N_0 gap (in dB) between these modulations and the reference BPSK(1) modulation for several receiver bandwidth.

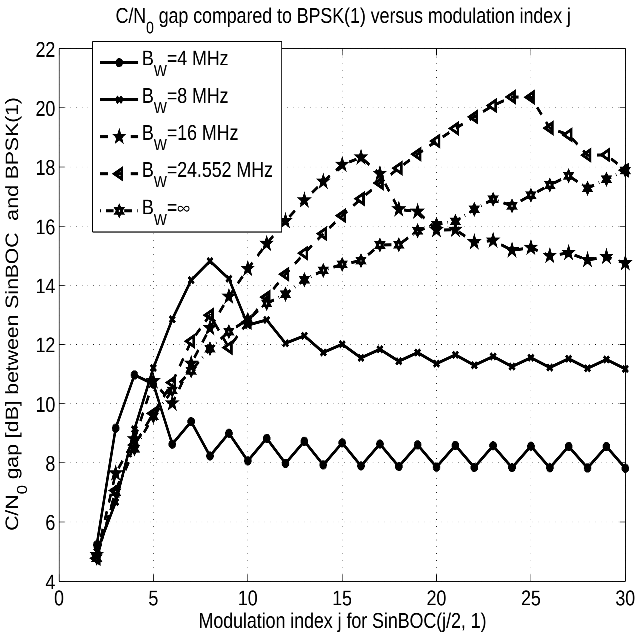

Fig. 1. Minimum C/N_0 gap between SinBOC($j/2, 1$) modulation and BPSK(1), such that BPSK(1) starts to have an equal or better performance than the SinBOC($j/2, 1$) modulation, single- path.

The figure can be interpreted as illustrated in the following example: assuming that both BPSK(1) and SinBOC$(1, 1)$ signals are available at the receiver and that the receiver has a double-sided bandwidth of 24.552 MHz, then the 'relevant' signal for positioning would be SinBOC$(1, 1)$ as long as its C/N0 is no more than 5 dB smaller than that of the BPSK(1) signal. Figure 2 shows the same comparison, this time for CosBOC($j/2, 1$) modulations.

For both SinBOC and CosBOC modulations, there is a clear saturation effect at small bandwidths which depends on the BOC-modulation order N_B: the higher the modulation order, the more bandwidth we need to take advantage of that particular modulation in tracking; otherwise, if the bandwidth is not sufficient, lower order modulations can offer the same or better performance at the same C/N_0. This fact is visible in both figures above if we compare for example the curve at 4 MHz bandwidth for $j = 6$ and $j = 7$: in both cases sine/cosine BOC$(3, 1)$ is worse than BOC$(3.5, 1)$ at the same C/N_0 (or even if the C/N_0 is up to 1 dB stronger for the higher-order modulation BOC$(3.5, 1)$). Another example, also visible in the above figures is for a typical mass-market receiver with 4 MHz

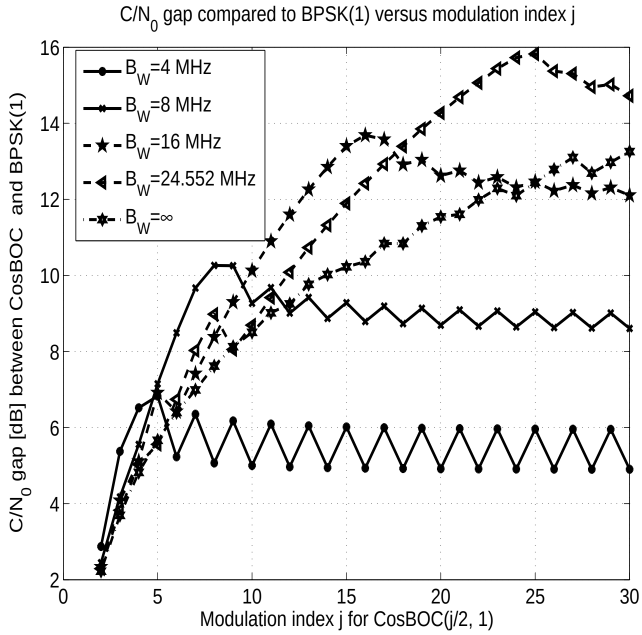

Fig. 2. Minimum C/N_0 gap between CosBOC($j/2, 1$) modulation and BPSK(1), single- path.

double-sided bandwidth: in here, if BPSK(1) signals are 11 dB stronger than the BOC-modulated signals, they will always offer the best performance, no matter on the BOC modulation index or on the BOC type (sine/cosine).

2) Multipath channels: The analysis in multipaths has been done numerically, based on the two-path delay tracking error variance derived in [14]. The main point in this subsection is to show that the findings are also valid in multipath environments. The channel impulse response is:

$$h(t) = A_1 e^{j\phi_1}\delta(t - \tau_0) + A_2 e^{j\phi_2}\delta(t - \tau_0 - \Delta\tau) \qquad (11)$$

where τ_0 is the Line Of Sight (LOS) delay, $\Delta\tau$ is the delay between first non-LOS and LOS, A_i is the i-th path amplitude, $i = 1, 2$ and ϕ_i is the i-th path phase, here assumed uniformly distributed. As shown in [14], the covariance matrix that defines the variances of the maximum likelihood joint amplitude-delay estimators is given by:

$$\Sigma = \begin{pmatrix} 1 & 0 & R(\Delta\tau) & A_2 R^{'}(\Delta\tau) \\ 0 & -A_1^2 R^{''}(0) & -A_1 R^{'}(\Delta\tau) & -A_1 A_2 R^{''}(\Delta\tau) \\ R(\Delta\tau) & -A_1 R^{'}(\Delta\tau) & 1 & 0 \\ R(\Delta\tau) & -A_1 A_2 R^{''}(\Delta\tau) & 0 & -A_2^2 R^{''}(\Delta\tau) \end{pmatrix}$$

and the estimator variances for joint amplitude-delay for first and second path is

$$var([A_1\ \tau_0\ A_2\ \tau_0 + \Delta\tau]^T) = \Sigma^{-1}((C/N_0)_{lin})^{-1} \qquad (12)$$

Above, $R(\Delta\tau) = \int_{-B_W/2}^{B_W/2} G_{BOC}(f) e^{+j2\pi f\Delta\tau} df$ is the autocorrelation function of a BOC modulated low-pass filtered with a bandwidth B_W. The delay tracking variance $var(\tau_0)$ can be derived numerically from the above for each modulation (since Σ depends on BOC type and index). An exact expression for the C/N_0 gap with multipaths is more difficult to obtain, but the numerical results are shown Figure 3, for 2 in-phase paths with second path being 3 dB lower than the first paths and situated at half chip apart from LOS. As expected, the multipath presence is increasing the C/N_0 gap, which means that higher-order modulations are better performing with respect to lower-order modulations in the presence of multipaths. Nevertheless, the performance is dependent on the multipath profile, and therefore, we recommend that the analysis is done in single path scenario and then the selection of the relevant signals is done with a small margin that compensates for the multipath presence.

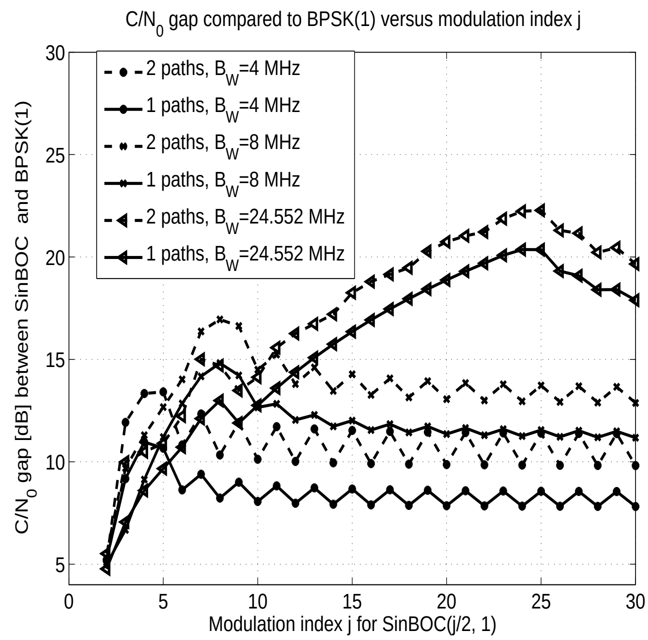

Fig. 3. Minimum C/N_0 gap between SinBOC$(j/2, 1)$ modulation and BPSK(1)for two-path channel.

IV. Conclusion

In this paper we have derived a simple formula for the C/N_0 gap that allows two different BOC signals to achieve the same positioning accuracy. This information is the first step towards a cognitive positioning engine, where the relevant signals are first identified and then fed into the navigation engine. Further research will focus on combinations of 2 or more relevant signals and on the performance with multipath channels with more than 2 paths.

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