Olli Aumala

Dithering in Analogue-to-Digital Conversion

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Thesis for the degree of Doctor of Technology to be presented with due permission
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Abstract

This study analyses properties of the analogue-to-digital converter, which is very central in modern measurement equipment. The work is mainly theoretical, and the theoretical models were verified partly with experimental study, partly with a virtual instrument developed for this purpose.

Fundamental properties of analogue-to-digital conversion are presented as background of the study. Limitations coming from the time-discrete nature of sampling and from the amplitude-discrete nature of quantisation are introduced.

The basic idea of dithering is presented. The condition for systematic correctness is developed. The random uncertainties of some practically interesting dithering schemes are developed. It is presented that dithering can be combined with suppression of network interference. Also the possibility to use unavoidable thermal noise of the measurand as dither signal is presented.

A short analysis on information transfer of the dithered conversion is presented. It is shown that dithering gives more accuracy, but this reduces the bandwidth of the conversion. Indeed, dithering may be characterised as changing bandwidth against resolution.

Two practical applications are briefly discussed. The first one shows how broad-range dithering can improve even the integral nonlinearity of the ADC. The second application is equipment for multichannel temperature measurement with very good resolution and speed.

Acknowledgements

Contents

Original publications

This thesis includes following original papers:

I Aumala, O. 1996a, Turning interference and noise into improved resolution. *Measurement* **19**, no. 1, p. 41-48.

II Aumala, O. 1996b, Optimal filtering for data acquisition. in: *IEEE Intrumentation and Measurement Technology Conference*, Brussels, 1996, p. 866-871

III Aumala, O., Holub, J. 1998a, Practical aspects of dithered quantizers. in: *Proc. Intl. On-line Workshop on Dithering in Measurement*, Prague, p. 11-18.

IV Aumala, O., Holub, J. 1998b, Dithering design for measurement of slowly varying signals. *Measurement* **23** no.4, p. 271-276

V Aumala, O., Holub, J. 2000, Information Transfer of Dithered A/D Converters. in: *Proc. XVI IMEKO World Congress*, Vol. X: EWADC´2000, 5th Workshop on ADC Modelling and Testing, Vienna, p. 45-50.

The author's contribution to the original publications was as follows. In all publications he was the first author, being the main study designer and writer of these publications. In publications III, IV and V he was working in co-operation with Dr. Jan Holub, who designed and performed the study for stochastic dithering and performed experimental analyses. He also authored relevant parts of those publications.

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Table of symbols and abbreviations

Symbols

b	normalised dither amplitude
B	bandwidth
C	channel capacity
d	dither signal
e	base number for the natural logarithm
F	cumulative distribution function
f	frequency
i	discrete variable
N	number of samples
n	dither signal; sequence number; bit number of a converter
P	probability
q	step size of a quantiser
R	range
S	spectrum
s, x	signal
t	time
U	total uncertainty
u	uncertainty
v	**error**
y	**output signal**
δ	impulse function
Φ	discrete cumulative distribution function
μ	mean value
Ψ	characteristic function
σ	standard deviation

Abbreviations

ADC	analogue-to-digital converter
DAQ	data acquisition unit
DSP	digital signal processing
LSB	least significant bit
RMS	root-mean-square

1 Introduction

Since the introduction of digital data processing, it has been used extensively in measurement. Digital computing is developing fast, and digital processors are included even in simple multimeters or in compact measuring transmitters for process variables. New virtual measuring instruments are common practice: they incorporate sensors, data acquisition (DAQ) units and personal computers.

The analogue-to-digital converter (ADC) is very central in most modern measurement arrangements (Aumala 1999). Conversion into digital form introduces quantisation and often means losses of precision. Dithering can be used for getting back at least a part of the loss. It has been used since 1960'ies (Veltman and Kwakernaak 1961), but without naming the method. According to the knowledge of the author, the first systematic presentation of dithering as a part of quantisation theory was published during the latest decade (Widrow *et al.* 1996). The first publication of this thesis was published approximately simultaneously.

As an introduction to the five published papers, the fundamental properties of analogue-digital conversion are discussed. After that, the basic idea and structure of dithering is presented. Application of a generated deterministic dither is analysed from the point of view of resolution, linearity and response time of the converter. After that stochastic dither signals such as thermal noise and sinusoidal interference are discussed. The information point of view is also touched. Finally, the results of the original papers are discussed. The main part of this thesis consists of the original published papers themselves.

2   Fundamental properties of analogue-to-digital conversion

Since the introduction of digital data processing, it has been used in measurement. Digital computing is developing fast, and digital processors are included even in simple multimeters or in compact measuring transmitters for process variables. Virtual measuring instruments are common practice today: they incorporate sensors, DAQ units and personal computers. There are examples of so-called microsensors that include every component mentioned in a single embedded microchip (Analog Devices 2001).

2.1   Sampling

The output of an analogue-to-digital converter (ADC) is a numeric value. By its nature, it cannot be moving continuously. Instead, it is based (ideally) on the analogous value of the input signal at a fixed point of time.

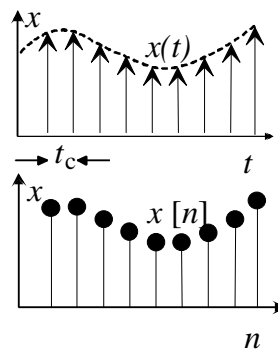An example of sampling is shown in Fig. 1.

Fig. 1. Sampling converts a continuous signal into a sample sequence. t time, n sequence number, x signal value. In this case the sampling is even; $x[n] = x(nt_c)$.

This time-discrete nature is not usually recognised when using a multimeter. But if the input signal moves continuously and one wishes to follow the movement of the measurand, the time-discrete nature becomes essential. It is the target of sampling theory. The main outcome of that theory is generally known as *sampling theorem*.

The sample sequence $x[n]$ may be described with a formula

$$x[n] = x(t) \cdot \sum \delta(t - it_{\text{c}}) = \sum x(it_{\text{c}}) \, \delta(t - it_{\text{c}}) \tag{1}$$

where $\delta(t - it_{\text{c}})$ is an impulse at it_{c}. Its spectrum is the Fourier transform

$$S_{\text{c}}(f) = \frac{1}{t_{\text{c}}} \sum S(f - \frac{i}{t_{\text{c}}}) \tag{2}$$

where S is the spectrum of the continuous signal $x(t)$ and S_{c} is the comb spectrum of the sample sequence $x(nt_{\text{c}})$.

In order to avoid aliasing (called also folding) of the signal spectrum it is required that the partial spectra for different i in (2) do not overlap. The highest frequency component $f_{\max}$ in the continuous signal has to be at most half of the sampling frequency f_{c}:

$$S(f) = 0 \quad \text{for} \, |f| \geq \frac{1}{2t_{\text{c}}} \tag{3}$$

It can be seen from (2) that if there is no aliasing, the sampling does not cause any deformation to the spectrum.

There is a recommended procedure to make sure that there is no aliasing (Aumala *et al.* 1998c). First, a low-pass filtering is arranged to the signal.

Then the sampling and conversion is made using a high sampling frequency. The spectrum is checked next to make sure that there is "enough empty bandwidth" in the spectrum of the sampled discrete signal. Finally, a suitable digital low-pass filtering and a corresponding decimation are arranged to make the final time series short enough and to allow fast processing.

2.2   Quantisation

The analogue-to-digital conversion produces a result on a discrete amplitude scale. Values on that scale correspond to the numerical values the converter can produce.

An ideal quantiser characteristic is in Fig. 2. The step size of this quantiser is q.
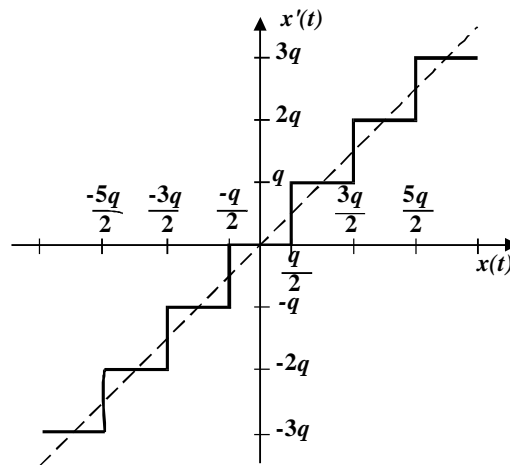

Fig. 2. Ideal quantiser characteristic. $x'(t)$ output of the quantiser.

An example of quantisation is given in Fig. 3 and Fig. 4. The original signal is here sinusoidal. Fig. 4 shows the quantisation error generated by the quantiser.
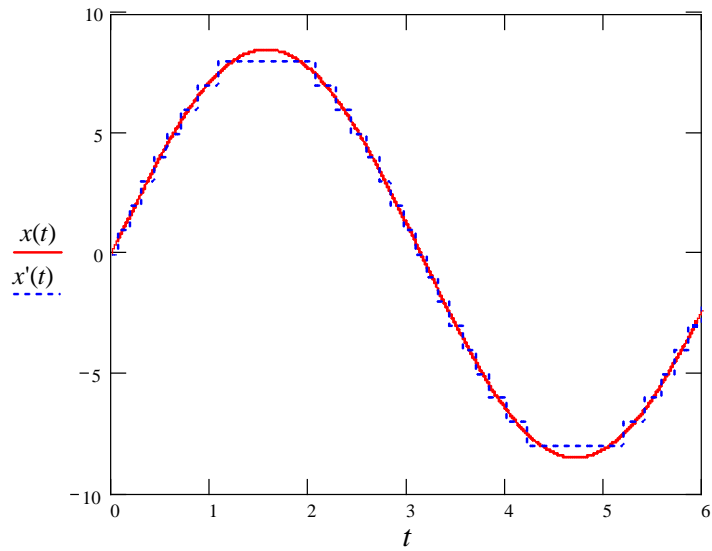
Fig. 3. A sinusoidal signal x and the correspondent quantised signal x'.
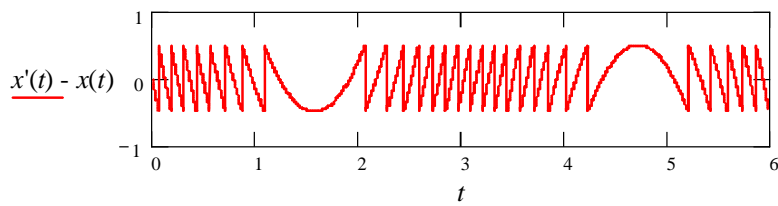
Fig. 4. Quantisation error for the signal of Fig. 3.

Quantisation may be analysed also as sampling in amplitude domain. For example, the cumulative distribution function $F(x)$ of a signal x is converted into a corresponding amplitude discrete cumulative distribution function $\Phi(x)$

$$\Phi(x) = \sum_i F(\frac{2i-1}{2})[\operatorname{sgn}(x - \frac{2i-1}{2}) - \operatorname{sgn}(x - \frac{2i+1}{2})] \qquad (4)$$

Quite analogously with the sampling theory, one may derive a condition for avoiding aliasing in amplitude domain (Widrow *et al.* 1996). The condition uses the characteristic function Ψ (Fourier transform of the distribution function F) and reads

$$\Psi_x(u) = 0 \quad \text{for } |u| \geq \frac{1}{2q} \tag{5}$$

In other words, the quantisation step q has to be smaller than half of the wavelength for the biggest wavenumber of the characteristic function.

The resolution of the converter introduces a quantisation uncertainty to the conversion result. In practically all cases the range of the input signal is much wider than the quantisation step q, and thus error may be assumed to have uniform distribution and the quantisation uncertainty may be calculated as a root mean square error of the conversion over one step:

$$u_q^2(x') = \frac{1}{q} \int_{-\frac{q}{2}}^{\frac{q}{2}} e^2(x)\, dx = \frac{1}{q} \int_{-\frac{q}{2}}^{\frac{q}{2}} x^2\, dx = \frac{q^2}{12} \tag{6}$$

The RMS error is the square root of (6).

(Dallet 1998) presents more concepts used for characterising the analogue-to-digital converters. The analysis includes statistical analysis as well as time domain and spectral analysis.

2.3 Nonideality of the conversion

The author has collected some nonidealities of the characteristics of the DAC in his textbook (Aumala 2000b). From the point of view of this study it is enough to take separately the differential nonlinearity (the

differences between consecutive quantisation step sizes) and to combine other types of nonlinearity into a single concept of integral nonlinearity.

Limitations in frequency domain are interesting in some cases. They are caused by the possible sample-and-hold operation and by the analogous filtering before the conversion, in some cases also by the conversion time of the converter unit itself (the so-called aperture error). Jittering is also important when the timing of the ADC is interfered e.g. inside a computer.

2.4 Need to modify the properties in amplitude and frequency domains

The application sets certain requirements for the analogue-to-digital conversion. This is particularly important for embedded systems where technology sets strict limits to the selection of units.

Sometimes the precision of each result is important, but the measurand moves slowly. If there is a possibility to improve the resolution, a decrease in the frequency bandwidth of the conversion is not harmful.

This is a good case for dithering. The resolution of the ADC can be improved considerably in a simple way, if the input signal is steady and noiseless. In some cases it is even possible to use the noise as dither signal and to improve the signal-to-noise figure simultaneously (Aumala 1996a).

Sometimes the speed is the most important factor, and amplitude resolution is not so important. One can then consider speeding up the conversion by reducing the number of digits. This is not discussed in this study.

For a signal corrupted by network (50 Hz) interference, dithering may be used according to the idea of integrating voltmeter. The interference acts as the dither signal, and the period of averaging conversion results equals to the network period. The spectral properties of this kind of dithering are comparable to the properties of the integrating voltmeter.

It should be noted that the analogue-to-digital conversion is often combined with a computer that is capable to apply powerful signal processing algorithms. If this is the case, an analysis of the signal properties is in order before deciding how to process the original conversion results of the dithered signal.

3 Dithering

One of the properties of the analogue-to-digital conversion is the amplitude discrete nature of the conversion result. This introduces a so-called quantisation error, which cannot be decreased by averaging several conversion results. The main idea of dithering is to add a suitable signal to the (amplitude continuous) signal to be converted. According to the distribution of this dither signal the conversion result becomes also distributed in some way. The quantisation error can be decreased then by processing a set of original conversion results.

The mentioned idea is depicted in Fig. 5.

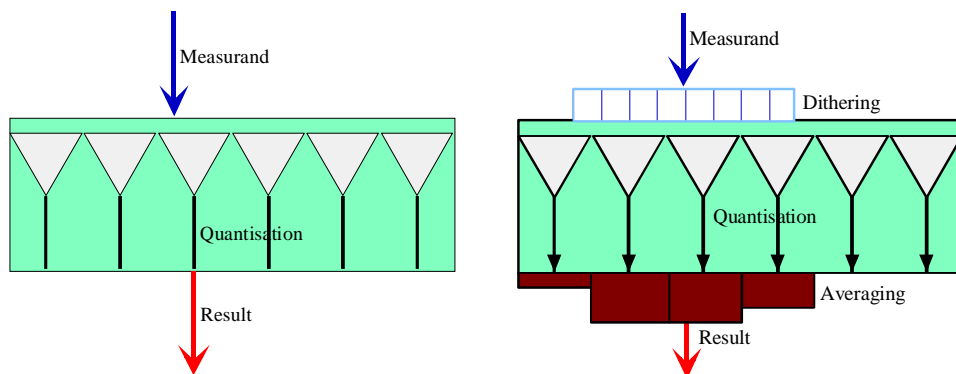Fig. 5. Dithering idea. Left undithered conversion, right dithered conversion. Dithering produces a distributed conversion result, and the final result is calculated by averaging a number of individual original results.

Petri (Petri 1996) presents topologies of dithering and analyses quantisation effects both for deterministic and for stochastic dither signals. They are shown in Fig. 6. One of the topologies is subtractive and

the other non-subtractive. The subtractive topology can be used when the dither signal is a generated one. In many cases the non-subtractive topology is the only practical possibility. This study considers only it.
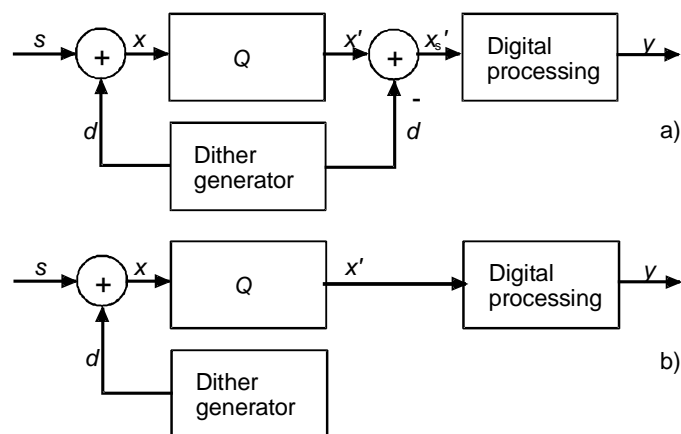
Fig. 6. Subtractive (a) and non-subtractive (b) dithering topologies. Q quantiser.

Dithering is used for improving the resolution of the conversion. In most cases the measurand is quasistatic: it is changing so slowly that it can be assumed to remain unchanged during the averaging period.

This study concentrates on this quasistatic case. One of the original papers discusses also faster changing signals. There the transfer of measurement information is the topic of discussion.

3.1 Condition for unbiased conversion result
The basic dithering structure includes averaging of a number of original conversion results. Averaging may decrease the standard uncertainty of

the final result, but the bias is retained. Therefore it is essential that the distribution of the dither signal d is chosen so that the bias (the systematic error) of the conversion becomes zero.

The range of the signal to be converted is usually much wider than the step size q of the converter. The bias can be calculated as a statistical mean over one quantisation step:

$$\mu(x') = \sum_i iq \cdot P(x_i')$$

$$\mu(x') = \sum_i iq \cdot \left[\Phi\left(\frac{2q+1}{2} - x'\right) - \Phi\left(\frac{2q-1}{2} - x'\right) \right] \tag{7}$$

where P denotes probability and Φ discrete cumulative distribution function.

If this mean equals x, the dithering is unbiased:

$$\mu(x') - x = 0 \tag{8}$$

There are several distributions that fulfil the condition (8). The classical dithering distribution is uniform with the range (- $q/2$, $q/2$). The same condition is fulfilled also for any sum of independent signals, where one of them is the mentioned uniform dither signal, and the other signal has a symmetrical distribution.

Two cases are worth mentioning. One is a sum of two independent uniformly distributed signals. Its distribution function is triangular over the range (- q, q). Because the distribution does not show any abrupt changes, it is not critical to the step size.

The other case is a dither signal with a comparatively wide distribution and a reasonably smooth distribution. This is not critical even to the form of the distribution function. Unfortunately this signal requires a big number of original conversion results to be averaged.

Fig. 7 (Aumala and Holub 2000a) presents three cases of unbiased dithering distributions: the uniform function, the triangular function, and the distribution produced by the uniform distribution and the distribution of a sinusoidal interference signal.
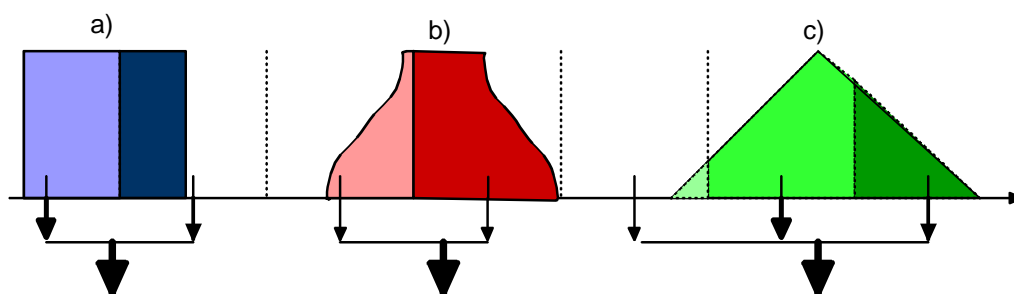

Fig. 7. Examples of dither distribution density functions. a) uniform distribution $(-q/2, q/2)$, b) uniform distribution plus sinusoidal interference, c) triangular distribution $(-q, q)$. For each case the arrows depict how the conversion result is formed.

The generated dither signals are usually deterministic and discrete. The dithering distribution is also discrete in this case. One can determine the values of the generated dither signal by using the desired type of cumulative distribution function. One such example is shown in Fig. 8 (Aumala and Holub 1998b).
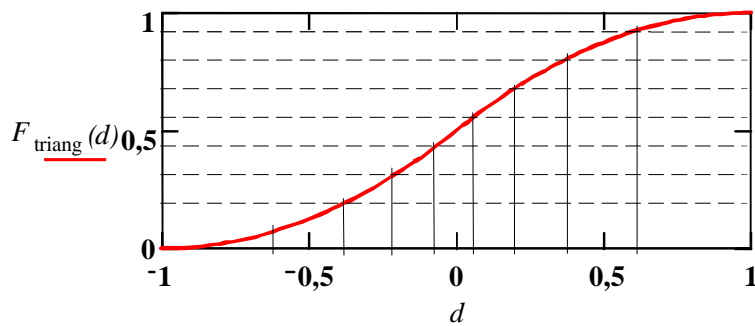
Fig. 8. Generating a dither signal with triangular distribution. $F_{\text{triang}}(d)$ cumulative distribution function, d dither signal. In this case values for a set of 7 values are shown.

3.2 Systematic error of the final result

The uncertainty may be evaluated using the generic principle given in the ISO Guide (Guide 1993). All uncertainty components are evaluated, possible correlations between them   are evaluated and component uncertainties are combined by summing their variances. It is assumed here that there are no mutual correlations.

An assumption is taken here that the range of the measurand is much wider than the quantisation step. It is assumed (see Fig. 4) that the quantisation error is distributed uniformly. The analysis concentrates on the uncertainty produced by the ideal quantiser according to Fig. 2. All other uncertainties (differential and integral nonlinearity of the converter, etc.) should be combined when estimating the total uncertainty of the analogue-to-digital converter.

This quantisation uncertainty is made up of two types: the uncertainty of the systematic error and the so-called type A uncertainty. The former one can be expressed

$$\sigma^2\left[\mu(x')-x\right]=\frac{1}{q}\int_{-q/2}^{q/2}\left[\mu(x')-x\right]^2 \, \mathrm{d}x \qquad (9)$$

where $\mu(x')$ is the mean of the final result and q is the step size of the quantiser.

Averaging may decrease type A uncertainty. The variance for the random error has to be deduced according to each case.

3.3   Deterministic dithering

Deterministic dithering is effective, when improvement in resolution is wanted with smallest number of averaging. The same objective may be expressed also: good resolution combined with a wide bandwidth or fast response time.

There are still other aspects that may determine the type of dither signal.

3.3.1   Uniform dithering

The traditional method of dithering is to add to the signal a series of N values distributed uniformly on the range (-1/2, 1/2) (Aumala and Holub 1998a). The individual values of the dither signal d are

$$d_i = -\frac{N+1}{2N}+\frac{i}{N} \quad i=1\ldots N \qquad (10)$$

It can be shown that this method applied to a steady measurand and averaging N samples for each result gives an unbiased result and an effective step size of $1/N$. An arrangement for generating this dither is shown in Fig. 9. Fig. 10 shows how the dithering amplitude affects the RMS error of the systematic error.
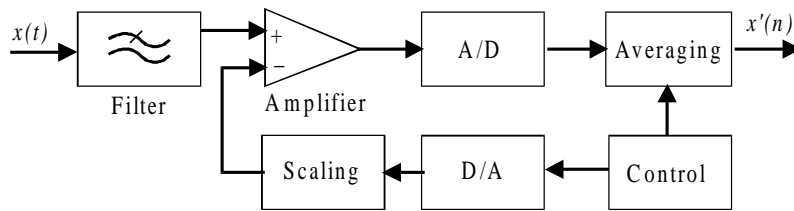
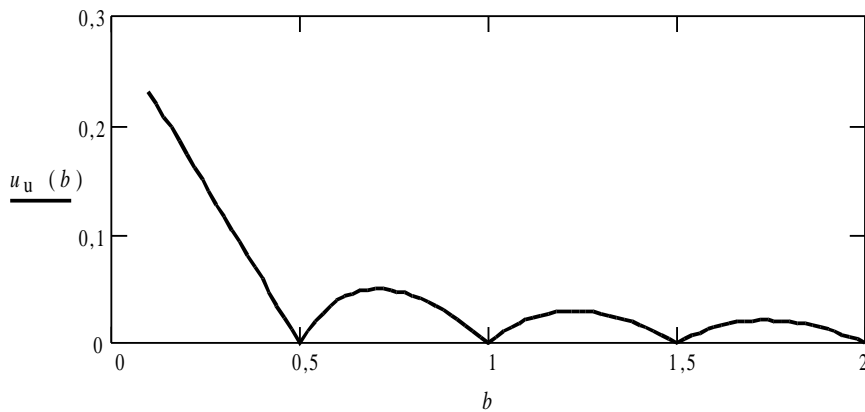Fig. 9. Classical uniform dithering arrangement. Some blocks of the arrangement may be found on the DAQ board.

Fig. 10. Inherent uncertainty of uniform dithering. b is the dithering range divided by the step size of the ADC.

3.3.2 Triangular distribution

Another distribution is better, if there is some differential nonlinearity in the converter. In other words: the step size q is not exactly the same everywhere. This is the case in most practical converters. It is advisable that the dithering distribution is smooth to avoid sensitivity to variations of the step size. The triangular distribution is good for this case. Generation of this dither signal was presented already in Fig. 8. The resulting RMS systematic error can be seen in Fig. 11.
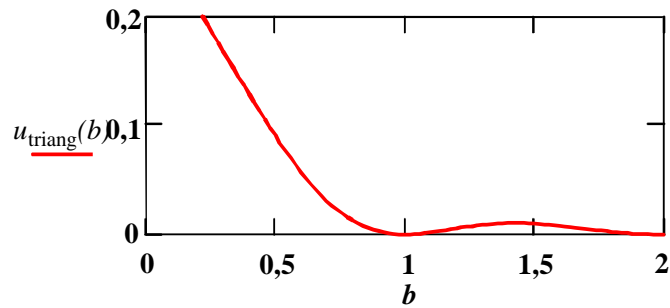
Fig. 11. RMS systematic error u of dither with triangular distribution on (- bq … bq).

3.3.3 Sinusoidal dithering

If the signal to be converted contains sinusoidal interference, the interference can be used as a dither signal. If there is a possibility to arrange so that N conversions to be averaged cover exactly the time of one network period, one can combine the ideas of dithering and integrating voltmeter. The interference is suppressed and the resolution is improved simultaneously.

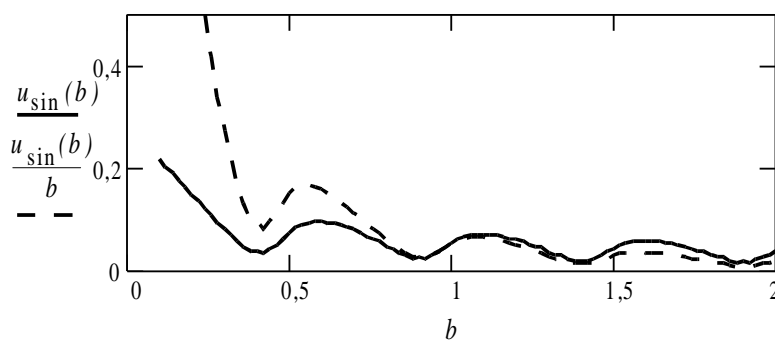The RMS systematic error of this arrangement is presented in Fig. 12.

Fig. 12. RMS systematic error of the sinusoidal dithering. b is the relative amplitude of the dithering sinusoid.

3.4 Stochastic dithering

The dither signals of chapter 3.3 are usually deterministic. This has a reason: the random error of the final result is usually smallest in this way. It is however possible to use stochastic dither signals also. Some commercial ADCs use this as their only dithering system.

A generated stochastic dithering can observe the condition (8) for unbiased dithering. More often there is a sort of stochastic dither signal present as a result of the noise in the system. In this case the distribution is most often normal and this does not fulfil exactly (8). There is however a possibility to have a bias that is small enough, if the variance of the noise is big enough. This is shown in Fig. 13.
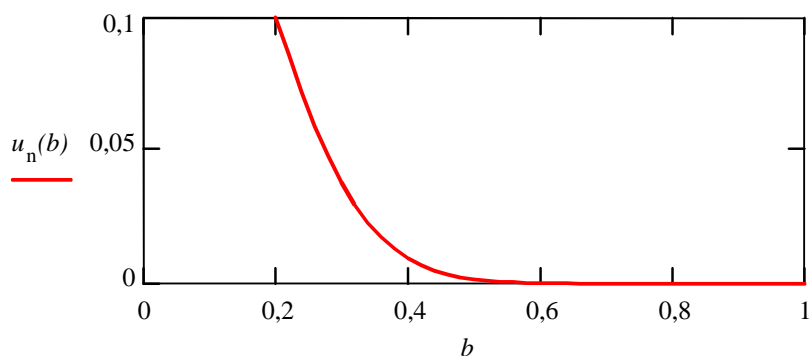
Fig. 13. RMS systematic error u of a normally distributed dithering. b is the normalised standard deviation of the dither.

3.5 Dithering in floating point conversion

Some computers use extensively floating point processing. There are also floating point ADCs, and the possibility for dithering may come out here, too.

Dunay *et al.* (Dunay *et al.* 1998) has presented a dithering system for this type of converter. The analysis is based on the characteristic of the converter in Fig. 14. The main idea is to dither with a distribution that is

uniform or triangular and scaled for the mantissa (to the actual step size). This is practical, because the signal is relatively seldom at the corner points of the characteristic where the step size changes. For a proper dithering, more bits are needed than the floating-point arithmetic usually provides.

A precise theory for this case is still to be developed. (Dunay *et al.* 1998) suggests for this purpose the model of Fig. 15.
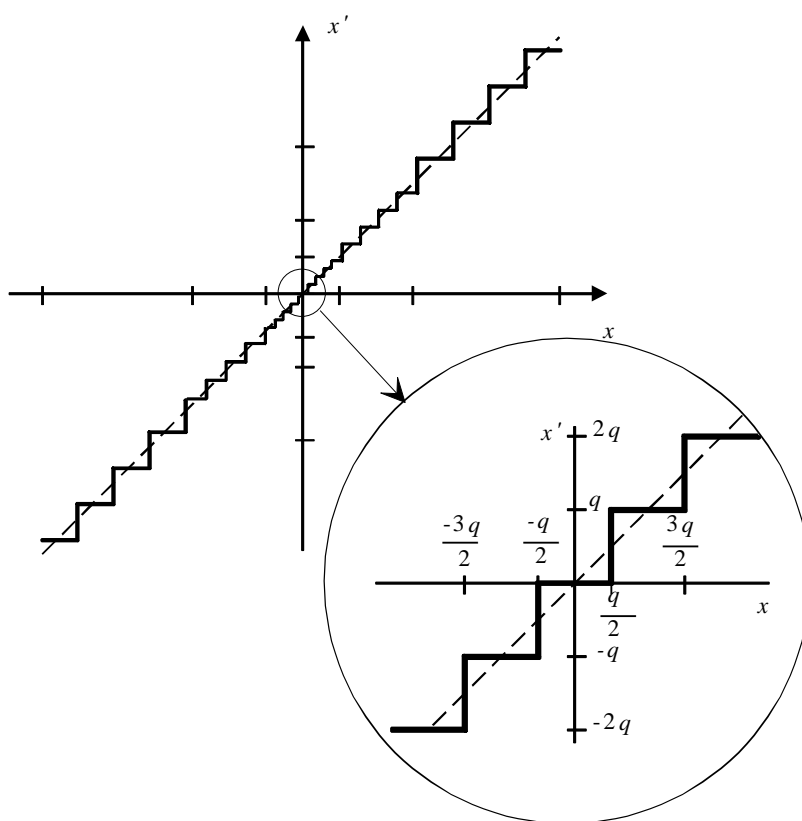

Fig. 14. The characteristic of a floating-point quantiser with a 3 bit mantissa.
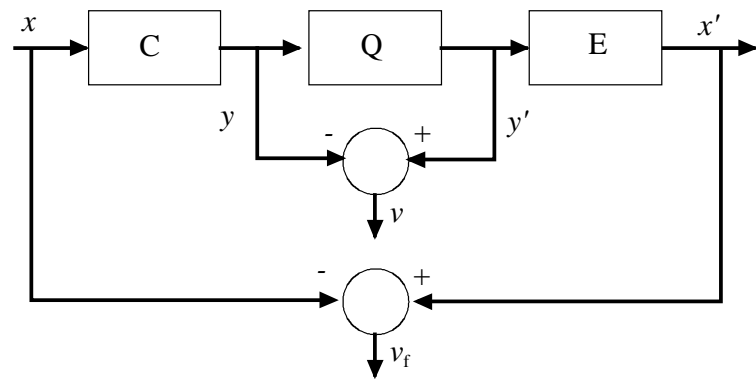
Fig. 15. A model of a floating point quantiser. x measurand, y and y' hidden signals, x' quantised signal, v hidden quantisastion error, v_f measurable quantisation error, C compressor, Q uniform quantiser, E decompressor.

4   Resolution, linearity and total uncertainty

The common dithering method is to add N values distributed uniformly over an interval $(-\frac{2k+1}{2}q, \frac{2k+1}{2}q)$, most often $(-q/2, q/2)$. In this "classical" case the systematic error is zero, and there is an uncertainty in one original conversion result only. The resulting resolution is q/N and the type A standard uncertainty of the final result is

$$u_{\mathrm{r\,unif}}(N) = \frac{q}{2N\sqrt{3}} \qquad (11)$$

It should be noted that in principle (11) applies regardless of k.

For other deterministic dithering cases the type A uncertainty can be determined according to the same principle. The number of original conversions, that are uncertain, is decisive. For the smallest useful triangular distribution, that number is 2 and the resulting random uncertainty is

$$u_{\mathrm{r\,triang}}(N) = \frac{q}{N\sqrt{6}} \qquad (12)$$

The formula (11) applies also to the sinusoidal dithering if the amplitude of the dither is less than $q/2$. For bigger amplitudes the corresponding number is bigger. For example the amplitude $b = 0,8\ q$ gives a local minimum for the RMS systematic error. Here the statistical mean of the above mentioned number of uncertain conversions is 1,6 and the type A uncertainty is

$$u_{r\sin}(1,6) = \frac{q}{N\sqrt{7,5}} \tag{13}$$

For stochastic dithering the uncertainty must be calculated assuming that each original conversion has an independent random uncertainty. (Holub 1997) gives the result for uniformly distributed stochastic dither with a maximum value $kq/2$

$$n_{eff} = n + 0,5\log_2 \frac{N}{1+k^2} \tag{14}$$

where n is the bit number of the converter, N is the number of averaging, and k is the dithering range divided by q. This corresponds for $k = 1$ to

$$u_{r\,stoch}(1) = \frac{q}{2\sqrt{6N}} \tag{15}$$

For a stochastic dither with normal distribution the same reference gives the same formula, but the value of k has to be calculated as follows:

$$k = \frac{U_{dith} \cdot 2^{n+1}\sqrt{3}}{R} \tag{16}$$

where U_{dith} is the RMS value of the dither signal, n is the number of bits in the conversion and R is the range of the converter.

The main difference between the deterministic and the stochastic dither is that the type A uncertainty is relative to $1/N$ for deterministic cases, but relative to $1/\sqrt{N}$ for stochastic cases.

The total uncertainty includes still other uncertainty components that are not analysed in this study. The integral nonlinearity is important here.

5   Viewpoint of measurement information

The most practical importance of a measurement is its information content (Aumala 1999). The classical information theory presented by Shannon (1948) analyses however only one aspect of the information: the syntactic information. Other aspects are the pragmatic information (meaningfulness, dependability, etc.) and the semantic information (usefulness, specificity, etc.). In this paper only the syntactic information is analysed, and it is discussed here as an equivalent to the measurement uncertainty. For dynamic measurements the channel capacity is discussed.

The syntactic information content of a single (final) measurement result may be expressed using the number of significant bits in it. This can be determined from values of the measurement range and total uncertainty.

The uncertainty analysis is made in Chapter 3. The results presented there are from (Aumala and Holub 2000a) in form of number of significant bits according to (Baccigalupi 1999). For example the classical uniform dithering has

$$n_{\text{eff}} = n + \log_2(N) \tag{17}$$

where n is the number of bit in the converter and N is the number of averaging for each final result. For a normally distributed stochastic dithering with $\sigma_{\text{dith}} = 0{,}7\ q$

$$n_{\text{eff}} = n + \frac{1}{2}\log_2\left(\frac{Nq^2}{q^2 + 12\sigma_{\text{dith}}^2}\right) \tag{18}$$

These cases are shown graphically in Fig. 16 for a commercial 10 bit converter. The values n_{eff} can be read on the left hand side scale. The figure contains also data for the channel capacity.
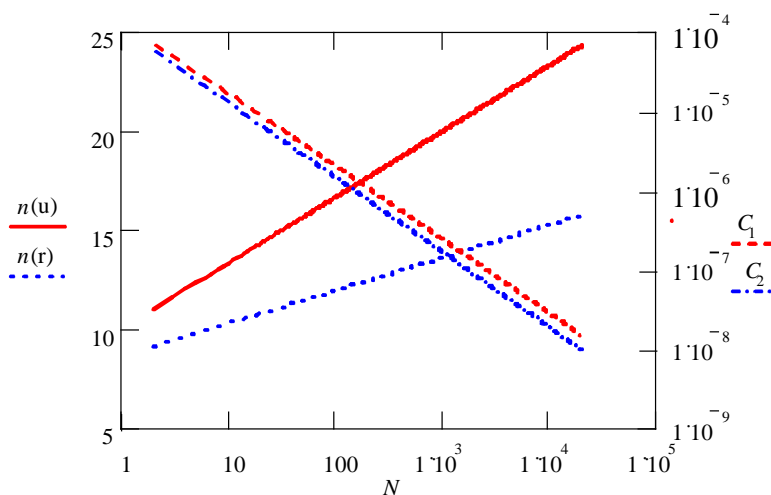

Fig. 16. Dithering increases the effective number of bits n_{eff} of an A/D converter. A/D board resolution 10 bits, conversion rate 80 kS/s. n(u) standard dithering (-q/2, q/2), n(r) dithering with normally distributed noise, $\sigma_{noise} = 0,7$ q. N is the averaging number. C_1 channel capacity of the standard dithering case, C_2 for the normally distributed noise dithering.

The channel capacity is a very important concept for information transfer. In measurement applications it can often be evaluated as the product of the effective number of bits and the bandwidth. For measurement of quasistatic signals the bandwidth is however not very important.

The bandwidth of a transfer device or system (amplifier etc.) used for sinusoidal signals is usually given as the –3 dB bandwidth. This is well suited for A/D converter tests with sinusoidal signals.

The characteristic operation of dithered converters is the averaging. If it is calculated as a moving average, the amplitude characteristic obeys the formula

$$\frac{Y(f)}{S(f)} = \frac{\sin(2\pi f N \Delta t)}{2\pi f N \Delta t} \qquad (19)$$

where Y is the spectrum of the output signal, S is the spectrum of the input signal, and Δt is the sampling period. The value corresponding to –3 dB is

$$f_{\text{-3 dB}} = \frac{0,227}{N \Delta t} \qquad (20)$$

For many measurement purposes the concept of response time is more useful than the –3 dB bandwidth. Because independent results within the total uncertainty can be available at a pace of averaging time, the bandwidth is taken here as the inverse of the averaging time

$$B = \frac{f_{\text{c}}}{N} \qquad (21)$$

Fig. 16 presents the theoretical channel capacity for the standard dithering case (uniform dithering $-q/2$, $q/2$) and for the stochastic dithering with normally distributed noise, $\sigma_{\text{noise}} = 0,7\ q$.

One can see immediately that the channel capacity depends mainly on the bandwidth. If the channel capacity is important, dithering should be restricted to the minimum needed.

Finally it should be noted that the ADC is often connected directly to a computer. In this case it is advisable to consider whether some dedicated

computing algorithm would be better than the simple averaging. For example, if the measurement is used for control purposes, possibly a Kalman filter designed to match the dithered original results could be applied.

6 Virtual dithering ADC

The theory of dithering and the presented cases were studied experimentally using a virtual instrument developed for this reason (Aumala. 1996c). The panel and the diagram are presented in Fig. 17 and Fig. 18. The instrument is currently available for downloading at http://mit.tut.fi/simulator/.
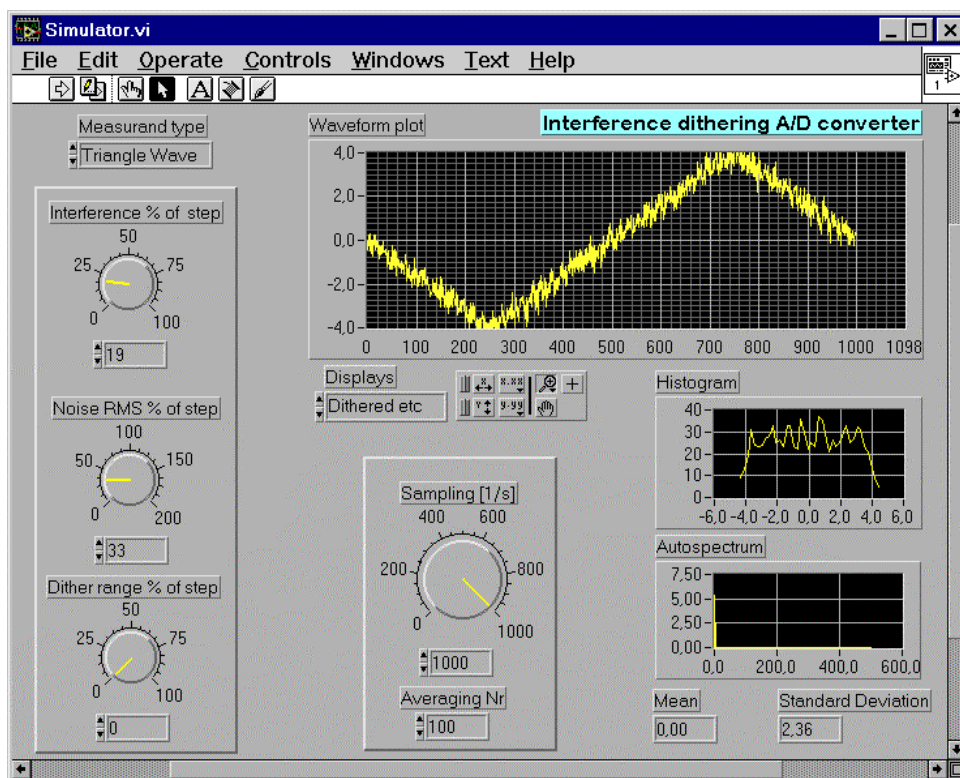

Fig. 17. The panel of the simulator. On the left side, setting devices for the measurand type and for the parameters of the dither signal. A combined dither is also available. Down in the centre, setting of sampling and averaging. All displays present the signal and its parameters chosen by the switch under the main display window.

The simulator is a LabVIEW virtual instrument having switches and knobs for setting amplitudes for various types of dither signals as well as

parameters of sampling and averaging. The displays present waveforms and some statistical parameters for the measurand, the dithered measurand, the original quantised signal, the error, and the final measurement result.
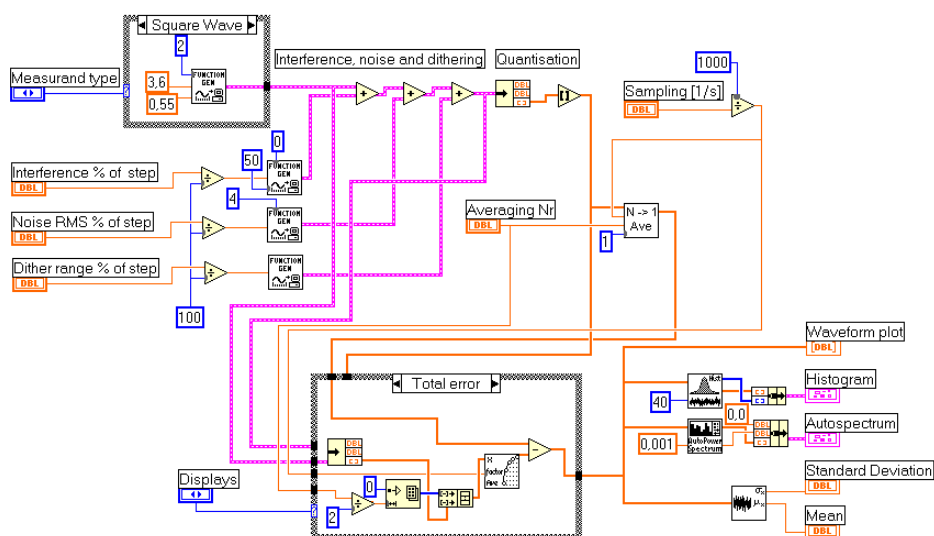

Fig. 18. The diagram of the virtual instrument. There is a full correspondence between blocks here and the knobs, switches and displays in Fig. 17. For switches of the panel only one of the alternative diagrams is visible.

By experimenting with this simulator it was easy to learn basic properties of dithering and to see that the theory works. One very useful observation was made: the assumption that the measurand is quasistatic has to be taken carefully. The first calculations of the behaviour of the systematic error of the normally distributed random dither were incorrect, because the author used numerical integration and the integration range was not broad enough. The simulator seemed to verify this result! Later on the error was detected. A further study showed that the wrong impression was a result from the (rather slow) movement of the measurand. The measurand was not steady enough during the averaging period!

7 Applications

The results given in this study were validated partly using a virtual dithering analogue-to-digital converter (Aumala 1996c), partly by analysing practical converters. The virtual converter was very suitable for analysing the systematic error and uncertainty for various dithering cases. There was a possibility to have displays for various phases of the operation as well as for certain statistical functions.

Two practical analyses were extremely helpful. One of them was the analysis for improving the integral nonlinearity of an analogue-to-digital converter and the other was a study of a PC Card Based DSP System for Temperature Measurement.

It can be noted that dithering is generally able to decrease the effect of nonlinearity of the characteristic if the dithering amplitude is at least in the order of this nonlinearity. This applies also to the integral nonlinearity of the analogue-to-digital converter. (Holub and Šmid 1998b) have analysed this effect using wavelet analysis (see Fig. 19 and Fig. 20). One can see in Fig. 20 that the dithering range should be rather broad, in this case 40 times the step size q.
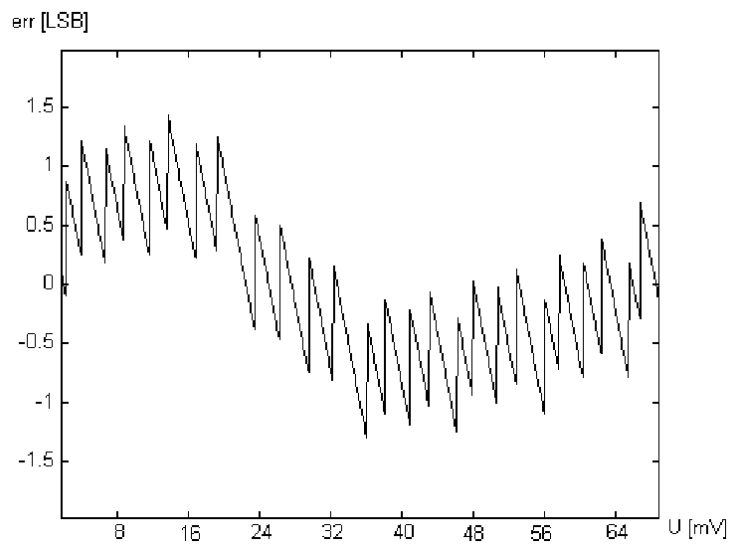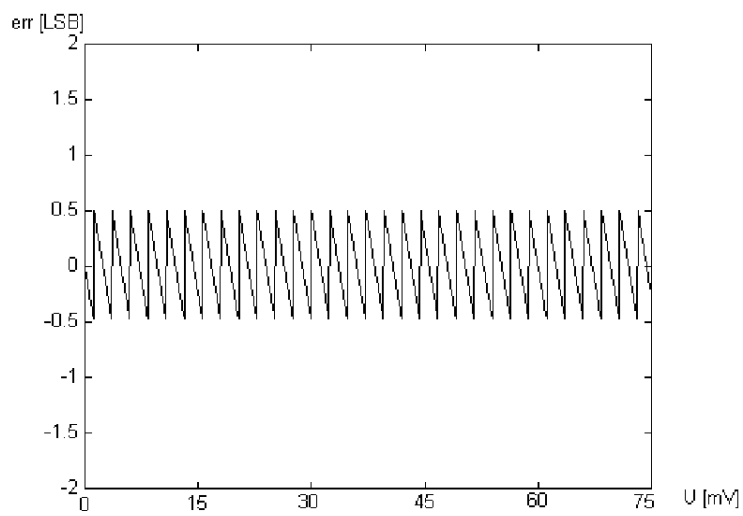
Fig. 19. A part of an ideal and a real characteristic of an analogue-to-digital converter (Holub and Šmid 1998b).
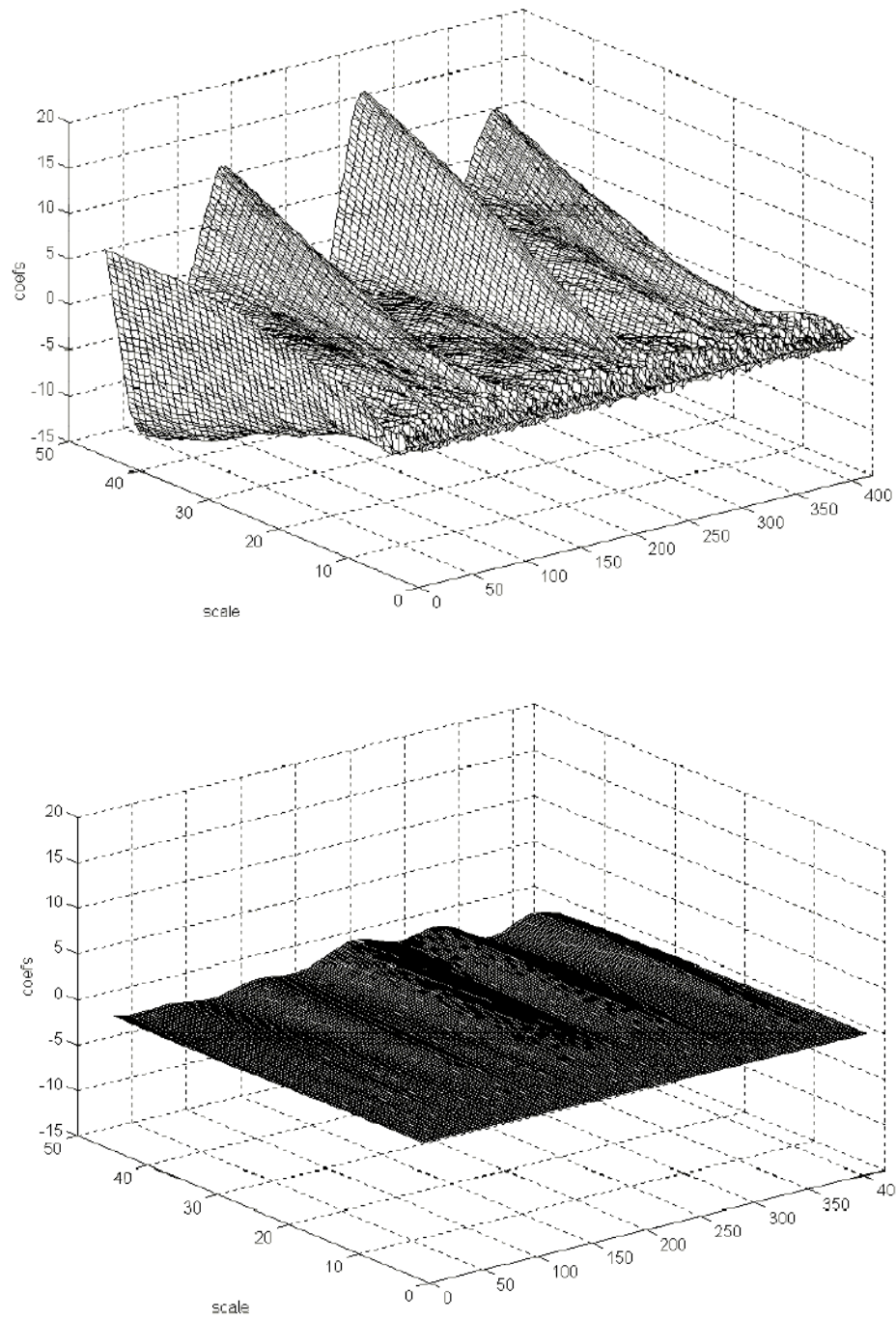
Fig. 20. Continuous Wavelet Transform of the error for the real analogue-to-digital converter. Above: without dithering, below: dithered uniformly, $k = 40$ (Holub and Šmid 1998b).

(Janásek 1998) presents a study on a multichannel PC card based DSP system for temperature measurement. This analysis gives a practical case for improving the resolution of a measurement system by dithering.

The sensor is a platinum temperature resistance element. In this case, the noise of the ADC system was with amplitude of 3 LSB, or with RMS value 0,5 LSB. The noise was used as a dither signal and the averaging showed also a good rejection of interference voltages. The system was able to measure up to 32 temperature channels with resolution of some hundredths of kelvin and the total measurement time of 20 or 40 ms. The resolution was 16 bits instead of the original 12 bits of the converter, and there was a good rejection of interference frequencies of 50 Hz and its harmonics.

8 Discussion

This study was performed during the years from 1995 to 2001. After publishing the first conference papers the author noted that the subject had raised a wider interest. The central role of the analogue-to-digital converter was recognised and a special Working Group was founded. This Working Group organised also a group for developing methodology for ADC testing.

This study analyses properties of the analogue-to-digital converter, which is very central in modern measurement equipment. The work is mainly theoretical, and the theoretical models were verified partly with experimental study, partly with a virtual instrument developed for this purpose.

Fundamental properties of analogue-to-digital conversion are presented as background of the study. Limitations coming from the time-discrete nature of sampling and from the amplitude-discrete nature of quantisation are analysed.

The basic idea of dithering is presented. The condition for freedom of bias is developed. It is found that the dither with a triangular distribution is advisable because it is not very sensitive to the differential nonlinearity of the converter. The uncertainties of some practically interesting dithering schemes are developed. It is presented that dithering can be combined with suppression of network interference. Also the possibility to use unavoidable thermal noise of the measurand as dither signal is presented.

A note for dithering in floating point conversion is given. There remains a need to develop a thorough theory in this topic.

A short analysis on information transfer of the dithered conversion is presented. It is shown that dithering gives more accuracy, but this reduces the bandwidth of the conversion. Indeed, dithering may be characterised as changing bandwidth against resolution.

Two practical applications are briefly discussed. The first one shows how broad-range dithering can improve even the integral nonlinearity of the ADC. The second application is equipment for multichannel temperature measurement with very good resolution and speed.

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