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Jussi Jalkanen

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Abstract

The topic of this thesis is discrete tubular truss optimization using heuristic multipurpose algorithms. The multicriteria topology, shape and sizing optimization problem has been formulated based on practical real life needs. This means that in addition the material cost also the manufacturing cost has been taken into account, design constraints satisfy the demands of steel design rules (Eurocode 3) and the selection of available tubular profiles does not have to be in a special way limited. Mass, cost or displacement have been used as the objective function criteria in the example problems. The design constraints ensure that truss members and joints are strong enough, single members or the whole truss do not buckle, certain displacements are small and natural frequencies do not locate in forbidden intervals.

The tubular truss optimization problem has been solved using four multipurpose heuristic algorithms. Two of them are local search algorithms simulated annealing (SA) and tabu search (TS) and two of them population based methods genetic algorithm (GA) and particle swarm optimization (PSO). The efficiency of heuristic algorithms has been studied empirically in several example problems. In the first academic ten-bar truss example problem heuristic algorithms have also been compared to a different type of branch&bound algorithm. The rest of the example problems deal with tubular plane and space trusses. Also the conflict of mass and cost and the effect of the number of different design variables to the final solution have been studied.

Preface

This thesis is based on work done as an assistant, a graduate school student and a senior assistant at the Institute of Applied Mechanics and Optimization, (Tampere University of Technology) during years 2000-2007. These eight years have not gone entirely to research since studying and teaching have also taken their time. I consider this thesis as the end of my rather long education.

First I would like to thank Professor Juhani Koski for the possibility to work at the Institute of Applied Mechanics and Optimization, his inspiring teaching and very competent guidance to structural optimization. The working environment has been excellent for academic research and free from exhausting responsibilities. I have always felt that Juhani has supported me in my work.

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Tampere, November 2007

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Nomenclature

Roman characters

A	cross-section area
A_0	cross-section area of chord
A_s	outer surface area
A_t	area inside centerline of profile wall
A_v, A_{vy}, A_{vz}	shear areas
a_w	size of fillet weld
a_{sp}, a_{gc}, a_{tc}	parameters in cost function
B	tabu list
b	width of profile
b_0	width of chord
b_1, b_2	width of bracing members
b_e	effective bracing member width
b_{ep}	effective punching shear width
$b_{e(ov)}$	effective width for overlapping bracing member
C	big positive constant
C_1, C_2	parameters in cost function
c_1, c_2	scaling parameters in PSO
d	difference of objective function values
d_n	depth of discrete neighborhood
E	Young's module
e	Neper's constant
e	eccentricity in joint
$\overline{F}$	fitness function
F	force

$\mathbf{f}$	global load vector
f	objective function
f^*	value of objective function in optimum
f^g	best known objective function value
$\tilde{f}$	penalized objective function
$\bar{f}$	scale factor in frequency constraint function
$f_1, f_2, \ldots$	natural frequencies
f_y	yield limit
f_{y0}	yield limit of chord
f_{y1}, f_{y2}	yield limits of bracing members
f_j^{max}	jth forbidden frequency interval upper limit
f_j^{min}	jth forbidden frequency interval lower limit
g	gap in joint
g^b	buckling strength constraint function
g^f	frequency constraint function
g^{gb}	global buckling constraint function
g_i	ith inequality constraint function
g^j	joint constraint function
g^{js}	joint strength constraint function
g^{jv}	joint validity constraint function
g^{kkgj}	KK-connection constraint function
g^s	strength constraint function
g^u	displacement constraint function
H	height of truss
$\overline{H}$	height of space truss
h	height of profile
h_0	height of chord
h_1, h_2	height of bracing member
I, I_y, I_z	moments of inertia

I_v	torsion modulus
i, j, k	indexes
i_r	radius of gyration
$\mathbf{K}$	global stiffness matrix
K	cost or cost function
K^{max}	maximum allowed cost
K_f	manufacturing cost
K_m	material cost
$\mathbf{k}$	element stiffness matrix
$\mathbf{K}_g$	global geometrical stiffness matrix
$\mathbf{k}_g$	element geometrical stiffness matrix
k_f	fabrication cost factor
k_m	material cost factor
k_n	parameter
k_y, k_z	parameters
L	length
L_b	buckling length
L_c	length of cut
L_w	length of weld
$\mathbf{M}$	global mass matrix
M	bending moment
$\overline{M}$	set of moves
$M_{Sd}, M_{y.Sd}, M_{z.Sd}$	design value for bending moment
$M_{0.Sd}$	design value for chord bending moment
$M_{c.Rd}, M_{c.y.Rd}, M_{c.z.Rd}$	bending moment capasities
$M_{pl.y.Rd}, M_{pl.z.Rd}$	plastic bending moment capasities
$M_{N.y.Rd}, M_{N.z.Rd}, M_{V.y.Rd}, M_{V.z.Rd}$	reduced bending moment capasities
$M_{t.Sd}$	design value for torsional moment
$M_{t.Rd}$	torsional moment capacity
$M_{t.pl.Rd}$	plastic torsional moment capacity

$\mathbf{m}$	element mass matrix
m	mass
$\overline{m}$	move to solution
N	normal force
N_{Sd}	design value for normal force
$N_{\mathrm{t.Rd}}$, $N_{\mathrm{c.Rd}}$	design capacities for normal force (tension and compression)
$N_{\mathrm{pl.Rd}}$	plastic design capacity for normal force
$N_{\mathrm{V.Rd}}$	reduced normal force capacity
$N_{\mathrm{b.Rd}}$	buckling capacity
$N_{i.\mathrm{Sd}}$	design value for ith joint bracing member
$N_{0.\mathrm{Sd}}$	design value for chord normal force
$N_{0.\mathrm{Rd}}$	chord normal force capacity
$N_{i.\mathrm{Rd}}$	ith joint bracing member capacity
$\overline{N}(\mathbf{x})$	discrete neighborhood of $\mathbf{x}$
n	parameter
n_{av_i}	number of allowed values for discrete x_i
n_{c}	number of criteria
n_{cs}	number of candidate solutions
n_{dv}	number of design variables
n_{df}	number of degrees of freedom
n_{f}	number of forbidden frequency intervals
n_{fix}	number of iteration rounds while x_i stays fixed
n_{gen}	number of generations
n_{ie}	number of constraints
n_{iter}	number of iteration rounds
n_{Lc}	number of load cases
n_{m}	number of truss members
n_{pop}	size of population
n_{s}	number of size design variables

$n_{\sec_i}$	number of allowed sections for *i*th member
n_{sh}	number of shape design variables
n_{swarm}	size of swarm
n_{T}	number of points in same temperature
n_{t}	number of topology design variables
n_{u}	number of displacement constraints
$\tilde{n}$	exponent
$\bar{n}$	number of candidate solutions
$\bar{n}_{\mathrm{gb}}$	safety factor against global buckling
P, p	probability
$\mathbf{p}_k^i$	best position for particle *i* during iteration round *k*
$\mathbf{p}_k^{\mathrm{g}}$	best position for swarm during iteration round *k*
p_{c}	crossover probability
p_{m}	mutation probability
p_i	selection probability of *i*th individual
q	overlap in joint
R	penalty for unfeasible constraint
r	radius of profile corner
r_1, r_2	random numbers
$\mathbf{S}$	constant matrix
S_0	initial population
S_k	*k*th population
$\bar{S}$	set of parents
s	length of tabu list
T	temperature
T^0	initial temperature
T^{final}	final temperature
T_i	duration of *i*th manufacturing stage

t	wall thickness of profile
t_0	wall thickness of chord
t_1, t_2	wall thickness of bracing member
$\mathbf{u}$	global node displacement vector
u	displacement in x-axis direction
$\overline{u}^{\max}$	maximum allowed displacement
$\overline{u}^{\min}$	minimum allowed displacement
V_0	initial set of velocities
V_k	set of velocities
V_{Sd}, $V_{y.\mathrm{S}d}$, $V_{z.\mathrm{Sd}}$	design values for shear force
V_{Rd}, $V_{y.\mathrm{Rd}}$, $V_{z.\mathrm{Rd}}$	shear force capacities
$V_{\mathrm{pl.Rd}}$, $V_{\mathrm{pl}.y.\mathrm{Rd}}$, $V_{\mathrm{pl}.z.\mathrm{Rd}}$	plastic shear force capacities
$\mathbf{v}$	velocity
v	displacement in y-axis direction
$\overline{v}^{\max}$	maximum allowed displacement
$v^{\max}$	maximum velocity
v_0^i	initial velocity for ith particle
W_0	section modulus of chord
$W_{\mathrm{el}.y}$, $W_{\mathrm{el}.z}$	elastic section moduli
$W_{\mathrm{pl}.y}$, $W_{\mathrm{pl}.z}$	plastic section moduli
W_{t}	section modulus in torsion
w	inertia
w_0	initial inertia
$\mathbf{x}$, $\mathbf{y}$, $\mathbf{s}$	design variable vector
$\mathbf{x}^*$	optimum design variable vector
$\mathbf{x}^0$	initial solution
$\mathbf{x}^k$	design variable vector during iteration round k
X	design space
X_0	initial set of design variable vectors

X_k	set design variable vectors
x, y, z	coordinates
x_i	ith design variable
x_i^{s}	ith size design variable
x_i^{sh}	ith shape design variable
x^{t}	topology design variable

Greek characters

α	imperfection factor
$\overline{\alpha}$	exponent
$\tilde{\alpha}$	parameter
β	parameter
β_{M}, $\beta_{\mathrm{M}y}$, $\beta_{\mathrm{M}z}$	equivalent uniform moment factors
γ	parameter
γ_{M0}, γ_{M1}, γ_{j1}	partial safety factors
δ	dynamic inertia reduction parameter
ε	temperature coefficient
η	correction factor
Θ_{d}	difficulty factor for preparation, assembly and tack welding
Θ_{dc}	difficulty factor for cutting
Θ_{dp}	difficulty factor for painting
Θ_{ds}	difficulty factor for surface preparation
θ_1, θ_2	angles between bracing members and chord
κ	number of structural elements
λ	load factor
λ_{ov}	overlap
λ_{cr}	critical load factor

$\bar{\lambda}$	non-dimensional slenderness
μ_y, μ_z	parameters
ρ	density
ρ_y, ρ_z	parameters
$\boldsymbol{\sigma}$	stress vector
σ	stress
σ^{max}	maximum allowed stress
ϕ	parameter
χ, χ_y, χ_z	reduction factors
χ_{min}	smaller reduction factor
ψ	factor
Ω	feasible set
ω, ω_1, ω_2, …	natural circular frequencies

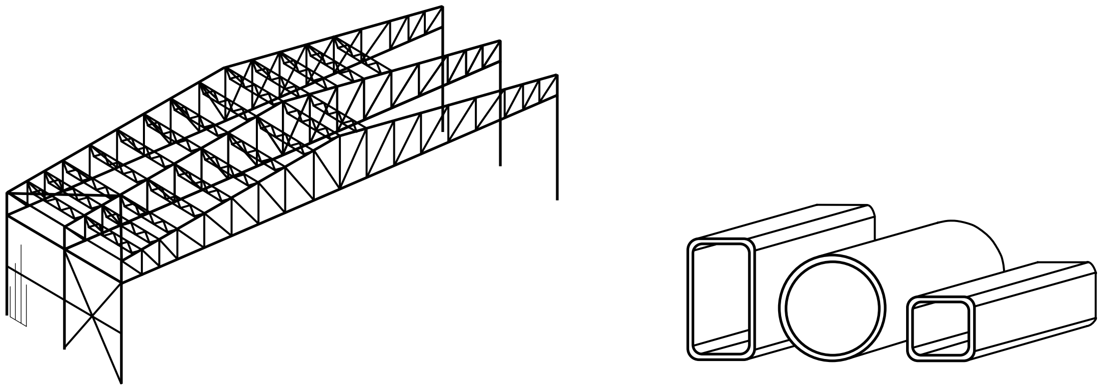
Acronyms

BB	branch&bound algorithm
CHS	circle hollow section
CO_2	carbon dioxide
GA	genetic algorithm
FEM	finite element method
PSO	particle swarm optimization
RHS	rectangular hollow section
SA	simulated annealing
SHS	square hollow section
SQP	sequential quadratic programming
TS	tabu search

1. Introduction

1.1 Background

Welded tubular steel trusses have become common in the applications of structural and mechanical engineering (Fig. 1.1). There are several reasons which have led to this and one of the most significant is the excellent mechanical properties of tubular members. Structural hollow sections have high bending and torsional rigidity compared to their weight and they are suitable for compressed members. Another reason that can be mentioned is the large amount of research which has been done to ensure the safety of the design codes of tubular members and joints. Also the selection of commercially available tubular profiles is large which makes it possible to choose appropriate profiles in a truss.



a) b)

Figure 1.1 a) Welded tubular roof trusses and columns. b) Rectangular (RHS), circle (CHS) and square (SHS) hollow section.

In the design of tubular trusses the next natural step is to move from analysis to optimization. Structural optimization offers a systematic way to go further than the traditional analysis of a few candidate structures that were selected based on designer’s experience and intuition. Modern optimization techniques with remarkably improved computer capacity help to find new better designs which would be otherwise left undiscovered.

The three basic approaches of structural optimization are sizing, shape and topology optimization (Fig. 1.2). In sizing optimization the idea is to change the cross-section dimensions or properties like plate thickness or cross-section area. In shape optimization the target is to find the best shape of the structure. In topology optimization the designer seeks the optimum structure by changing the amount and the location of material or components in the structure. The combination of these approachies is also possible and it can be assumed to give the best final result. In the literature the fundamental concepts of structural optimization have been presented in the text books of Arora [4], Farkas & Jármai [17], Haftka & Gürdal [23], Kirsch [36] and Vanderplaats [66].

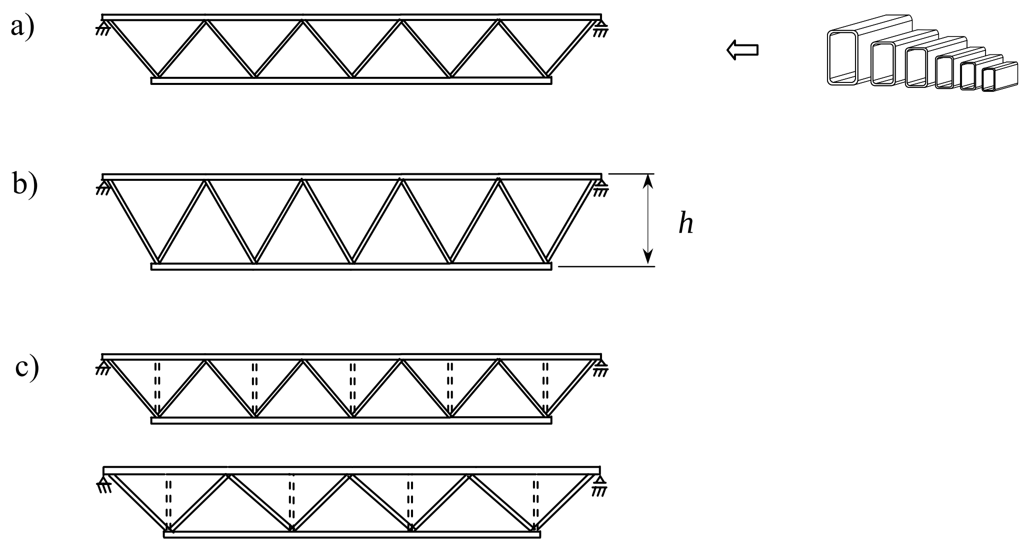


Figure 1.2. a) The sizing optimization, b) shape optimization and c) topology optimization of tubular truss.

The type of design variables affects strongly on chances to solve a structural optimization problem. If all design variables can have any value between certain lower and upper bounds the problem is said to be continuous. In a discrete or integer optimization problem there is only a certain set of possible values available for each design variables. Mixed-integer problem is a combination of the previous ones containing both continuous and discrete design variables. Discrete or mixed-integer optimization problems are usually much harder to solve than the ones that contain only continuous design variables. In the optimization of tubular trusses at least some of the design variables are typically discrete. For example in the sizing optimization the task is to select suitable profiles for each truss member from the given finite selection of structural hollow sections and arbitrary sizes can not be selected. Nemhauser & Wolsey [48] and Floudas [20] deal with discrete optimization from mathematical point of view in their books. The articles of Arora, Huang & Hsieh [5], Arora [6], Bauer & Gutkowski [8], Huang & Arora [24] and Thaeder & Vanderplaats [64] offer reviews concerning discrete structural optimization. Gutkowski's book [22] introduces to discrete structural optimization and also e.g. Aho [3] and Turkkila [65] treat discrete structural optimization problems in their dissertations.

In structural optimization the ultimate target is usually the minimization of the cost of structure and the role of design constraints is to take care that the structure remains useful and it is possible to manufacture. Since the cost can be approximated up to certain point based on the material consumption the mass is traditionally chosen as the minimized objective function in metal structures. The manufacturing cost is not considered separately and it is assumed to be directly proportional to the mass of structure. However the amount of material cost from the total cost varies depending on the type of the welded steel structure and the minimum mass structure is not necessarily always the same as the minimum cost structure. Earlier the cost optimization of welded steel structures has been considered in the articles of Jármai & Farkas, [18], [30] and [31]. In all those papers the approach is basically the same and the form of cost function is modified depending on desired accuracy and application (welded box beam [30] and [31], stiffened welded plate [18] and [30]). Pavlovčič, Krajnc & Beg [54] have presented more detailed cost function than Jármai & Farkas and applied it to plane frames made of welded I-beams. Also Klanšek & Kravanja [37] and [38] use a rather detailed cost function for composite floor systems. Sarma & Adeli give a chronological review of the journal articles [60] and concern the fuzzy cost optimization of steel structures [61]. In addition Adeli & Sarma deal with the cost optimization in their book [2].

Although the designer is usually interested in the minimization of the cost of tubular truss it is worthwhile to formulate the optimization problem as a multicriteria problem. In multicriteria optimization two or more conflicting criteria can be considered at the same time. For example the designer may want to reach the minimum mass tubular truss which is also as stiff as possible. The solution for a multicriteria optimization problem is a set of Pareto optimal points which can be achieved by solving single criterion problems. Since the solution of multicriteria optimization problem is not unique the designer (or the so called decision-maker) can choose the final design from a set of mathematically equal optimal solutions based on some additional information (e.g. the pleasantness of appearance) which was not possible to be included to the original optimization problem. The set of Pareto optima also enables the trade-off between competing criteria in the final decision-making and thus brings more potential to the optimization. Miettinen gives a good mathematician's view to multicriteria optimization in her book [46] and Koski [40], Marler & Arora [45] and Osyczka [51] introduce to multicriteria structural optimization. Previously Kere [33], Koski [39] and Turkkila [65] have applied multicriteria optimization in their dissertations.

In tubular truss optimization the demands of appropriate steel design code such as Eurocode 3 [63] and other relevant recommendations [42], [53], [58], [69], [70] and [56] have to be taken into account so that the optimized structure would be usable. The use of design code and recommendations brings on several constraints and some of them are inconvenient from many optimization algorithms' point of view due to e.g. their piecewise definition. Heuristic multipurpose optimization algorithms like simulated annealing (SA), tabu search (TS), genetic algorithm (GA) and particle swarm optimization (PSO) are simple methods for solving such problems with numerous awkward constraints and typically discrete design variables. For these algorithms it is enough if the values of the objective function and constraints can be calculated in a given point in the design space. Heuristic methods can not guarantee the global optimum in reasonable time but it is possible to find solutions that are good enough in many applications.

Heuristic algorithms are rather popular optimization methods in literature. Introductions to SA, TS and GA can be found from books [1] (Aarts & Lenstra) and [57] (Reeves). Balling [7] has used SA in the optimization of steel frames and also articles of Chen & Su [12], Leite & Topping [43] and Moh & Chiang [47] consider structural optimization using SA. Bennage & Dhingra [9] have used TS to the topology optimization of trusses, Bland [10] to discrete truss sizing optimization and Dhingra & Bennage [16] to the single and multicriteria optimization of trusses with continuous and discrete design variables. Ohsaki, Kinoshita & Pan [49] have used special multicriteria SA and TS techniques in seismic design of steel frames. GA is definitely the most popular heuristic method among the four above mentioned and there are hundreds of studies concerning structural optimization with GA. Pezeshk & Camp [55] offer an extensive summary concerning previous works in the optimization of steel structures with GA. Multicriteria genetic algorithms have been introduced e.g. in Deb's and Osyczka's books [14] and [52]. PSO is a rather new heuristic optimization algorithm which has gained increasing attention during last few years. Venter & Sobieszczanski-Sobieski have used it in a simple cantilever beam example [67] and also in a very large transport aircraft wing multidisciplinary optimization problem [68]. Fourie & Groenwold [21] and Schutte & Groenwold [62] in turn have used PSO in several truss sizing optimization problems and one shape optimization problem. A multicriteria particle swarm optimization algorithm has been introduced in Coello's and Lechuga's paper [13].

For a designer it can be difficult to choose a proper heuristic algorithm in a new optimization problem. In the literature the number of different available methods is big and several versions and parameter values have been used. The common factor for the most of the papers seems to be that they focus on only one algorithm at a time without considering other rather similar methods.

Besides the development of individual algorithms also comparisons between different algorithms are needed to clarify the mutual efficiency and the applicability of rival heuristic methods. Previously Jalkanen & Koski [28] have compared the efficiency of SA, TS, GA and PSO in two space frame optimization examples. Manoharan & Shanmuganathan [44] have compared SA, TS and GA by using simple truss examples and Botello, Marroquin, Onate & van Horebeek [11] have studied the efficiency of SA, GA and their combination in truss problems. Also Degertekin [15] has compared SA and GA in nonlinear steel frame optimization.

Trusses are typical applications in structural optimization literature. However, most of the existing studies are more academic than practical. They do not consider structural hollow sections and ignore design codes. Previously Jármai, Snyman & Farkas [32] have studied CHS truss mass minimization problem with the size and shape design variables. Saka [59] has minimized the mass of geometrically nonlinear latticed dome made of circle hollow sections by chancing topology, shape and size. Also Kilkki has dealt with tubular truss mass minimization in his dissertation [35]. Farkas & Jármai have discussed the cost minimization of tubular trusses in their book [17] and article [19] which deals with simple CHS truss supporting an oil pipe. Kripakaran, Gupta & Baugh [41] use two phase method to tubular truss sizing and shape optimization in which the minimum cost is the target but the cost per truss member is supposed to be known. Iqbal & Hansen [25] discuss the cost optimization and use CHS truss size and shape optimization problem as an example. Klansek & Kravanja [38] have minimized the cost of composite floor systems in which a concrete slab is reinforced using a tubular truss.

1.2 Goal and outline of study

The first goal of this thesis is to formulate an optimization problem for tubular trusses in such a way that it has not only academic value but it is also applicable to the real life purposes. This means that in addition to the material cost (mass) also the manufacturing cost is taken into account, design constraints are based on steel design rules and the selection of available structural hollow sections is not limited to few selected sizes. The presented approach is meant to be general and it is not connected to any specific truss type or manufacturer.

The second goal is to find a solution for the formulated tubular truss optimization problem soundly and efficiently. Multipurpose heuristic optimization algorithms simulated annealing, tabu search, genetic algorithm and particle swarm optimization are chosen as the solution methods due to their great flexibility and versatility. The aim is to compare their mutual efficiency and to find out practical guidelines to the use of heuristic methods in the current problem. The tubular truss example problems are fictitious. The comparison is also done using an academic benchmark problem.

The content of this thesis is divided into six chapters. The first one provides a brief background and motivation for optimizing tubular trusses. It also reviews existing articles which are relevant in tubular truss optimization and presents the goals of study.

The second chapter deals with the design of tubular trusses. The first half brings out the benefits of structural hollow sections, introduces typical truss and joint types and considers some design aspects. The second half concentrates on the design code requirements of tubular trusses and the demands due to strength, buckling strength and static joint strength are presented for square and rectangle hollow sections in cross-section classes 1 and 2. In order to limit the amount of work circle hollow sections and cross-section classes 3 and 4 are not considered.

The third chapter discusses heuristic multipurpose algorithms simulated annealing, tabu search, genetic algorithm and particle swarm optimization. The common advantages and disadvantages are presented before local search algorithms (SA and TS) and population based methods (GA and PSO) are dealt in more detail.

The tubular truss optimization problem is introduced in chapter four. The design variables are introduced and objective function criteria and constraints are explained. In the end the optimization problem is presented in the standard form.

Chapter five considers example problems. At first the mutual efficiency of heuristic methods is compared in an academic test problem which is taken from literature. Then the same comparison is repeated using plane tubular truss which is also used to study the conflict of mass and cost and the simultaneous minimization of cost and deflection. The last example truss is a multiplanar tower.

The results of thesis are summarized in chapter six. It offers also some ideas for the future research.

2. Design of tubular trusses

2.1 Structural hollow sections

Structural hollow sections have several advantageous properties which have made them popular in the design of steel structures like roofing trusses and frameworks. In a closed thin walled cross-section material is distributed far away from centroid and bending rigidities and torsional rigidity are high compared to e.g. I-beam or angle section which has the same mass. Structural hollow sections are especially suitable for compression and torsion members. Lateral buckling and torsional buckling are not usually limiting phenomenon.

The design rules and recommendations for tubular steel trusses have been developed based on extensive theoretical and experimental research which has taken place over the last decades. Designer can find direct guidelines for the designing of typical trusses with various different details from Eurocode 3 [63] and CIDECT’s guides ([42] [53], [58], [69] and [70]). The large selection of commercially available structural hollow sections with exact design rules makes it possible to reach a safe and economical design in each particular case. Tubular trusses can be manufactured efficiently in machine workshop and transported in large parts to a construction site for the erection.

The closed tubular shape of cross-section reduces the area of outer surface and thus the need of painting and fire protection compared to open cross-sections. In the corners of hollow section the sufficient paint thickness is also easy to achieve due to big enough rounding. The void inside the tube can be exploited by filling it with concrete to increase the strength and the fire protection without changing tube dimensions. Other possibilities are to use it e.g. as a rain water pipe or as a part of water circulation for the fire protection. The air drag of tubular members is also smaller than the air drag of open cross-sections which reduces wind load. The last mentioned but in many cases not the least beneficial is the pleasant external appearance of structural hollow sections.

2.2 Some design aspects

In the tubular truss design the number of joints should be kept as small as possible to reduce the needed labor in machine workshop and to reach a low final cost. On the other hand it is advantageous if all the loads act in the joints so that the bending moment stays small in chords. In the Warren truss with K-joints the number of bracing members is smaller than in the Pratt truss with N-joints but the buckling length of the compressed upper chord is long (Fig. 2.1). In the modified Warren truss with KT-joints the compressed upper chord is supported better but the design of joints is more difficult and it is necessary to use overlap joints. In Pratt truss compressed bracing members are shorter than in modified Warren truss.

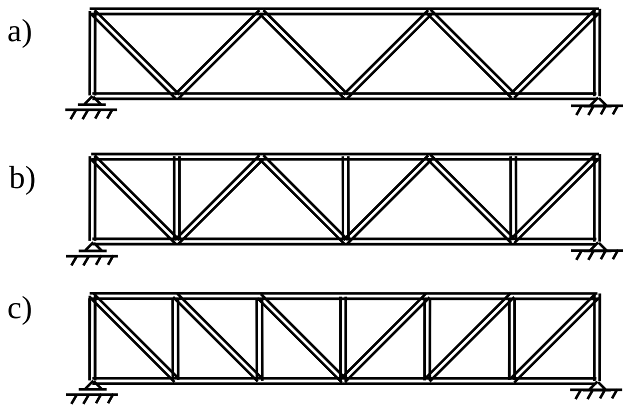


Figure 2.1. Typical tubular trusses: a) Warren truss with K-joints, b) modified Warren truss with KT-joints and c) Pratt truss with N-joints.

The overlap joints (Fig. 2.2) of RHS or SHS sections are more expensive than corresponding gap joints because the end of the bracing member has to be cut in two directions while in a gap joint only one cut is enough. In overlap joint dimensions have to be precise and there is no possibility for small adjustments as there is in gap joints. The strength of a fully overlap joint is usually better than the strength of a gap joint. In the gap joint the gap g has to be big enough and in the overlap joint the overlap q has to be sufficient.

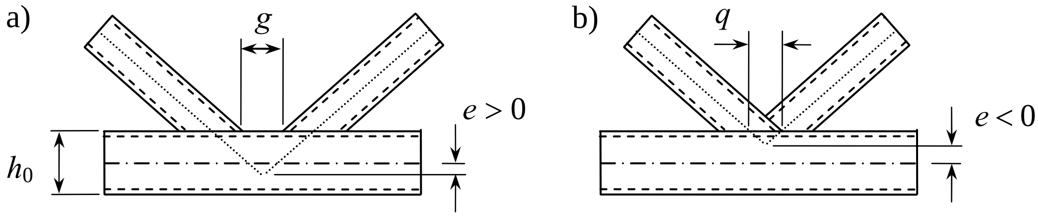


Figure 2.2. a) A gap joint with positive eccentricity *e*. *g* is the magnitude of gap. b) An overlap joint with negative eccentricity. *q* is the magnitude of overlap.

The joints of a tubular truss should be designed so that there is no need to reinforce or to stiffen the joints afterwards. Strengthening causes extra work and thus more cost. In a tubular truss all the members are welded directly together and the dimensions of the members have to be chosen in such a way that the strength requirements for members and joints are considered at the same time.

2.3 Structural analysis

The members of steel structures are divided into four different cross-section classes (1, 2, 3 and 4) depending on section dimensions and loading. In this thesis only the classes 1 and 2 hollow sections are considered purely due to simplicity. It is assumed that the whole hollow section belongs to bigger class determined by flange and web in compression. For a truss that is made of classes 1 and 2 profiles the structural analysis can be done according to the elasticity theory. In cross-section class 1 also the plasticity theory would be applicable but this choice is ignored.

In the structural analysis of tubular trusses a common practice is to assume that all connections are pinned joints and loads affect directly on joints. The only non-zero internal force is the normal force in the members. Since in real life chords are continuous bended structures and distributed loads may act along chords a more accurate way is to considerer chords as a beam and to assume that bracing members are connected using pinned joints. The real rotational stiffness between chords and bracing members can be neglected if the joints have sufficient rotation capacity. This capacity can be achieved by limiting the dimensions of members in the joint as it is done in the design codes.

In tubular truss design the eccentricity of connections should be limited because it causes an additional bending moment to chord. A small eccentricity can be permitted to ease the manufacturing. If the eccentricity e fulfils demand

$$-0,55h_0 \le e \le 0,25h_0 \tag{2.1}$$

for RHS and SHS hollow sections (Fig. 2.2) it can be ignored in the design of a tension chord, bracing members and joints but it has to be taken into account in the case of a compressed chord.

If the buckling in the plane of tubular truss is considered without taking into account the end restraint the buckling length of a compressed chord is the distance between joints. In the plane perpendicular to truss the buckling length is the distance between lateral support points. For bracing members the buckling length is the length of the member in both directions. Due to the rotational stiffness at the ends of members above chord buckling lengths can be multiplied by 0,9 and bracing member buckling length by 0,75 in lattice girders [53].

In this thesis the structural analysis is done using finite element method and simple linear beam element which has 6 degrees of freedom in plane case and 12 in space case (Appendix A). In bracing members the rotational degrees of freedom are ignored due to the pinned joints. The global stiffness matrix **K**, consistent mass matrix **M** and geometrical stiffness matrix $\mathbf{K}_g$ can be formed using corresponding element matrixes **k**, **m** and $\mathbf{k}_g$. Unknown global node displacements **u** can be solved using system of equations

$$\mathbf{Ku} = \mathbf{f} \tag{2.2}$$

where **f** includes known and equivalent nodal loads. The natural circular frequencies ω_1, ω_2, ω_3, … can be calculated from eigenvalue problem

$$\mathbf{Ku} = \omega^2 \mathbf{Mu} \ . \tag{2.3}$$

The natural frequencies f_1, f_2, f_3, ... are

$$f_i = \frac{\omega_i}{2\pi} \ , \qquad i = 1, 2, \ldots, n_{\text{df}} \ . \tag{2.4}$$

n_{df} is the number of unknown degrees of freedom in FEM-model. The critical load factor λ_{cr} according to linear stability theory is the smallest positive eigenvalue for problem

$$\mathbf{Ku} = -\lambda \mathbf{K}_{\text{g}} \mathbf{u} \ . \tag{2.5}$$

λ_{cr} is directly the safety factor against the global buckling of a structure.

2.4 Design code requirements

The safe and usable tubular truss has to fulfill the demands of appropriate steel design code. The individual members of the truss have to be strong enough and they can not buckle. Also welded joints have to carry loads without failure. Following design criteria are for RHS and SHS sections (Fig. 2.3) in cross-section classes 1 and 2 and they are based on Eurocode 3 [63] and CIDECT design guide [53].

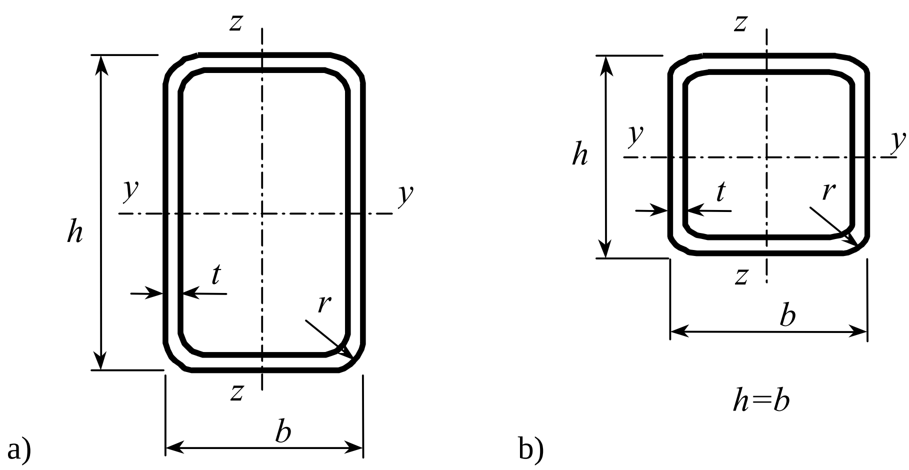


Figure 2.3. a) Rectangle hollow section (RHS). b) Square hollow section (SHS).

The design values for normal force, shear force, bending moment and torsional moment are N_{Sd}, V_{Sd} ($V_{y.\mathrm{Sd}}$ and $V_{z.\mathrm{Sd}}$), M_{Sd}, ($M_{y.\mathrm{Sd}}$ and $M_{z.\mathrm{Sd}}$) and $M_{\mathrm{t.Sd}}$.

2.4.1 Strength

Presented design criteria should be effective in every cross-section for all the members of the truss. If loads are point forces in joints and there are no distributed forces present, the shear force distributions are constant and the bending moment distributions are linear. Thus the most critical cross-section is always in the beginning or at the end of each chord member. Normal force is the only internal force in bracing members. In the cross-section classes 1 and 2 the design capacities are calculated based on plasticity theory.

The design criteria for tension and for compression without buckling are

$$N_{\mathrm{Sd}} \le N_{\mathrm{t.Rd}} \quad \text{and} \quad N_{\mathrm{Sd}} \le N_{\mathrm{c.Rd}} \ . \tag{2.6}$$

The design capacities $N_{\mathrm{t.Rd}}$ and $N_{\mathrm{c.Rd}}$ can be calculated using cross-section area A and yield limit f_{y}

$$N_{\mathrm{t.Rd}} = N_{\mathrm{c.Rd}} = N_{\mathrm{pl.Rd}} = A\frac{f_{\mathrm{y}}}{\gamma_{\mathrm{M0}}} \ . \tag{2.7}$$

$\gamma_{\mathrm{M0}} = 1{,}1$ is the partial safety factor.

The design criteria for bending is

$$M_{\mathrm{Sd}} \le M_{\mathrm{c.Rd}} \tag{2.8}$$

or if the bending is about both principal axis

$$\left(\frac{M_{y.\mathrm{Sd}}}{M_{\mathrm{c.}y.\mathrm{Rd}}}\right)^{1,66} + \left(\frac{M_{z.\mathrm{Sd}}}{M_{\mathrm{c.}z.\mathrm{Rd}}}\right)^{1,66} \le 1 \ . \tag{2.9}$$

The bending moment capacities $M_{\mathrm{c.}y.\mathrm{Rd}}$ and $M_{\mathrm{c.}z.\mathrm{Rd}}$ depend on plastic section modulus $W_{\mathrm{pl.}y}$ and $W_{\mathrm{pl.}z}$

$$M_{\mathrm{c.}y.\mathrm{Rd}} = M_{\mathrm{pl.}y.\mathrm{Rd}} = W_{\mathrm{pl.}y}\frac{f_{\mathrm{y}}}{\gamma_{\mathrm{M0}}} \quad \text{and} \quad M_{\mathrm{c.}z.\mathrm{Rd}} = M_{\mathrm{pl.}z.\mathrm{Rd}} = W_{\mathrm{pl.}z}\frac{f_{\mathrm{y}}}{\gamma_{\mathrm{M0}}} \ . \tag{2.10}$$

The design criterion for shear is

$$V_{\mathrm{Sd}} \le V_{\mathrm{Rd}} \tag{2.11}$$

or if the shear affects in the both directions of principal axis

$$V_{y.\mathrm{Sd}} \le V_{y.\mathrm{Rd}} \quad \text{and} \quad V_{z.\mathrm{Sd}} \le V_{z.\mathrm{Rd}} . \tag{2.12}$$

The shear force capacities $V_{y.\mathrm{Rd}}$ and $V_{z.\mathrm{R}d}$ depend on shear areas $A_{\mathrm{v}y}$ and $A_{\mathrm{v}z}$

$$V_{y.\mathrm{Rd}} = V_{\mathrm{pl}.y.\mathrm{Rd}} = \frac{A_{\mathrm{v}y} f_{\mathrm{y}}}{\sqrt{3}\,\gamma_{\mathrm{M0}}} \quad \text{and} \quad V_{z.\mathrm{Rd}} = V_{\mathrm{pl}.z.\mathrm{Rd}} = \frac{A_{\mathrm{v}z} f_{\mathrm{y}}}{\sqrt{3}\,\gamma_{\mathrm{M0}}} \tag{2.13}$$

$A_{\mathrm{v}y}$ and $A_{\mathrm{v}z}$ are connected to the cross-section areas of webs

$$A_{\mathrm{v}z} = A\frac{h}{b+h} \quad \text{and} \quad A_{\mathrm{v}y} = A\frac{b}{b+h} . \tag{2.14}$$

The design criterion for torsion is

$$M_{\mathrm{t.Sd}} \le M_{\mathrm{t.Rd}} \tag{2.15}$$

where the torsional moment capacity $M_{\mathrm{t.Rd}}$ is

$$M_{\mathrm{t.Rd}} = M_{\mathrm{t.pl.Rd}} = \frac{f_{\mathrm{y}}}{\sqrt{3}\,\gamma_{\mathrm{M0}}} W_{\mathrm{t}} \approx \frac{f_{\mathrm{y}}}{\sqrt{3}\,\gamma_{\mathrm{M0}}} 2A_{\mathrm{t}}\, t \ . \tag{2.16}$$

W_{t} is the section modulus in torsion, t is the wall thickness and A_{t} is the area inside the center line of the wall.

If normal force N_{Sd}, shear forces $V_{y.\mathrm{Sd}}$ and $V_{z.\mathrm{Sd}}$, bending moments $M_{y.\mathrm{Sd}}$ and $M_{z.\mathrm{Sd}}$ and torsion moment $M_{\mathrm{t.Sd}}$ act in the same cross-section following interaction formula can be used

$$\left(\frac{M_{y.\mathrm{Sd}}}{M_{\mathrm{N}.y.\mathrm{Rd}}}\right)^{\bar{\alpha}} + \left(\frac{M_{z.\mathrm{Sd}}}{M_{\mathrm{N}.z.\mathrm{Rd}}}\right)^{\bar{\alpha}} + \frac{M_{\mathrm{t.Sd}}}{M_{\mathrm{t.Rd}}} \le 1 . \tag{2.17}$$

The last term in Eq. (2.17) is presented in [56].

The exponent $\bar{\alpha}$ can be calculated using equation

$$\overline{\alpha} = \frac{1,66}{1 - 1,13\left(\frac{N_{\mathrm{Sd}}}{N_{\mathrm{V.Rd}}}\right)^2} \leq 6 \; . \tag{2.18}$$

If the design values of shear forces $V_{y.\mathrm{Sd}}$ and $V_{z.\mathrm{Sd}}$ exceed 50% of the plastic shear capacities $V_{\mathrm{pl}.y.\mathrm{Rd}}$ and $V_{\mathrm{pl}.z.\mathrm{Rd}}$, they have to be taken into account and reduced yield limits $(1-\rho_y)f_{\mathrm{y}}$ and $(1-\rho_z)f_{\mathrm{y}}$ have to be used for shear areas $A_{\mathrm{v}y}$ and $A_{\mathrm{v}z}$. In that case the reduced normal force capacity is

$$N_{\mathrm{V.Rd}} = \left(A - \rho_y\, A_{\mathrm{v}y} - \rho_z\, A_{\mathrm{v}z}\right) \frac{f_{\mathrm{y}}}{\gamma_{\mathrm{M0}}} . \tag{2.19}$$

Parameters ρ_y and ρ_z depend on the shear forces $V_{y.\mathrm{Sd}}$ and $V_{z.\mathrm{Sd}}$ in a following way

$$\rho_y = \begin{cases} 0 & , \quad V_{y.\mathrm{Sd}} < 0,5\, V_{\mathrm{pl}.y.\mathrm{Rd}} \\ \left(\frac{2V_{y.\mathrm{Sd}}}{V_{\mathrm{pl}.y.\mathrm{Rd}}} - 1\right)^2 & , \quad 0,5\, V_{\mathrm{pl}.y.\mathrm{Rd}} \leq V_{y.\mathrm{Sd}} < V_{\mathrm{pl}.y.\mathrm{Rd}} \\ 1 & , \quad V_{y.\mathrm{Sd}} \geq V_{\mathrm{pl}.y.\mathrm{Rd}} \end{cases} \tag{2.20}$$

$$\rho_z = \begin{cases} 0 & , \quad V_{z.\mathrm{Sd}} < 0,5\, V_{\mathrm{pl}.z.\mathrm{Rd}} \\ \left(\frac{2V_{z.\mathrm{Sd}}}{V_{\mathrm{pl}.z.\mathrm{Rd}}} - 1\right)^2 & , \quad 0,5\, V_{\mathrm{pl}.z.\mathrm{Rd}} \leq V_{z.\mathrm{Sd}} < V_{\mathrm{pl}.z.\mathrm{Rd}} \\ 1 & , \quad V_{z.\mathrm{Sd}} \geq V_{\mathrm{pl}.z.\mathrm{Rd}} \end{cases} \tag{2.21}$$

For RHS section $M_{\mathrm{N}.y.\mathrm{Rd}}$ and $M_{\mathrm{N}.z.\mathrm{Rd}}$ can be calculated using equations

$$M_{\mathrm{N}.y.\mathrm{Rd}} = 1,33 M_{\mathrm{V}.y.\mathrm{Rd}} \left(1 - \frac{N_{\mathrm{Sd}}}{N_{\mathrm{V.Rd}}}\right) \leq M_{\mathrm{V}.y.\mathrm{Rd}} \tag{2.22}$$

$$M_{\mathrm{N}.z.\mathrm{Rd}} = M_{\mathrm{V}.z.\mathrm{Rd}} \left(\frac{1 - \frac{N_{\mathrm{Sd}}}{N_{\mathrm{V.Rd}}}}{0,5 + \frac{ht}{A}}\right) \leq M_{\mathrm{V}.z.\mathrm{Rd}} \tag{2.23}$$

and for SHS section using equations

$$M_{\mathrm{N}.y.\mathrm{Rd}} = 1,26 M_{\mathrm{V}.y.\mathrm{Rd}} \left(1 - \frac{N_{\mathrm{Sd}}}{N_{\mathrm{V.Rd}}}\right) \quad \text{and} \quad M_{\mathrm{N}.z.\mathrm{Rd}} = 1,26 M_{\mathrm{V}.z.\mathrm{Rd}} \left(1 - \frac{N_{\mathrm{Sd}}}{N_{\mathrm{V.Rd}}}\right) . \tag{2.24}$$

If $V_{y.\mathrm{Sd}}$ and $V_{z.\mathrm{S}d}$ exceed 50% of the plastic shear capacities $V_{\mathrm{pl}.y.\mathrm{Rd}}$ and $V_{\mathrm{pl}.z.\mathrm{Rd}}$ yield limits for the shear areas are $(1-\rho_y)f_\mathrm{y}$ and $(1-\rho_z)f_\mathrm{y}$. The reduced bending capacities $M_{\mathrm{V}.y.\mathrm{Rd}}$ and $M_{\mathrm{V}.z.\mathrm{Rd}}$ are

$$M_{\mathrm{V}.y.\mathrm{Rd}} = \frac{\left(W_{\mathrm{pl}.y} - \dfrac{\rho_y A_{\mathrm{v}z}^2}{8t}\right) f_\mathrm{y}}{\gamma_{\mathrm{M0}}} \leq M_{\mathrm{pl}.y.\mathrm{Rd}} \tag{2.25}$$

$$M_{\mathrm{V}.z.\mathrm{Rd}} = \frac{\left(W_{\mathrm{pl}.z} - \dfrac{\rho_z A_{\mathrm{v}y}^2}{8t}\right) f_\mathrm{y}}{\gamma_{\mathrm{M0}}} \leq M_{\mathrm{pl}.z.\mathrm{Rd}} \ . \tag{2.26}$$

For the tension or compression bracing members, without considering possible buckling, it is enough to check only the design criteria (2.6). In chord member the normal force, the bending moment(s) and the shear force(s) and in multiplanar case also the torsion moment are usually nonzero. Thus for chord members interaction formula (2.17) should be used. In addition to criterion (2.17) also (2.12) and

$$N_{\mathrm{Sd}} \leq N_{\mathrm{V.Rd}} \tag{2.27}$$

should be checked in order to prevent the yielding due to normal force and shear forces alone.

2.4.2 Buckling strength

The design criterion (2.6) for compressed members is not usually active because buckling is a more restrictive phenomenon for such members than yielding. In tubular trusses the compressed members should fulfill not only design criterion (2.6) but also design criterion

$$N_{\mathrm{Sd}} \leq N_{\mathrm{b.Rd}} \ . \tag{2.28}$$

The buckling capacity $N_{\mathrm{b.Rd}}$ can be calculated using formula

$$N_{\mathrm{b.Rd}} = \chi A \frac{f_\mathrm{y}}{\gamma_{\mathrm{M1}}} \ . \tag{2.29}$$

$\gamma_{M1} = 1{,}1$ is the partial safety factor for buckling and the formula for the reduction factor χ is

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} \leq 1 \tag{2.30}$$

where

$$\phi = 0{,}5 \cdot \left[1 + \alpha \cdot \left(\bar{\lambda} - 0{,}2\right) + \bar{\lambda}^2\right] \tag{2.31}$$

and α is the imperfection factor and $\bar{\lambda}$ is the non-dimensional slenderness

$$\bar{\lambda} = \frac{L_b}{\pi i_r}\sqrt{\frac{f_y}{E}} \; . \tag{2.32}$$

E is Young's modulus, L_b is the buckling length and $i_r = \sqrt{I/A}$ is the corresponding radius of gyration. $\bar{\lambda}$ can be calculated about both principal axis and the bigger one should be used in Eq. (2.31) and (2.32). According to [56] a simple and safe way is to use nominal yield strength f_y and value $\alpha = 0{,}49$.

In combined compression, bending and torsion a tubular truss member should fulfill the criterion

$$\frac{N_{Sd}}{N_{b.Rd}} + \frac{k_y M_{y.Sd}}{M_{y.Rd}} + \frac{k_z M_{z.Sd}}{M_{z.Rd}} + \frac{M_{t.Sd}}{M_{t.Rd}} \leq 1 \; . \tag{2.33}$$

The last term in the above interaction equation is presented in [56].

The buckling capacity $N_{b.Rd}$ is

$$N_{b.Rd} = \chi_{min} A \frac{f_y}{\gamma_{M1}} \tag{2.34}$$

where χ_{min} is the smaller one from reduction factors χ_y and χ_z.

The bending moment capacities $M_{y.Rd}$ and $M_{z.Rd}$ are equal to the plastic moment capacities $M_{pl.y.Rd}$ and $M_{pl.z.Rd}$ Eq. (2.10) and torsional moment capacity $M_{t.Rd}$ is equal to plastic torsional moment capacity (2.16).

Formulas for parameters k_y and k_z are

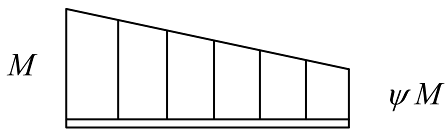
$$k_y = 1 - \frac{\mu_y N_{\mathrm{Sd}}}{\chi_y A f_{\mathrm{y}}} \leq 1{,}5 \quad \text{and} \quad k_z = 1 - \frac{\mu_z N_{\mathrm{Sd}}}{\chi_z A f_{\mathrm{y}}} \leq 1{,}5 \tag{2.35}$$

where

$$\mu_y = \bar{\lambda}_y \left(2\beta_{\mathrm{M}y} - 4\right) + \frac{W_{\mathrm{pl}.y} - W_{\mathrm{el}.y}}{W_{\mathrm{el}.y}} \leq 0{,}9 \tag{2.36}$$

$$\mu_z = \bar{\lambda}_z \left(2\beta_{\mathrm{M}z} - 4\right) + \frac{W_{\mathrm{pl}.z} - W_{\mathrm{el}.z}}{W_{\mathrm{el}.z}} \leq 0{,}9 \ . \tag{2.37}$$

$W_{\mathrm{el}.y}$ and $W_{\mathrm{el}.y}$ are elastic section moduli and $\beta_{\mathrm{M}y}$ and $\beta_{\mathrm{M}z}$ are the equivalent uniform moment factors that depend on bending moment distributions. For the linear distribution they can be seem from Fig. 2.4.



$$\beta_{\mathrm{M}} = 1{,}8 - 0{,}7\psi$$

$$-1 \leq \psi \leq 1$$

Figure 2.4. The equivalent uniform moment factor β_{M} for linear bending moment distribution.

The interaction formula (2.33) has to be checked at both ends of compressed chord member. In bracing members N_{Sd} is constant and it is enough to check Eq. (2.28).

2.4.3 Static joint strength

The most economical way to connect RHS or SHS members is to weld them together without any additional reinforcements. Depending on loads and the dimensions of connected members joints may be damaged in several different ways. Figure 2.5 represents the failure modes of N- and K-joints which are the only joints considered in this thesis.

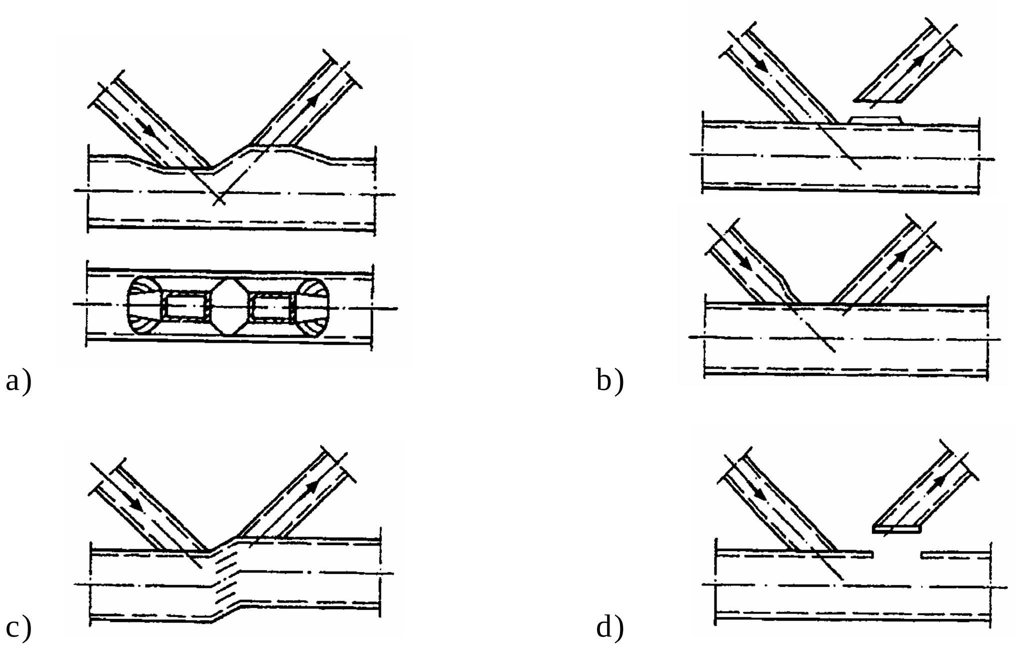


Figure 2.5. Failure modes for N- and K-joints between RHS and SHS sections [53]. a) The chord face plastification. b) The brace failure (tension failure or local buckling failure). c) The chord shear failure. d) The chord punching shear failure.

In the design of joints it is assumed that normal forces are the only internal forces in the bracing members and the normal force and the bending moment in the plane of truss are the only internal force and moment in the chord member. The shear forces, the bending moment out of the plane of the truss and the torsion moment are ignored. It is also assumed that the eccentricity e fulfills the demand (2.1), angles θ_1 and θ_2 (Fig. 2.6) and the angle between bracing members are at least 30°, the minimum wall thickness is 2,5 mm and yield limit is no more than 355 MPa.

The magnitude of gap g can be calculated using equation

$$g = (e + \tfrac{1}{2} h_0) \frac{\sin(\theta_1 + \theta_2)}{\sin\theta_1 \sin\theta_2} - \frac{h_1}{2\sin\theta_1} - \frac{h_2}{2\sin\theta_2} \qquad (2.38)$$

and the magnitude of overlap is $q = -g$.

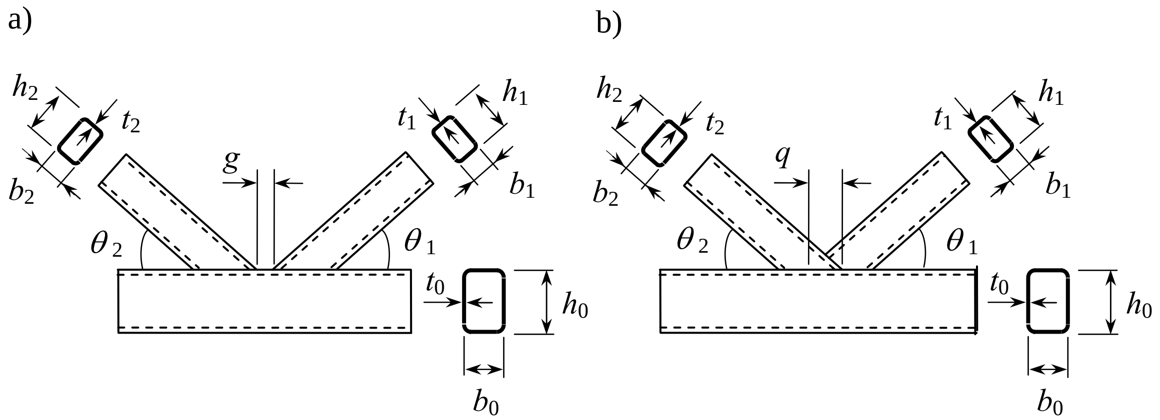


Figure 2.6. The dimensions of the a) gap N- and K-joints and b) the overlap N- and K-joints.

In the N- and K-type gap joints between a rectangular or square bracing member and a rectangular chord section (Fig. 2.6a)) the design criterion for the normal force of the bracing member is

$$N_{i,\mathrm{Sd}} \le N_{i.\mathrm{Rd}} \quad , \quad i = 1,2 \tag{2.39}$$

The joint bracing member capacity is according to [53] and [56]

$$N_{i.\mathrm{Rd}} = 8{,}9 \frac{f_{\mathrm{y}0}\, t_0^2}{\sin\theta_i} \left[\frac{b_1 + b_2 + h_1 + h_2}{4b_0} \right] \sqrt{\gamma}\, k_n \frac{1{,}1}{\gamma_{\mathrm{Mj}}\gamma_{\mathrm{M0}}} \quad \text{(chord face plastification)} \tag{2.40}$$

$$N_{i.\mathrm{Rd}} = \frac{f_{\mathrm{y}0} A_{\mathrm{v}}}{\sqrt{3}\sin\theta_i} \frac{1{,}1}{\gamma_{\mathrm{Mj}}\gamma_{\mathrm{M0}}} \quad \text{(chord shear failure)} \tag{2.41}$$

$$N_{i.\mathrm{Rd}} = f_{\mathrm{y}_i} t_i \left(2h_i - 4t_i + b_i + b_{\mathrm{e}_i} \right) \frac{1{,}1}{\gamma_{\mathrm{Mj}}\gamma_{\mathrm{M0}}} \quad \text{(brace failure)} \tag{2.42}$$

$$N_{i.\mathrm{Rd}} = \frac{f_{\mathrm{y}0} t_0}{\sqrt{3}\sin\theta_i} \left(\frac{2h_i}{\sin\theta_i} + b_i + b_{\mathrm{ep}_i} \right) \frac{1{,}1}{\gamma_{\mathrm{Mj}}\gamma_{\mathrm{M0}}} \quad \text{(punching shear failure)} \ \text{(only if } \beta \le 1 - \frac{1}{\gamma}\text{)} \tag{2.43}$$

The dimensions b_0, b_1, b_2, t_0, t_1, t_2, h_1, h_2, θ_1 and θ_2 can be seen from Fig. 2.6a), f_{y0} and f_{yi} are chord and bracing members yield limits and $\gamma_{\mathrm{Mj}} = 1{,}1$ is the joint partial safety factor. Formulas for the parameters β and γ are

$$\beta = \frac{b_1 + b_2}{2b_0} \tag{2.44}$$

$$\gamma = \frac{b_0}{2t_0} \quad . \tag{2.45}$$

k_n can be calculated using equation

$$k_n = \begin{cases} 1,0 & , \quad n \geq 0 \\ 1,3 + \dfrac{0,4}{\beta} n & , \quad n < 0 \end{cases} \leq 1,0 \tag{2.46}$$

where n depends on the normal stress in the chord face connected to bracing members

$$n = \left(\frac{N_{0.\mathrm{Sd}}}{A_0} + \frac{M_{0.\mathrm{Sd}}}{W_0} \right) \frac{\gamma_{\mathrm{M0}}}{f_{\mathrm{y0}}} \quad . \tag{2.47}$$

n is negative if the chord face is compressed, A_0 is the cross-section area of the chord and W_0 is the section modulus of the chord. The formula for shear area is

$$A_{\mathrm{v}} = (2h_0 + \tilde{\alpha}\, b_0)\, t_0 \tag{2.48}$$

where

$$\tilde{\alpha} = \sqrt{\frac{1}{1 + \dfrac{4g^2}{3t_0^2}}} \quad . \tag{2.49}$$

The effective width of a bracing member b_{e_i} and the effective punching shear width b_{ep_i} can be calculated using equations

$$b_{\mathrm{e}_i} = \frac{10\, b_i t_0^2 f_{\mathrm{y0}}}{b_0 t_i f_{\mathrm{y}_i}} \leq b_i \tag{2.50}$$

$$b_{\mathrm{ep}_i} = \frac{10\, t_0 b_i}{b_0} \leq b_i \quad . \tag{2.51}$$

In order to prevent a chord shear failure, the normal force of the chord $N_{0.Sd}$ in the gap between bracing members has to fulfill the criterion

$$N_{0.\mathrm{Sd}} \leq N_{0.\mathrm{Rd}} \quad . \tag{2.52}$$

According to [56] the joint chord member capacity is

$$N_{0.\mathrm{Rd}} = \begin{cases} \dfrac{A_0 f_{\mathrm{y}0}}{\gamma_{\mathrm{M}0}} & , \quad V_{\mathrm{Sd}} \leq 0,5\, V_{\mathrm{pl.Rd}} \\ \left[A_0 - A_{\mathrm{v}} \left(\dfrac{2\, V_{\mathrm{Sd}}}{V_{\mathrm{pl.Rd}}} - 1 \right)^2 \right] \dfrac{f_{\mathrm{y}0}}{\gamma_{\mathrm{M}0}} & , \quad V_{\mathrm{Sd}} > 0,5\, V_{\mathrm{pl.Rd}} \end{cases} \tag{2.53}$$

where V_{Sd} is the shear force of the chord in the gap and $V_{\mathrm{pl.Rd}}$ is

$$V_{\mathrm{pl.Rd}} = \frac{f_{\mathrm{y}0} A_{\mathrm{v}}}{\sqrt{3}\, \gamma_{\mathrm{M}0}} \quad . \tag{2.54}$$

Formulas (2.40), ..., (2.43) and (2.53) for joint bracing and chord member capacities are not valid unless certain dimensional terms hold. According to [53] these ranges of validity are

$$\frac{b_i}{b_0}, \frac{h_i}{b_0} \;\geq\; 0,1 + 0,01 \frac{b_0}{t_0} \qquad , \; i = 1,2 \tag{2.55}$$

$$\beta \;\geq\; 0,35 \tag{2.56}$$

$$\frac{b_i}{t_i}, \frac{h_i}{t_i} \;\leq\; \begin{cases} 1,25 \sqrt{\dfrac{E}{f_{\mathrm{y}_i}}} & , \quad \text{compression} \\ 35 & , \quad \text{tension} \end{cases} \leq 35 \qquad , \; i = 1,2 \tag{2.57}$$

$$\frac{b_0}{t_0}, \frac{h_0}{t_0} \;\leq\; 35 \tag{2.58}$$

$$0,5 \;\leq\; \frac{h_i}{b_i} \;\leq\; 2 \qquad , \; i = 1,2 \tag{2.59}$$

$$0,5(1-\beta) \;\leq\; \frac{g}{b_0} \;\leq\; 1,5(1-\beta) \tag{2.60}$$

$$g \geq t_1 + t_2 \qquad (2.61)$$

$$\theta_1 \, , \theta_2 \; \geq \; 30° \qquad (2.62)$$

Reference [56] represents also following ranges of validity

$$\beta \; \geq \; 0{,}1 + 0{,}01 \frac{b_0}{t_0} \qquad (2.63)$$

$$\frac{b_i + h_i}{t_i} \geq 25 \quad , \quad i = 0{,}1{,}2 \qquad (2.64)$$

$$0{,}5 \leq \frac{h_0}{b_0} \leq 2 \quad . \qquad (2.65)$$

In addition it is demanded that bracing members can not be wider than chords

$$b_1 \, , b_2 \; \leq \; b_0 \; . \qquad (2.66)$$

If the cross-sections of a chord and a bracing member are square in the N- and K-type gap joint, according to [53], the only failure mode which has to be taken into account is the chord face plastification. However reference [56] uses similar formulas as with a rectangular chord section and this approach has been selected.

In the overlap N- and K-joints between a rectangular or square bracing member and a rectangular or square chord section (Fig. 2.6b)) the only failure mode which has to be checked is the brace failure. The design criterion for the normal force of overlapping bracing member *i* is

$$N_{i.\mathrm{Sd}} \leq N_{i.\mathrm{Rd}} \; . \qquad (2.67)$$

The overlapping bracing member should be the one with smaller wall thickness or yield limit. According to [53] the overlapping bracing member capacity is

$$N_{i.\mathrm{Rd}} = \begin{cases} f_{yi} t_i \left[2\lambda_{\mathrm{ov}}(2h_i - 4t_i) + b_{\mathrm{e}_i} + b_{\mathrm{e(ov)}}\right] \dfrac{1,1}{\gamma_{\mathrm{Mj}}\gamma_{\mathrm{M0}}} & , \quad 0,25 \le \lambda_{\mathrm{ov}} < 0,5 \\ f_{yi} t_i \left[2h_i - 4t_i + b_{\mathrm{e}_i} + b_{\mathrm{e(ov)}}\right] \dfrac{1,1}{\gamma_{\mathrm{Mj}}\gamma_{\mathrm{M0}}} & , \quad 0,5 \le \lambda_{\mathrm{ov}} < 0,8 \\ f_{yi} t_i \left[2h_i - 4t_i + b_i + b_{\mathrm{e(ov)}}\right] \dfrac{1,1}{\gamma_{\mathrm{Mj}}\gamma_{\mathrm{M0}}} & , \quad 0,8 \le \lambda_{\mathrm{ov}} < 1,0 \end{cases} . \qquad (2.68)$$

λ_{ov} can be calculated using overlap q and the height h_i and angle θ_i of overlapping bracing member i

$$\lambda_{\mathrm{ov}} = \frac{q \sin\theta_i}{h_i} \quad . \qquad (2.69)$$

The effective width of a bracing member b_{e_i} can be calculated using equation (2.50). $b_{\mathrm{e(ov)}}$ is the effective width for overlapping bracing member i connected to overlapped bracing member j

$$b_{\mathrm{e(ov)}} = \frac{10 b_i f_{yj} t_j^2}{b_j f_{yi} t_i} \le b_i \quad . \qquad (2.70)$$

In reference [53] the ranges of validity for overlapping N- and K-joints are (2.59) and (2.62) and

$$\frac{b_i}{b_0}, \frac{h_i}{b_0} \ge 0,25 \qquad , \ i = 1,2 \qquad (2.71)$$

$$\frac{b_i}{t_i}, \frac{h_i}{t_i} \le \begin{cases} 1,1\sqrt{\dfrac{E}{f_{y_i}}} & , \quad \text{compression} \\ 35 & , \quad \text{tension} \end{cases} \le 35 \qquad , \ i = 1,2 \qquad (2.72)$$

$$\frac{b_0}{t_0}, \frac{h_0}{t_0} \le 40 \qquad (2.73)$$

$$0,25 \le \lambda_{\mathrm{ov}} \le 1,0 \qquad (2.74)$$

$$\frac{t_i}{t_j} \leq 1{,}0 \qquad (i \text{ is overlapping and } j \text{ overlapped}) \qquad (2.75)$$

$$\frac{b_i}{b_j} \geq 0{,}75 \qquad (i \text{ is overlapping and } j \text{ overlapped}) \qquad (2.76)$$

Reference [56] represents also demands (2.64) and (2.65) and replaces (2.73) with

$$\frac{b_0}{t_0}, \frac{h_0}{t_0} \leq 35 \qquad (2.77)$$

which has been used in this study as well as (2.66).

In tubular truss joint the welding should be done around the perimeter of a bracing member by using a butt weld, a fillet weld or a combination of these two. The size of the weld has to be chosen so that the weld is not weaker than the bracing member. This demand is fulfilled if the size of the weld is $a_{\mathrm{w}} \geq 0{,}95\,t$ ($f_{\mathrm{y}} = 235$ MPa) or $a_{\mathrm{w}} \geq 1{,}07\,t$ ($f_{\mathrm{y}} = 355$ MPa). In partially overlapped N- and K-joints the overlapped part under the overlapping member does not need to be welded unless $\lambda_{\mathrm{ov}} = 1{,}0$ or bracing member forces differ more than 20%.

The design of multiplanar tubular truss joints can be done based on previous uniplanar static joint strength requirements, considering one plane truss separated from the whole structure at a time. However there are geometrical and loading effects in multiplanar joints which affect the joint capacity compared to uniplanar joints. The bracing members of other planes than the considered current plane of truss stiffen the joint depending on the dimensions of the connected members. Also the normal forces in these bracing members can increase or decrease the considered joint capacity depending on their directions. According to [53] the correction factor $\eta = 0{,}9$ can be used for the member capacities in KK-connections (two Warren trusses together) if the angle between planes of Warren trusses is more than 60° and less than 90°. Also following criterion should be checked in the gap of a KK-joint

$$\left(\frac{N_{0.\mathrm{Sd}}}{A_0 f_{\mathrm{y}0}}\right)^2 + \left(\frac{\sqrt{3}V_{\mathrm{Sd}}}{A_0 f_{\mathrm{y}0}}\right)^2 \leq 1 \tag{2.78}$$

where V_{Sd} is the total shear force. It is assumed that the same correction factor $\eta = 0{,}9$ can be used for all the multiplanar truss joint capacities.

3. Heuristic optimization algorithms

Traditionally the solution methods of structural optimization problems are classified to direct and indirect methods. The idea of direct methods is to use directly the algorithms of mathematical optimization theory to solve a structural optimization problem. In indirect methods a structure that fulfills some kind of indirect demand based on strength of materials, is considered to be an optimal structure (e.g. a fully stressed design). The direct methods are more universal than the indirect methods that depend on the problem and the type of structure and do not necessarily lead to the optimum. On the other hand the indirect methods are usually more straightforward and efficient than the direct methods.

Heuristic optimization algorithms are a different approach to solving a structural optimization problem (or any other optimization problem) than the above mentioned direct and indirect methods. Heuristic means a deduction which does not fulfill all strict logical requirements but still often leads to a correct or a good answer. This means a stochastic or deterministic optimization algorithm which usually works but does not necessarily produce the optimum and may sometimes fail badly. Also indirect methods could be considered heuristic algorithms but due to their limited applicability they are not taken into account at this point. The selection of heuristic optimization algorithms is large and there are also several different combinations. In many versions the idea is taken from nature like the evolution or the behavior of a swarm. The wide applicability and the ability to solve, at least approximately, computationally hard discrete and mixed-integer problems have made heuristic algorithms popular in structural optimization.

This thesis considers tabu search (TS), simulated annealing (SA), genetic algorithm (GA) and particle swarm optimization (PSO) which belong to two different classes of heuristic algorithms: SA and TS are local search algorithms and GA and PSO are population based algorithms. TS, SA, GA and PSO are probably the four most widely used heuristic optimization algorithms in structural optimization.

There are several common features between heuristic optimization algorithms even thought their backgrounds are different (Table 3.1). Some of the characteristics can be though as clear benefits and some unfortunate drawbacks from practitioner's point of view. The main reason that makes heuristic algorithms tempting is that they are rather simple and very flexible and still suitable for a variety of hard discrete or combinatorial optimization problems. The major limitation is the big number of objective and constraint function evaluations which means that structural analysis has to be done typically several hundred or thousand times during the optimization.

Table 3.1. The common advantages and disadvantages of heuristic optimization algorithms.

Advantages:	Disadvantages:
- Simple idea and implementation.	- A lot of function evaluations (FEM-analysis) needed.
- Very flexible.	- The quality of result unknown.
- Suitable for computationally hard discrete problems.	- A lot of algorithms and many parameters connected to them.
- No need for sensitivity analysis.	- No sensitivity information available.
- The ability to avoid weak local optima.	- How to deal with constraints?
- Easy to use parallel processing.	- Stochastic algorithms demand several optimization runs.
- Rapid improvement at the early iteration rounds.	- The similar way to solve easy and hard problems (e.g. linear-nonlinear, convex-nonconvex).
- Analysis tools can be “blackbox”-type.	

In examples calculated results are achieved by solving pure discrete single criterion optimization problem

$$\min \ f(\mathbf{x}), \quad \mathbf{x} \in \Omega \subset X \ , \tag{3.1}$$

where $\mathbf{x} = \left[x_1,\ x_2, \ldots,\ x_{n_{\mathrm{dv}}}\right]^{\mathrm{T}}$ is a design variable vector, $f(\mathbf{x})$ is an objective function, $\Omega = \left\{ \mathbf{x} \in X \mid g_i(\mathbf{x}) \le 0 \ , \forall \ i = 1, 2, \ldots, n_{\mathrm{ie}} \right\}$ is a feasible set and $X = \left\{ \mathbf{x} \in \Re^{n_{\mathrm{dv}}} \ \middle| \ x_i \in \left\{ x_i^1, x_i^2, \ldots, x_i^{n_{\mathrm{av}i}} \right\} \forall \ i = 1, 2, \ldots, n_{\mathrm{ie}} \right\}$ is a design space. n_{dv} is the number of design variables, $g_i(\mathbf{x})$ is the inequality constraint function, n_{ie} is the number of constraints and n_{av_i} is the number of allowed values for x_i. Since all the design variables are discrete there is the certain finite set of possible candidate solutions i.e. $\mathbf{x} \in \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{n_{\mathrm{cs}}}\}$. The number of candidate solutions n_{cs} is assumed to be so big that it is not possible to check them all and to choose the best feasible one (direct enumeration).

For the solution $\mathbf{x} \in X$ the discrete neighborhood $\overline{N}(\mathbf{x})$ means all such solutions that are in some sense close to $\mathbf{x}$ (Fig. 3.1). A new solution $\mathbf{y}$ near $\mathbf{x}$ can be formed by making a move $\overline{m}$ to $\mathbf{x}$ which is marked $\mathbf{y} = \mathbf{x} \oplus \overline{m}$. The discrete neighborhood $\overline{N}(\mathbf{x})$ contains all the points that are formed by making one move to $\mathbf{x}$

$$\overline{N}(\mathbf{x}) = \left\{ \mathbf{y} \in X \;\middle|\; \mathbf{y} = \mathbf{x} \oplus \overline{m}, \overline{m} \in \overline{M} \right\}. \tag{3.2}$$

$\overline{M}$ is the set of all moves and the size of the discrete neighborhood means the number of elements in $\overline{M}$. The move $\overline{m}$ is feasible if $\mathbf{y} = \mathbf{x} \oplus \overline{m}$ is feasible.

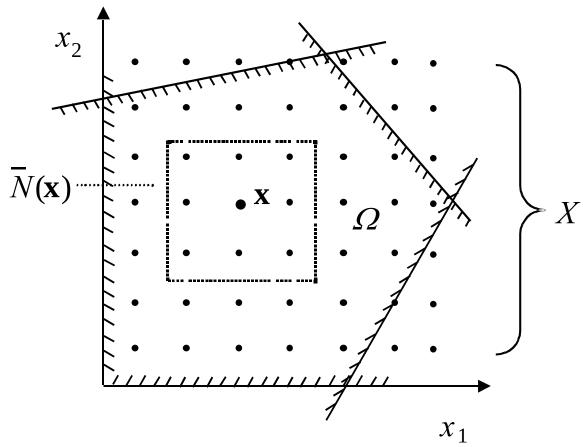


Figure 3.1. The solution $\mathbf{x}$, the discrete neighborhood $\overline{N}(\mathbf{x})$, the feasible set Ω and the design space X.

In the discrete optimization problem (3.1) the local minimum is a point $\mathbf{x}^* \in \Omega$ that $f(\mathbf{x}^*) \le f(\mathbf{x}) \;\; \forall\, \mathbf{x} \in \Omega \cap \overline{N}(\mathbf{x}^*)$ i.e. there is no better feasible solution than $\mathbf{x}^*$ in the discrete neighborhood $\overline{N}(\mathbf{x}^*)$. In the global minimum point $\mathbf{x}^* \in \Omega$ it holds $f(\mathbf{x}^*) \le f(\mathbf{x}) \quad \forall\, \mathbf{x} \in \Omega$ which means that $\mathbf{x}^*$ is the best solution in the feasible set Ω.

Table 3.2 gives the summary of heuristic optimization algorithms.

Table 3.2. The characteristic of heuristic optimization algorithms.

	SA	TS	GA	PSO
Group/single solution	single	single	group	group
Discrete neighborhood	yes	yes	no	no
Stochastic/ deterministic	stochastic	deterministic	stochastic	stochastic
Initial guess	yes	yes	no	no
Design variables originally	continuous /discrete	discrete	discrete	continuous
Direct parallelization	no [1)]	yes	yes	yes
Coding	no	no	yes	no
Implementation	easy	easy	longer	simplest
Popularity in structural optimization	several articles	least used	very popular	increasingly popular
Original idea	physics	?	evolution	swarm
Introduced	1983	1986	1975	1995

[1)] Leite & Topping [43].

3.1 Local search algorithms

Simulated annealing and tabu search belong to the class of local search algorithms that improve the value of the objective function based on local information. The idea is to start from an initial guess $\mathbf{x}^0$ and during each iteration round k to look for a new better solution $\mathbf{x}^{k+1}$ from the discrete neighborhood of current solution $\overline{N}(\mathbf{x})$ (Fig. 3.2). In the basic version of local search the iteration process stops in the local minimum i.e. when the discrete neighborhood does not contain any better feasible points. The local search algorithm can be considered as a discrete version of the deepest descent method.

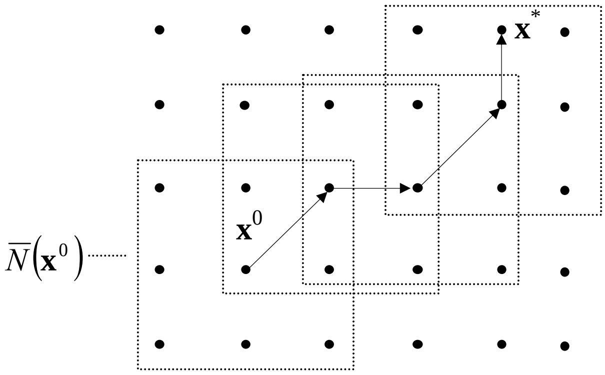


Figure 3.2. The iteration through the design space by using discrete neighborhoods in the local search.

The weakness of the basic version of local search is that it gets stuck to the nearest local optimum. If the problem has several local optima the chosen initial guess and the definition of the discrete neighborhood determine which one the algorithm ends up. Usually the value of objective function is rather poor in the first local optimum. The main difference between advanced local search algorithms is how the weak local optima can be avoided. Algorithms contain different mechanisms to escape from the local minimum "uphill" toward better solutions and hopefully all the way to the global optimum.

3.1.1 Discrete neighborhood

The structure of the discrete neighborhood affects the possibilities to find a good solution to the current optimization problem. Unfortunately there are no general rules to specify an efficient discrete neighborhood because it depends on the problem. It is possible to give only some common guidelines.

The discrete neighborhood should be connected or at least weakly connected. $\overline{N}(\mathbf{x})$ is called connected if for each pair $\mathbf{x}_1, \mathbf{x}_2 \in \Omega$ a sequence of solutions $\mathbf{x}_1 = \mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_j = \mathbf{x}_2$ exists so that $\mathbf{s}_i \in \overline{N}(\mathbf{s}_{i-1})$, $i = 2,3,\ldots, j$ and it is called weakly connected if for each $\mathbf{x} \in \Omega$ a sequence of solutions $\mathbf{x} = \mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_j = \mathbf{x}^*$ exists so that $\mathbf{s}_i \in \overline{N}(\mathbf{s}_{i-1})$, $i = 2,3,\ldots, j$ and $\mathbf{x}^*$ is the global optimum. For the connected discrete neighborhood each solution is reachable from all other solutions and for the weakly connected discrete neighborhood it is possible to reach the global optimum from the initial guess.

The size of the discrete neighborhood should be suitably chosen compared to the current optimization problem. If the size is big it takes longer to evaluate the values of the objective and constraint functions in the points of discrete neighborhood and the number of iteration rounds remains modest. On the other hand too small discrete neighborhood weakens possibilities to find a good solution because the search is not so wide. Besides the correct size discrete neighborhood should also be simple to form and problem specific properties should be taken into account.

In this study the discrete neighborhood is formed so that each discrete design variable may change in turn no more than d_{n} step bigger and smaller (so called depth of the discrete neighborhood is d_{n}) while the other variables remain the same. If the number of design variables is n_{dv}, the size of the discrete neighborhood remains reasonable being $2d_{\mathrm{n}}n_{\mathrm{dv}}$ (Fig. 3.3). The discrete neighborhood would be more detailed if the increasing and decreasing were allowed for several design variables at the same time but the size becomes rapidly too large as the number of variables increases.

$$x_i \in \{1, 2, \ldots, 50\} \quad \forall\, i = 1, 2, 3$$
$$n_{\mathrm{dv}} = 3$$
$$d_{\mathrm{n}} = 2$$

$$\mathbf{x} = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix} \Longrightarrow \left.\begin{matrix} \begin{bmatrix} 8 \\ 20 \\ 30 \end{bmatrix} \begin{bmatrix} 9 \\ 20 \\ 30 \end{bmatrix} \begin{bmatrix} 11 \\ 20 \\ 30 \end{bmatrix} \begin{bmatrix} 12 \\ 20 \\ 30 \end{bmatrix} \\ \begin{bmatrix} 10 \\ 18 \\ 30 \end{bmatrix} \begin{bmatrix} 10 \\ 19 \\ 30 \end{bmatrix} \begin{bmatrix} 10 \\ 21 \\ 30 \end{bmatrix} \begin{bmatrix} 10 \\ 22 \\ 30 \end{bmatrix} \\ \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ 29 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ 31 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ 32 \end{bmatrix} \end{matrix}\right\} \overline{N}(\mathbf{x})$$

Figure 3.3. The discrete neighborhood $\overline{N}(\mathbf{x})$ in the case of three design variables and the depth of two.

3.1.2 Simulated annealing, SA

Simulated annealing is a stochastic optimization algorithm the idea of which comes originally from statistical physics. It tries to imitate the annealing process in which a liquid metal is cooled slowly to a solid state. If the cooling is slow enough molecules are organized so that the energy function reaches the global minimum and if the cooling is too rapid the result is a local minimum for the energy function.

The strategy of SA is to select randomly a new point **y** in the discrete neighborhood of the current solution $\overline{N}\left(\mathbf{x}^k\right)$. If the value of the objective function $f(\mathbf{y})$ is better than $f(\mathbf{x}^k)$, **y** is accepted automatically as the next iteration point $\mathbf{x}^{k+1}$. Otherwise **y** can be selected by probability

$$P(T,d) = \mathrm{e}^{-\frac{d}{T}} \tag{3.3}$$

where T is the so called temperature and $d = f(\mathbf{y}) - f\left(\mathbf{x}^k\right)$. Only slightly poorer point **y** has a better chance to be selected than worse points. If point **y** is unfeasible it can be automatically rejected and a new one can be chosen. Another way is to penalize the objective function according to the unfeasibility

$$\tilde{f}(x) = f(\mathbf{x})\left[1 + \sum_{g_i(\mathbf{x})>0} R\, g_i(\mathbf{x})\right] . \tag{3.4}$$

$\tilde{f}(\mathbf{x})$ is the penalized objective function and R is penalty. The value of R should be able to be chosen suitably: If it is too small, unfeasible solutions become too good and if it is too big only slightly unfeasible but otherwise good solutions become too poor. If the penalty is the same for all constraint functions they have to be scaled so that function values are in the same order of magnitude.

Temperature T is decreased during the optimization process according to the cooling schedule. As the optimization process proceeds and the temperature decreases the likelihood to accept worse solutions approaches zero and only the solutions that improve the objective function are accepted. The value of objective function varies during optimization and that is why the best feasible solution that was found has to be kept in memory.

In the beginning the initial temperature T^0 should be so high that almost every neighbor point **y** is selected. The idea is to try to prevent getting stuck to a poor local optimum point. A suitable initial temperature can be determined based on Eq. (3.3) by calculating the average difference of the values of the objective functions between the randomly selected pairs of points in the design space and using probability which is almost one.

The suitable cooling schedule is important for the efficiency of SA. If the cooling is too fast the algorithm ends up most likely to a poor final solution. On the other hand slow cooling increases the calculation time. The decrease of temperature can be done in several ways but in the most common case the new temperature is the old one multiplied by a constant $\varepsilon = 0{,}80 \ldots 0{,}99$. Usually the decrease of temperature does not happen every round and n_{T} points are considered in the same temperature. Parameter n_{T} can be increased as the optimization proceeds.

The optimization can be terminated when the temperature has reached some selected level T^{final}. The final temperature should be so low that the probability to accept worse solution than the current one is almost zero. Temperature T^{final} can be calculated using the same kind of approach as in the initial temperature with a small chosen probability (~0,01). The flowchart of the basic SA is presented in Appendix B.

3.1.3 Tabu search, TS

The idea of deterministic tabu search is to move from the current iteration point $\mathbf{x}^k$ to the next best point $\mathbf{x}^{k+1}$ in the discrete neighborhood $\overline{N}(\mathbf{x}^k)$ as was case in the basic form of local search algorithm. However the best point in the discrete neighborhood of $\mathbf{x}^k$ is not always allowed because there is a so called tabu list B which contains forbidden solutions. As soon as a new iteration point is found it will be added to the tabu list and usually the oldest forbidden solution is released. In the practical size of structural optimization problems all previous points can be stored to the tabu list. The constraints can be taken into the account in the same way as in SA i.e. an unfeasible solution is automatically rejected or the value of objective function is penalized according to the unfeasibility (3.4).

In the beginning tabu search works like the basic local search algorithm seeking its way to the nearest local optimum. In that point there are no better neighbor points left and the next one will be the best non-tabu point. In the minimization problem this means that the value of objective function increases and TS starts to climb "uphill". Thus the tabu list makes it possible to escape from a weak local optimum and to find later a better local or even the global optimum point. Usually the optimization continues until a certain number of iteration rounds is full. As it was in SA the value of the objective function gets worse occasionally during the optimization and the best feasible solution that was found has to be kept in memory.

A bit different way to carry out the tabu list is to prohibit the feature of solution instead of the actual solution. This means that the group of same kind of solutions becomes forbidden during one iteration round. The purpose is to direct the search toward new regions in the design space which hopefully contain new better solutions. This kind of practice may lead to a situation in which a new better feasible solution is rejected even if the search has never been in that point.

The type of the tabu list can be static or dynamic. The length of the static list remains constant but the length of the dynamic tabu list depends on the information received during the optimization process. For example if the best known objective function value improves the tabu list can be shorter and vice versa.

The tabu list can be considered as a memory in TS. The next iteration point does not depend only on the current one and the discrete neighborhood around it but also the way the search has reached that point. If all previous iteration points are not included to the tabu list it is possible (at least in theory) that

TS falls into an endless loop. The flowchart of the basic TS is presented in Appendix B.

3.2 Population based methods

Genetic algorithm and particle swarm optimization differ from the local search algorithms in such a way that there is a group of solutions instead of a single solution in GA and PSO. In population based methods it is assumed that a group can offer some extra benefits compared to a single individual. Several solutions can spread out into the search space and it will be explored more widely than by using only one solution (explicit parallelism). On the other hand parallel solutions increase the amount of calculations per iteration round and thus the number of rounds will be smaller. In GA the efficiency of a group is based on competition and in PSO co-operation between individuals.

The analyses of population members are independent from each other. This makes it easy to use parallel processing which reduces the calculation time efficiently and economically. The optimization algorithm is running in one computer and it divides analysis tasks to several other computers to be done simultaneously as it is represented e.g. in [34]. Also TS can be parallelized directly because the analyses of the discrete neighborhood members are not connected.

In population based methods there is no need to choose an initial guess because the first group of solutions is usually selected randomly. In local search algorithms the efficiency of the selected method depends on the initial guess. If it is poor the result will also probably be poor and if the initial guess is close to the optimum the convergence may be fast. This gives a possibility to an experienced user to use his or her professional skills and intuition in such a way that is not possible in population based methods. In some cases it can be really difficult to find a feasible initial solution and population based methods with randomly selected initial population make starting easier.

3.2.1 Genetic algorithm, GA

In genetic algorithm the population of individuals becomes to better and better during the optimization process by the basic operators selection, crossover and mutation. The idea is to follow the principle of nature that only the fittest individuals survive. The origin of GA is in mid seventies and it has been the most popular heuristic optimization method.

The individuals of the population are the encoded solutions of the optimization problem. The most common way is to use binary coding in which a binary string corresponds to each solution. The other way to carry out encoding is to number the allowed discrete design variable values from one forward and use these integers in encoding. This method increases the number of different codes, decreases the length of individuals and prevents turning up such coded individuals that do not have a counterpart among possible candidate solutions.

The fitness function is a mechanism that determines the quality of an individual. It is derived from the objective function and gets only positive values. In the minimization problem the simplest way to form fitness function is

$$\overline{F}(\mathbf{x}) = C - f(\mathbf{x}) \ , \qquad (3.5)$$

where C is a big positive constant. In constraint optimization problem the feasibility of a solution has to be considered also in the fitness function. This can be done again by penalizing the unfeasible solutions according to the unfeasibility (Eq. (3.4)).

The purpose of the selection is to choose good individuals as the parents of the next generation (iteration round). The better fitness function value an individual has the bigger the probability of selection and the better chance to be a parent. For individual i the probability of selection p_i depends on the values of the fitness function

$$p_i = \frac{\overline{F}_i}{\sum_j \overline{F}_j} \ . \qquad (3.6)$$

The above roulette-wheel selection is one of the so called proportional selection. Other possibilities are e.g. ranking selection and tournament selection. In ranking selection the proportions of the fitness function values are insignificant and the order of magnitude determines directly the probability of selection. In tournament selection the idea is to pick the group of individuals at random and select the best individual as a new parent. In binary tournament selection there are only two candidates. If one candidate is feasible and the other unfeasible the feasible one is selected. In such a case that both individuals are unfeasible it is not necessary to use penalty function approach because the less unfeasible can be selected otherwise. This means that there is no need to choose proper penalty parameters in binary tournament selection.

The crossover produces new individuals or offspring to the next generation. The idea is to randomly cut two parents into pieces and join two offspring from these parts. The number of pieces can vary and there are e.g. one point and two point crossovers. The crossover happens by some probability p_c and by probability $1-p_c$ parents are copied directly to the next generation. Usually the magnitude of crossover probability is $p_c = 0,6\ldots0,8$. In most cases the size of the population is kept constant and each pair of parents produce two offspring.

The selection and the crossover combine properties that already exist in an initial population. They do not bring any additional information to the system and cause more and more homogeneous population. The idea of the mutation is to randomly change individuals slightly different. This can be done by turning one bit opposite in binary encoding by mutation probability p_m. The mutation brings diversity to the population that hopefully leads to a better final result. Usually the mutation probability is $p_m = 0,005\ldots0,05$.

Since GA is a stochastic optimization algorithm it is possible that the best individual will not always survive. This can be prevented by using elitism in which the best individual is automatically copied to the next generation. For example the worst individual in population can be replaced by the best one from the previous generation.

The initial population is usually selected at random in GA. The usual stopping criterion is to wait that a certain number of generations is full and optimization can be terminated. Other possibilities are to observe fitness function values or the diversity of population and if the fitness function value does not improve anymore or all individuals are too similar the optimization can be stopped. The flowchart of the basic GA is presented in Appendix B.

3.2.2 Particle swarm optimization, PSO

The basic idea of stochastic PSO is to model the social behavior of a swarm (e.g. birds or fish) in nature. A swarm of particles tries to adapt to its environment by using previous knowledge based on the experience of individual particles and the collective experience of the swarm. It is useful for a single member, and at the same time for the whole swarm, to share information among other members to gain some advantage.

In PSO for particle *i* the new position $\mathbf{x}_{k+1}^{i}$ depends on the current position $\mathbf{x}_{k}^{i}$ and so called velocity $\mathbf{v}_{k+1}^{i}$

$$\mathbf{x}_{k+1}^{i} = \mathbf{x}_{k}^{i} + \mathbf{v}_{k+1}^{i} \tag{3.7}$$

where the velocity is calculated as follows

$$\mathbf{v}_{k+1}^{i} = w\mathbf{v}_{k}^{i} + c_1 r_1 \left(\mathbf{p}_{k}^{i} - \mathbf{x}_{k}^{i}\right) + c_2 r_2 \left(\mathbf{p}_{k}^{g} - \mathbf{x}_{k}^{i}\right) . \tag{3.8}$$

$\mathbf{p}_{k}^{i}$ is the best position for particle *i* and $\mathbf{p}_{k}^{g}$ is the best feasible position for the whole swarm. *w* is so called inertia, r_1 and r_2 are uniform random numbers $r_1, r_2 \in [0,1]$ and c_1 and c_2 are the scaling parameters. The value of *w* controls how widely the search process is done in the search space. The value of c_1 indicates how much a particle trusts itself and c_2 how much it trusts the swarm. The idea of the last two terms connected to c_1 and c_2 in the Eq. (3.8) is to direct the optimization process towards good potential areas in the search space (Fig. 3.4). Usually $0,8 < w < 1,4$ and $c_1 = c_2 = 2$ are selected. The value of w can be changed dynamically so that it is bigger during early iteration rounds and becomes smaller later when there is time to focus on promising areas.

Basically PSO is an algorithm for continuous unconstrained optimization problems. Discrete design variables can be taken into account by simply rounding each design variable to the closest allowed value in Eq. (3.7). Constraints can be handled again by penalizing unfeasible solutions according to the unfeasibility.

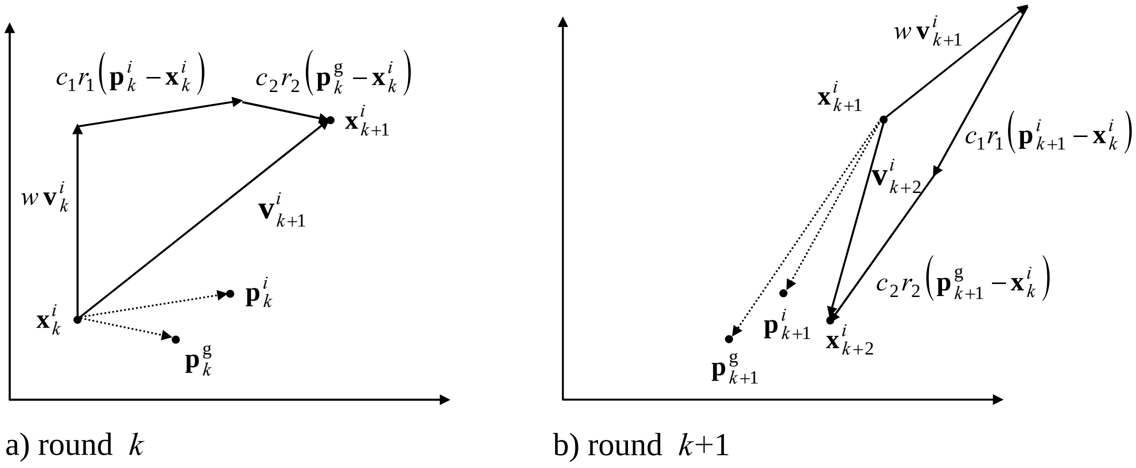


Figure 3.4. a) Step from point $\mathbf{x}_k^i$ to $\mathbf{x}_{k+1}^i$ and b) from point $\mathbf{x}_{k+1}^i$ to $\mathbf{x}_{k+2}^i$ in PSO. Inertia term (e.g. $w\mathbf{v}_k^i$) widen the optimization process and terms connected to $\mathbf{p}_k^g$ and $\mathbf{p}_k^i$ direct the search towards known good solutions.

The initial swarm and velocities can be selected randomly in PSO. As a terminating criterion the given number of iteration rounds can be used or the best known feasible objective function value can be observed and if there is no improvement during few last rounds the optimization is terminated. The flowchart of the basic PSO is presented in Appendix B.

4. Tubular truss optimization problem

In the current tubular truss optimization problem topology, shape and the sizes of truss members can be changed during optimization. The topology of the truss should be chosen from the group of selected topologies and the sizes of profiles from the given selection of standard cold-formed RHS or SHS sections. The change in the dimensions of a truss affects its shape. The minimized objective function can be e.g. mass, cost or displacement or there can be conflicting criteria in a multicriteria problem at the same time. Constraints take care the demands of steel design rules (Eurocode 3). The truss can be plane or space truss and the problem can include several load cases. For the sake of simplicity only N- and K-joints and profiles in cross-section classes 1 and 2 are considered.

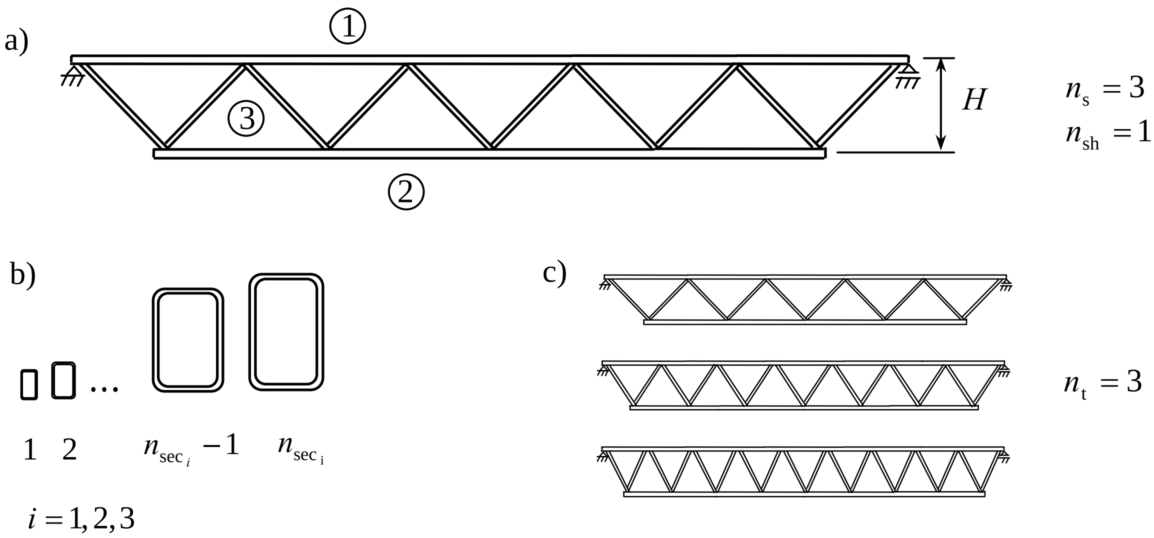


Figure 4.1. An example tubular truss problem: a) The lower chord, the upper chord and the bracing members form the three groups of members. In each group the size of the profile is constant. The height *H* is the only dimension which can be changed. b) The sets of allowed sections for the each group of members. c) Three possible truss topologies.

In the optimization problem the number of different members or groups of members is n_s in the truss (Fig. 4.1). A truss member means a web bar or a piece of chord between two joints. The number of allowed sections for each member or group of members i is n_{sec_i}. It is assumed that the number and the size distribution of hollow sections can be arbitrary. The numbers of convertible truss dimensions and available truss topologies are n_{sh} and n_{t} which means that the shape and the topology of truss can change only limited depending on designer's choices.

4.1 Discrete design variables

There are usually several different ways to choose design variables in an optimization problem and this choice affects the nature of the problem and thus applicable algorithms. For example in the tubular truss sizing optimization problem one cross-section property (typically area A) can be taken as the only design variable per each member and all the other needed cross-section properties ($W_{\mathrm{el}.y}$, $W_{\mathrm{pl}.y}$, etc.) are expressed as the function of the selected variable. The number of design variables stays small but it is difficult to form needed expressions in arbitrary case. Other possibilities are to choose directly all cross-section properties (A, $W_{\mathrm{el}.y}$, $W_{\mathrm{pl}.y}$, etc.) or the dimensions of hollow section (h, b and t) as the design variable. In both cases design variables are connected: Fixing one value reduces the possible values of some other variables.

In this study there are one topology design variable x^{t}, n_{sh} shape design variables x_i^{sh} and n_{s} size design variables x_i^{s} in the tubular truss optimization problem. The form of the design variable vector is $\mathbf{x} = \left[\, x^{\mathrm{t}} \;\vdots\; x_1^{\mathrm{sh}}\; x_2^{\mathrm{sh}} \ldots x_{n_{\mathrm{sh}}}^{\mathrm{sh}} \;\vdots\; x_1^{\mathrm{s}}\; x_2^{\mathrm{s}} \ldots x_{n_{\mathrm{s}}}^{\mathrm{s}} \,\right]^{\mathrm{T}}$. The set of available topologies is selected in advance and it is arranged into some kind of order. The discrete topology design variable $x^{\mathrm{t}} \in \{1, 2, \ldots, n_{\mathrm{t}}\}$ indicates directly the ordinal number of the chosen topology in this set. The discrete shape design variables x_i^{sh} ($i = 1, 2, \ldots, n_{\mathrm{sh}}$) are certain dimensions of the truss which determine the shape the of truss (e.g. height, width or joints locations). In many cases these variables could be continuous but they are assumed to be discrete. The discrete size design variable $x_i^{\mathrm{s}} \in \{1, 2, \ldots, n_{\mathrm{sec}_i}\}$ ($i = 1, 2, \ldots, n_{\mathrm{s}}$) is the ordinal number of the section of member (or the group of members) i in the set of allowed hollow sections. The set of allowed sections is arranged in some sense in the order of magnitude which is not unique in arbitrary case. The needed cross-section properties can be picked up from manufacturer's table.

4.2 Objective function

In the optimization of steel structures the ultimate target is mainly low cost. The cost of the welded steel structure like tubular truss consists of design, material, fabrication, transportation, erection and maintain costs. The mass of the structure is often used as a substitute for the real cost and it is chosen as the minimized objective function. The cost is not the only target and other possibilities are e.g. stiffness (deflections), the lowest natural frequency or the maximum stress in fatigue. In this thesis mass, cost and displacement are considered in the objective function.

In many cases it is useful to consider several conflicting criteria at the same time and to use the methods of multicriteria optimization instead of traditional single criterion optimization. The solution of multicriteria optimization problem is not unique which leaves room for the final decision making.

4.2.1 Mass

The total mass of truss m depends on density ρ and the cross-section areas A_i and the lengths of all members L_i

$$m = \rho \sum_{i=1}^{n_{\mathrm{m}}} A_i L_i \quad . \qquad (4.1)$$

n_{m} is the number of members in the truss.

4.2.2 Cost

In this thesis only the material and the manufacturing costs are considered and all the other costs are ignored. Since steel has a certain price per kilogram, the material cost can be calculated simply multiplying the mass of the structure by the current material cost factor k_m. It is assumed that k_m is constant for different size hollow sections.

In the calculation of the manufacturing cost the common approach is to divide the manufacturing process into several production stages and to consider the cost of each stage. Typical production stages of welded steel structures are the cutting and edge grinding, surface preparation, welding including preparation, assembly and tack welding and surface finishing. The manufacturing cost depends on the technical level of a machine workshop, the level of labor cost and the volume of production. Different production conditions make it difficult to obtain a single universal cost function and there will always be some case-specific factors in the cost function.

Jármai and Farkas ([17] and [30]) have presented the following cost function for welded steel structures

$$K = K_\mathrm{m} + K_\mathrm{f} = k_\mathrm{m} m + k_\mathrm{f} \sum_i T_i \qquad (4.2)$$

where K_m and K_f are the material and the manufacturing costs and k_m and k_f are the corresponding cost factors. T_i is the time [min] for production stage i. Instead of Eq. (4.2) the minimized cost function can be also expressed in the form

$$\frac{K}{k_\mathrm{m}} = m + \frac{k_\mathrm{f}}{k_\mathrm{m}}\left(T_1 + T_2 + T_3 + T_4 + T_5 + T_6\right) \ . \qquad (4.3)$$

According to [30] the ratio $k_\mathrm{f}/k_\mathrm{m}$ varies between 0–2 kg/min and it is 2 kg/min in USA and Japan, 1-1,5 kg/min in Western Europe and 0,5 kg/min in developing countries. There is only one cost factor k_f for all production stages which helps the application but it is also a simplification at the same time.

T_1 is the time for preparation, assembly and tack welding and it can be calculated using formula

$$T_1 = C_1 \Theta_\mathrm{d} \sqrt{\kappa m} \ . \qquad (4.4)$$

C_1 is a constant, Θ_d is a difficulty factor ($\Theta_\mathrm{d} = 1, \ldots, 4$) and κ is the number of structural elements to be assembled.

T_2 and T_3 are the times for welding and additional fabrication costs like changing the electrode, deslagging and chipping. The formula for the sum of T_2 and T_3 is

$$T_2 + T_3 = 1{,}3 \cdot \sum_i C_{2_i} a_{w_i}^{\tilde{n}} L_{w_i} \tag{4.5}$$

where a_{w_i} is the size [mm] and L_{w_i} the length [m] of the weld i. Constant C_{2_i} and exponent $\tilde{n}$ depend on the welding technology, the type of the weld and welding position as presented in [30].

T_4 is the time for surface preparation which means the surface cleaning, sand-spraying, etc. The formula for T_4 is

$$T_4 = \Theta_{ds} a_{sp} A_s \quad . \tag{4.6}$$

Θ_{ds} is a difficulty factor, $a_{sp} = 3 \cdot 10^{-6}$ min/mm^2 is a constant and A_s means the area of surface to be cleaned [mm^2].

T_5 means the painting time of ground and topcoat. It can be calculated using the formula

$$T_5 = \Theta_{dp} \left(a_{gc} + a_{tc} \right) A_s \tag{4.7}$$

where Θ_{dp} is the difficulty factor for painting ($\Theta_{dp} = 1, 2, 3$), $a_{gc} = 3 \cdot 10^{-6}$ min/mm^2 and $a_{tc} = 4{,}15 \cdot 10^{-6}$ min/mm^2 are constants and A_s is the area of surface [mm^2].

The time for cutting and edge grinding T_6 can be calculated using formula

$$T_6 = \Theta_{dc} \sum_i L_{c_i} \left(4{,}5 + 0{,}4\, t_i^2 \right) \quad . \tag{4.8}$$

It is a slightly modified version of the formula presented in [19]. Θ_{dc} is the difficulty factor, L_{c_i} is the length of cut i and t_i is the wall thickness.

4.2.3 Displacement

Since the structural analysis is done by using finite element method the value of displacement u_i connected to a certain degree of freedom i can be selected as the minimized displacement. The correct element can easily be picked up from global displacement vector **u**. If there are several load cases the biggest value of u_i is chosen. The deflection is not a practical objective function in single criterion problem unless the maximum allowed mass or cost is limited.

4.3 Constraints

The purpose of constraints is to make sure that the optimized structure remains usable. Truss members and joints have to be strong enough, single members or the whole structure may not buckle, deflections should be small and natural frequencies can not be located in the forbidden intervals. There are also demands due to the dimensions of truss, manufacturing and possible appearance of truss, transportation etc.

From the practical point of view the most important thing is to fulfill the demands of an appropriate steel design code. In this thesis Eurocode 3 is used and thus the strength constraint for each member i without considering buckling is

$$g_i^{\mathrm{s}} = \max\left\{ \left(\frac{M_{y.\mathrm{Sd}}}{M_{\mathrm{N}.y.\mathrm{Rd}}}\right)^{\alpha} + \left(\frac{M_{z.\mathrm{Sd}}}{M_{\mathrm{N}.z.\mathrm{Rd}}}\right)^{\alpha} + \frac{M_{\mathrm{t.Sd}}}{M_{\mathrm{t.Rd}}} - 1 \ , \ \ldots \right. \quad . \qquad (4.9)$$
$$\left. \frac{V_{y.\mathrm{Sd}}}{V_{y.\mathrm{Rd}}} - 1 \ , \ \frac{V_{z.\mathrm{Sd}}}{V_{z.\mathrm{Rd}}} - 1 \ , \ \frac{N_{\mathrm{Sd}}}{N_{\mathrm{V.Rd}}} - 1 \right\} \le 0$$

In bracing members bending moments, torsional moment and shear forces vanish and $N_{\mathrm{V.Rd}} = N_{\mathrm{pl.Rd}}$ which leads to form

$$g_i^{\mathrm{s}} = \frac{N_{\mathrm{Sd}}}{N_{\mathrm{pl.Rd}}} - 1 \le 0 \ . \qquad (4.10)$$

If loads are point forces in joints the constraint (4.9) should be checked at both ends of each chord member and the bigger value from these two chosen. Because in topology optimization the number of truss members varies and there can not be one strength constraint per each truss member the final strength constraint is

$$g^{\mathrm{s}} = \max\left\{ g_1^{\mathrm{s}}, g_2^{\mathrm{s}}, \ldots, g_{n_{\mathrm{m}}}^{\mathrm{s}} \right\} \le 0 \ . \qquad (4.11)$$

If one buckling constraint is taken per each truss member it is unnecessary to know in advance whether the normal force is tension or compression. Only compressed members can buckle and the buckling constraint which takes this into account is

$$g_i^{\mathrm{b}} = \begin{cases} \dfrac{N_{\mathrm{Sd}}}{N_{\mathrm{b.Rd}}} + \dfrac{k_y M_{y.\mathrm{Sd}}}{M_{y.\mathrm{Rd}}} + \dfrac{k_z M_{z.\mathrm{Sd}}}{M_{z.\mathrm{Rd}}} + \dfrac{M_{\mathrm{t.Sd}}}{M_{\mathrm{t.Rd}}} - 1 & , \quad \text{compression} \\ -1 & , \quad \text{tension} \end{cases} \leq 0 \tag{4.12}$$

Since bending and torsional moments vanish in bracing members, Eq. (4.12) takes the form

$$g_i^{\mathrm{b}} = \begin{cases} \dfrac{N_{\mathrm{Sd}}}{N_{\mathrm{b.Rd}}} - 1 & , \quad \text{compression} \\ -1 & , \quad \text{tension} \end{cases} \leq 0 \ . \tag{4.13}$$

It is assumed that the joints of a compressed chord are always laterally supported and thus a chord member means a piece of chord between sequential joints.

n_{m} buckling constraints (4.12) and (4.13) can be merged into a final single buckling constraint

$$g^{\mathrm{b}} = \max\left\{ g_1^{\mathrm{b}}, g_2^{\mathrm{b}}, \ldots, g_{n_{\mathrm{m}}}^{\mathrm{b}} \right\} \leq 0 \quad . \tag{4.14}$$

The static joint strength constraint for a N- and K-type joint *k* is

$$g_k^{\mathrm{js}} = \max \left\{ \begin{array}{ll} \dfrac{N_{1.\mathrm{Sd}}}{N_{1.\mathrm{Rd}}} - 1 \, , \, \dfrac{N_{2.\mathrm{Sd}}}{N_{2.\mathrm{Rd}}} - 1 \, , \, \dfrac{N_{0.\mathrm{Sd}}}{N_{0.\mathrm{Rd}}} - 1 & , \quad g \geq 0 \\ \dfrac{N_{i.\mathrm{Sd}}}{N_{i.\mathrm{Rd}}} - 1 & , \quad g < 0 \end{array} \right\} \leq 0 \tag{4.15}$$

where the joint bracing members capacities $N_{1.\mathrm{Rd}}$ and $N_{2.\mathrm{Rd}}$ are the smallest ones given by equations (2.40), (2.41), (2.42) and (2.43). The joint chord member capacity $N_{0.\mathrm{Rd}}$ can be calculated using equation (2.53) and the normal force capacity of overlapping bracing member *i* can be calculated using (2.68).

Due to the ranges of validity (gap joint (2.55), …, (2.66) and overlap joint (2.59), (2.62), (2.64), (2.65), (2.66) and (2.71), …, (2.77)) constraint (4.15) is not alone sufficient and in addition to it an extra joint validity constraint $g_k^{\mathrm{jv}} \leq 0$ is needed. The form of this constraint is such that in the case of gap joint demands (2.55), …, (2.66) are transformed into "less or equal to zero"-type (≤0–type) inequalities and the maximum value of these new 22 single inequalities is chosen as the value of the constraint function g_k^{jv}. In the case of overlapping joint corresponding demands are treated the same way and the form of g_k^{jv} is

$$g_k^{\mathrm{jv}} = \max \begin{Bmatrix} 0,1+0,01\dfrac{b_0}{t_0}-\dfrac{b_1}{b_0} \;,\; 0,1+0,01\dfrac{b_0}{t_0}-\dfrac{b_2}{b_0} \;,\dots,\; b_2-b_0 & , & g \geq 0 \\ 0,5-\dfrac{h_1}{b_1} \;,\; \dfrac{h_1}{b_1}-2 \;,\; \dots \;,\; 0,75-\dfrac{b_i}{b_j} & , & g < 0 \end{Bmatrix} \leq 0 \tag{4.16}$$

The joint strength and validity constraints (4.15) and (4.16) can be merged into a single joint constraint

$$g^{\mathrm{j}} = \max \left\{ g_1^{\mathrm{js}}, g_2^{\mathrm{js}}, \dots, g_{n_{\mathrm{j}}}^{\mathrm{js}} \;,\; g_1^{\mathrm{jv}}, g_2^{\mathrm{jv}}, \dots, g_{n_{\mathrm{j}}}^{\mathrm{jv}} \right\} \leq 0 \;. \tag{4.17}$$

For a multiplanar gap KK-connection k should also use constraint

$$g_k^{\mathrm{kkgj}} = \begin{cases} \left(\dfrac{N_{0.\mathrm{Sd}}}{A_0 f_{\mathrm{y0}}}\right)^2 + \left(\dfrac{\sqrt{3}V_{\mathrm{Sd}}}{A_0 f_{\mathrm{y0}}}\right)^2 - 1 & , \quad g \geq 0 \\ -1 & , \quad g < 0 \end{cases} \leq 0 \tag{4.18}$$

to prevent failure in the gap. The final KK-connection constraint is

$$g^{\mathrm{kkgj}} = \max \left\{ g_1^{\mathrm{kkgj}}, g_2^{\mathrm{kkgj}}, \dots, g_{n_{\mathrm{j}}}^{\mathrm{kkgj}} \right\} \leq 0 \;. \tag{4.19}$$

Eurocode 3 gives some demands for the deflections of a steel structure in vertical and horizontal directions depending on the span and the height of the structure (L/500 … L/200 and h/500 … h/150). These or some other proper values based on experience can be used as limits in the displacement constraint. In the constraint the value of displacement u_i should be more than the minimum allowed value $\overline{u}_i^{\min}$ or less than the maximum allowed value $\overline{u}_i^{\max}$. This can be presented in the forms

$$g^{\mathrm{u}} = 1 - \frac{u_i}{\bar{u}_i^{\min}} \le 0 \quad \text{and} \quad g^{\mathrm{u}} = \frac{u_i}{\bar{u}_i^{\max}} - 1 \le 0 \ . \tag{4.20}$$

The global buckling constraint makes sure that the whole truss will not lose it stability in the sense of linear stability theory. The form of global buckling constraint is

$$g^{\mathrm{gb}} = 1 - \frac{\lambda_{\mathrm{cr}}}{\bar{n}_{\mathrm{gb}}} \le 0 \ . \tag{4.21}$$

λ_{cr} is the lowest positive eigenvalue of problem (2.5) and $\bar{n}_{\mathrm{gb}}$ is the safety factor against global buckling. The linear stability theory gives only an approximation for the real critical load factor λ_{cr} which is usually clearly less than this approximation. Due to this reason safety factor has to be selected a big enough ($\bar{n}_{\mathrm{gb}} \ge 3$) in order to prevent global buckling.

In the frequency constraint it is demanded for each forbidden interval $j = 1, 2, \ldots, n_{\mathrm{f}}$ (Fig. 4.2) that

$$f_i \le f_j^{\min} \ \vee \ f_i \ge f_j^{\max} \qquad \forall \ \ i = 1, 2, \ldots, n_{\mathrm{df}} \ . \tag{4.22}$$

n_{df} is the number of the degrees of freedom in FEM-model which is at the same time also the number of the natural frequencies.

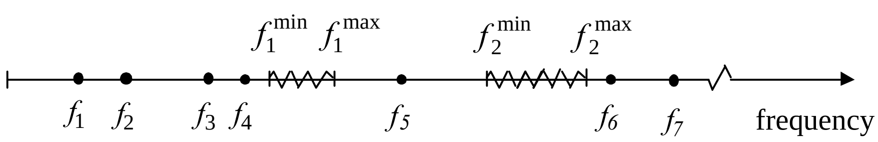


Figure 4.2. The natural frequencies are not allowed to fall into forbidden intervals.

Inequalities in Eq. (4.22) can be merged into a single constraint

$$g^{\mathrm{f}} = \max_{j=1,2,\ldots,n_{\mathrm{f}}} \left\{ \max_{i=1,2,\ldots,n_{\mathrm{df}}} \left\{ \bar{f} \cdot \left(\frac{f_i}{f_j^{\min}} - 1 \right) \left(1 - \frac{f_i}{f_j^{\max}} \right) \right\} \right\} \le 0 \ . \tag{4.23}$$

$\bar{f}$ is a suitable scale factor.

4.4 Optimization problem in standard form

In the mathematical standard form the tubular truss optimization problem is

$$
\begin{aligned}
&\min \begin{bmatrix} f_1(\mathbf{x}) \\ \vdots \\ f_{n_\mathrm{c}}(\mathbf{x}) \end{bmatrix} \\
&g_j^{\mathrm{s}}(\mathbf{x}) \le 0 && j = 1, \ldots, n_{\mathrm{Lc}} \\
&g_j^{\mathrm{b}}(\mathbf{x}) \le 0 && j = 1, \ldots, n_{\mathrm{Lc}} \\
&g_j^{\mathrm{j}}(\mathbf{x}) \le 0 && j = 1, \ldots, n_{\mathrm{Lc}} \\
&g_j^{\mathrm{kkgj}}(\mathbf{x}) \le 0 && j = 1, \ldots, n_{\mathrm{Lc}} \\
&g_{i,j}^{\mathrm{u}}(\mathbf{x}) \le 0 && i = 1, \ldots, n_{\mathrm{u}} \quad , \quad j = 1, \ldots, n_{\mathrm{Lc}} \\
&g_j^{\mathrm{bg}}(\mathbf{x}) \le 0 && j = 1, \ldots, n_{\mathrm{Lc}} \\
&g^{\mathrm{f}}(\mathbf{x}) \le 0 \\
&\mathbf{x} \in \left\{ \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{n_{\mathrm{cs}}} \right\}
\end{aligned}
\tag{4.24}
$$

$f_1(\mathbf{x})$, …, $f_{n_\mathrm{c}}(\mathbf{x})$ are the minimized criteria which can be e.g. the mass or the cost of the truss or some suitable displacement and n_c is the number of criteria. Also a single criterion objective function can be used. $g_j^{\mathrm{s}}(\mathbf{x}) \le 0$ are the strength constraints and $g_j^{\mathrm{b}}(\mathbf{x}) \le 0$ are the buckling strength constraints. $g_j^{\mathrm{j}}(\mathbf{x}) \le 0$ are the constraint for joints and $g_j^{\mathrm{kkgj}}(\mathbf{x}) \le 0$ for multiplanar gap KK-connections. n_j is the number of joints. $g_{i,j}^{\mathrm{u}}(\mathbf{x}) \le 0$ are the displacement constraints and n_u the number of these constraints. $g_j^{\mathrm{bg}}(\mathbf{x}) \le 0$ and $g^{\mathrm{f}}(\mathbf{x}) \le 0$ are the global stability and the frequency constraints. There are n_{Lc} load cases and constraints have to hold in all of them. Because all design variables are discrete the number of possible candidate solutions n_{cs} is finite.

5. Examples

5.1 Ten-bar truss

The first numerical example problem deals with the discrete optimization of a ten-bar plane truss presented in Fig. 5.1. The idea is to compare the mutual efficiency of simulated annealing, tabu search, genetic algorithm and particle swarm optimization in a simple academic optimization problem without taking into account the requirements of design rules. Due to the simplicity and small size the problem can be solved also using branch&bound algorithm and the performance of heuristic algorithms can be compared to this different type of method. Two alternative cases have been considered: A good initial guess is not known or it can be used in optimization. The example problem has been presented in Ref. [50].

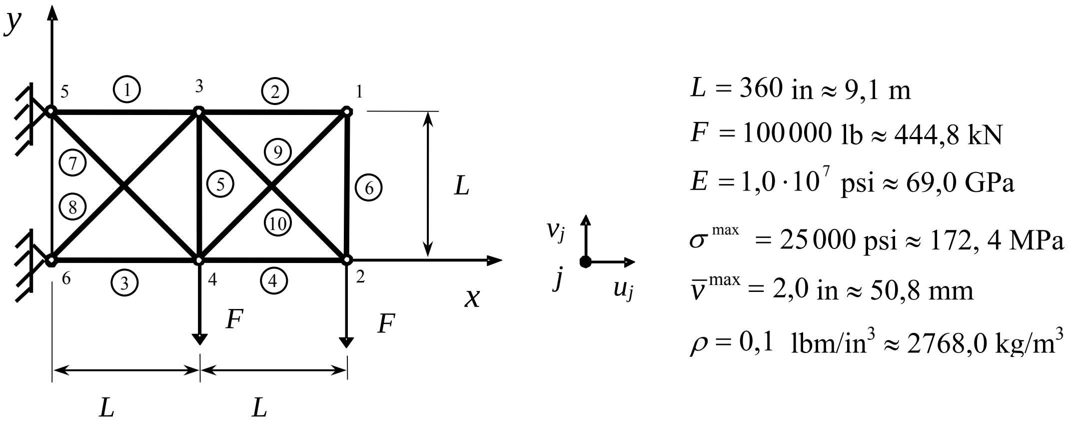


Figure 5.1. The ten-bar aluminium plane truss.

The mass of the truss should be minimized so that the normal stress is more than $-\sigma^{max}$ and less than σ^{max} in all the bars and the deflection in nodes 1, 2, 3 and 4 is less than the maximum allowed value $\bar{v}^{max}$. The cross section areas A_i are the design variables and their values should be chosen from a set which includes 81 evenly distributed values.

In the mathematical form the ten-bar plane truss optimization problem is

$$
\begin{aligned}
&\min\, m(\mathbf{x}) = \sum_{i=1}^{10} \rho L_i A_i \\
&g_i(\mathbf{x}) = -1 - \frac{\sigma_i(\mathbf{x})}{\sigma^{\max}} \le 0 && \forall\ i = 1,\ \ldots,\ 10 \\
&g_i(\mathbf{x}) = \frac{\sigma_i(\mathbf{x})}{\sigma^{\max}} - 1 \le 0 && \forall\ i = 11,\ \ldots,\ 20 \qquad (5.1)\\
&g_{j+20}(\mathbf{x}) = -\bar{v}^{\max} - v_j(\mathbf{x}) \le 0 && \forall\ j = 1,\ 2,\ 3,\ 4 \\
&A_i \in \{0{,}1\ ,\ 0{,}5\ ,\ 1{,}0\ ,\ 1{,}5\ ,\ 2{,}0\ ,\ldots,\ 39{,}5\ ,\ 40{,}0\}\quad \text{in}^2 \\
&\mathbf{x} = [\,A_1\ A_2\ \ldots\ A_{10}\,]^{\mathrm{T}}
\end{aligned}
$$

The versions and the needed parameter values of SA, TS, GA and PSO have been chosen according to literature and test runs ([26] and [27]). In this thesis all heuristic algorithms including branch&bound are self implemented and they run in Matlab. The structural analysis is done using finite element method and a linear four degree of freedom bar element in example 5.1.

In SA the initial temperature is $T^0 = 10000$ and the final temperature is $T^{\text{final}} = 0{,}01$. The depth of the discrete neighborhood is $d_{\text{n}} = 12$. The coefficient for the decrease of temperature is $\varepsilon = 0{,}9$ and the number of iteration rounds is $n_{\text{T}} = 40$ in the same temperature. An unfeasible point is always automatically rejected and a new one is selected.

The tabu list has been implemented so that all the previous solutions and solutions which are similar to them are prohibited. The design variable that changed last will stay fixed $n_{\text{fix}} = 5$ following iteration rounds. Also in TS an unfeasible point is always automatically rejected and the depth of the discrete neighborhood is $d_{\text{n}} = 15$. The number of iteration rounds is $n_{\text{iter}} = 200$ in TS.

In GA binary encoding, binary tournament selection, two points crossover and elitism have been used. The size of the population is $n_{\text{pop}} = 80$ individuals, the number of generations is $n_{\text{gen}} = 120$ and the probabilities of crossover and mutation are $p_{\text{c}} = 0{,}8$ and $p_{\text{m}} = 0{,}05$.

In PSO the size of the swarm is $n_{\text{swarm}} = 50$ particles but it will be increased by one randomly chosen particle every time the best known feasible objective function value has not improved during 5 previous iteration rounds. However, if the best known objective function value stays the same 20 iteration rounds the increase of the swarm stops. Initial velocities v_0^i , $i = 1, \ldots, 50$, are set to zero and velocities can not exceed value $v^{\max} = 12$ in^2 during optimization. The initial inertia is $w_0 = 1{,}4$ and the new inertia for the next 5 iteration rounds is calculated from previous one by multiplying it with $\delta = 0{,}8$. The values of scaling parameters are $c_1 = c_2 = 2$. The number of iteration rounds is $n_{\text{iter}} = 200$ and the penalty for unfeasible constraints is $R = 2$.

The truss has to be analyzed thousands of times during the optimization. In a discrete optimization problem it is useful to keep in memory some time the values of the objective function and constraints after they have been calculated. Before a new analysis the memory is checked and if the solution is not there, the values of the objective function and the constraints are calculated. In this study the number of solutions in the memory is 500. In GA and PSO the memory shows its strength but it seems to be unnecessary in SA and TS.

Heuristic methods TS, SA, GA and PSO have been compared to branch&bound algorithm (BB) which is based on implicit enumeration. In BB the original discrete problem is systematically replaced by a set of subproblems in which the design variables are relaxed to continuous ones with certain lower and upper bounds. Based on the solutions of subproblems it can be concluded what kind of discrete solutions can not be optimal. This reduces efficiently the set of potential candidate solutions. BB is guaranteed to find the global optimum for a discrete optimization problem if all the solutions of subproblems are global optima. The number of subproblems and the needed solution time increases rapidly as the size of discrete optimization problem increases and BB is suitable algorithm only for limited size problems. In this study subproblems are solved using Matlab Optimization

Toolbox's SQP-algorithm (sequential quadratic programming) and needed derivatives are calculated using analytical expressions (Appendix C). The use of BB is discussed more detailed e.g. in Ref. [5].

5.1.1 Comparison of algorithms without a good initial guess

SA, TS and BB need an initial guess before the optimization can be started. At first a good initial guess is not assumed to be known even though it is easy to find a feasible solution just by selecting big enough members to the truss. The feasible initial guess is such that A_i = 20 in^2 = 129,0 cm^2 $i = 1,\ldots,10$ (m = 8392,9 lbm = 3807,0 kg, max $\left\{ |\sigma_i| \mid i = 1,\ldots,10 \right\}$ = 10231,8 psi = 70,5 MPa and max $\left\{ -v_j \mid j = 1,\ldots,4 \right\}$ = 1,97 in = 50,2 mm) which corresponds an average randomly selected individual. In GA the initial population and in PSO the initial swarm are selected randomly in the beginning of each run.

The optimization problem is solved once using TS and BB and 1000 times using SA, GA and PSO in several microcomputers. The duration of one FEM-analysis is approximately 0,004 – 0,005 seconds and the total time for one optimization run is 35 – 190 seconds. Figure 5.2 represents the improvement of the objective function in individual optimization runs and the average improvement as the function of expended FEM-analysis. In this thesis FEM-analysis includes the calculation of displacements, stresses, critical load factor and natural frequencies. If heuristic algorithms are used the most time consuming phase is the analysis of the structure and other calculations need less time unless the analysis model is small. Thus it is more useful to consider the number of needed FEM-analysis than the actual time in an academic test problem so that conclusions would be useful also with the bigger structures. Since stochastic methods can sometimes fail quite badly it is better to focus on the median of the objective function than the average value of the objective function. The median curves are drawn by calculating the median of the objective function and the median of the number of expended FEM-analysis up to each iteration step.

Table 5.1 represents the best found final results and Fig. 5.3 a) the mutual efficiency of TS, SA, GA, PSO and BB during optimization. Figure 5.3 b) shows the divergence of results using the upper and lower quartiles and Fig. 5.4 the distribution of the optimization results compared to the best found solution. The system of units is the same as in the original paper [50].

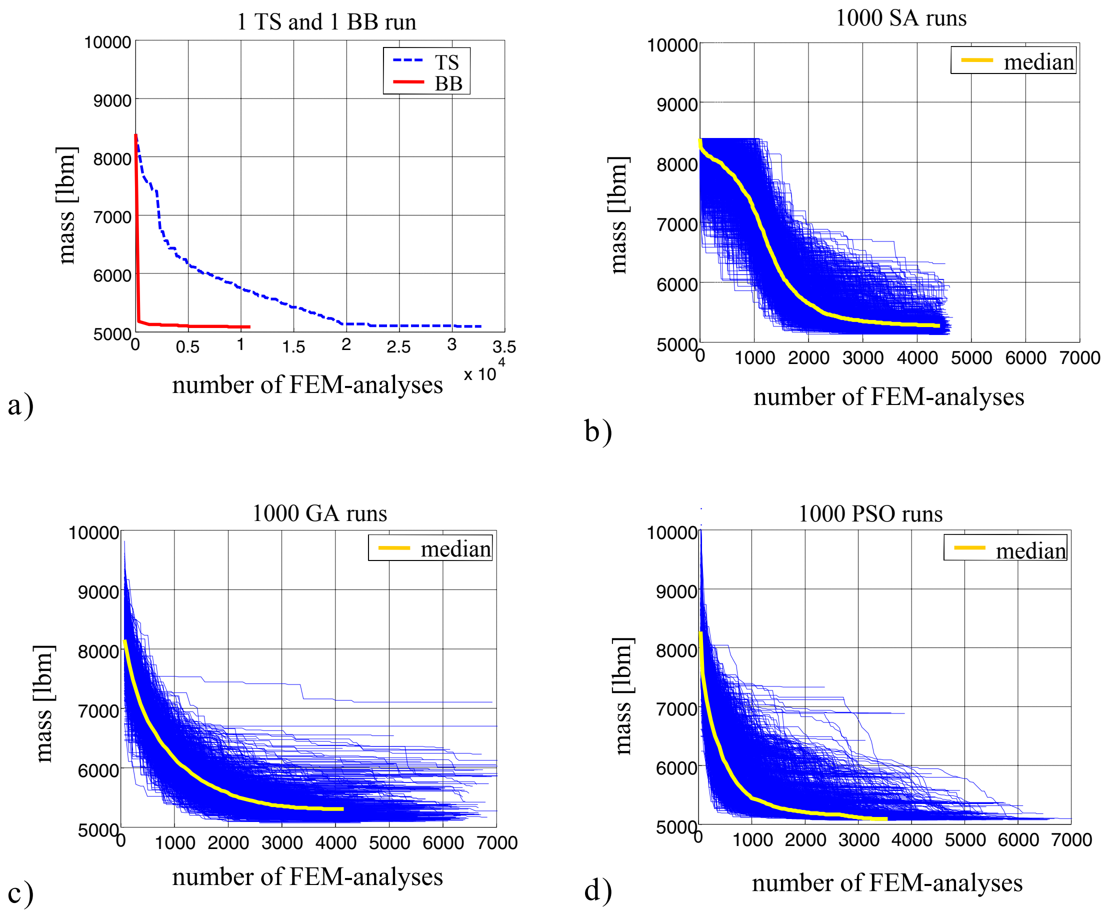


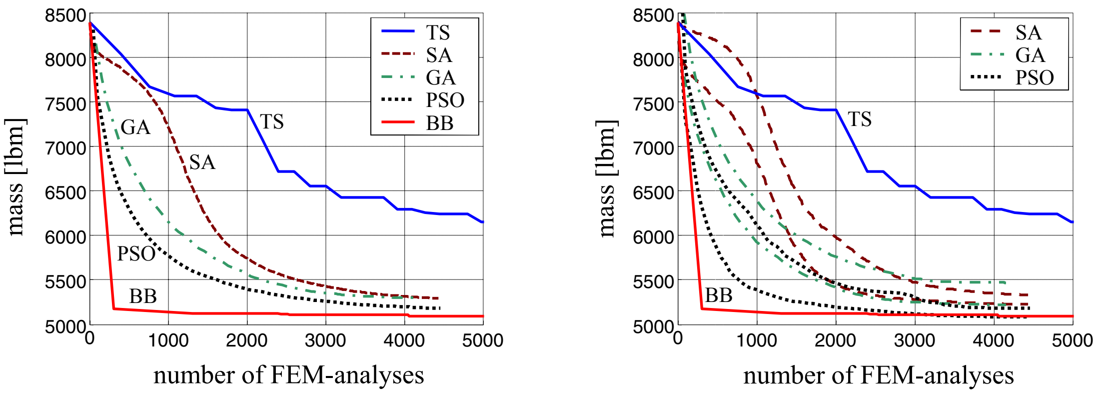
Figure 5.2. The decrease of mass in individual optimization runs and the improvement of the median of mass: a) Tabu search and branch&bound, b) Simulated annealing, c) Genetic algorithm, d) Particle swarm optimization.

An efficient heuristic optimization algorithm finds a good solution using as few FEM-analysis as possible. The comparison of the final results is not enough and the numbers of expended analysis have to be taken into account. In practical applications the fast improvement of the objective function is often more important than a good final solution which is laborious to achieve. If stochastic algorithm is used the reliability is also important. The number of optimization runs can be reduced by using reliable algorithm which gives uniform results in consecutive runs.

Table 5.1. The best found final results in optimization.

	cross section areas [in^2]										mass	number
	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	[lbm]	of runs
SA	28,5	0,1	27,5	16,5	0,1	1,0	8,0	20,0	21,0	0,1	5120,6	1
TS	31,0	0,1	25,0	17,5	0,1	0,5	7,5	19,5	20,5	0,1	5094,6	
GA	31,5	0,1	24,5	15,5	0,1	0,5	7,5	20,0	21,0	0,1	5073,5	1
PSO	**29,5**	**0,1**	**23,0**	**16,0**	**0,1**	**0,5**	**7,5**	**21,5**	**21,5**	**0,1**	**5067,3**	**2**
	31,0	**0,1**	**23,0**	**14,5**	**0,1**	**0,5**	**7,5**	**20,5**	**22,5**	**0,1**	**5067,3**	**3**
	30,0	**0,1**	**23,0**	**15,5**	**0,1**	**0,5**	**7,5**	**21,0**	**22,0**	**0,1**	**5067,3**	**3**
	29,5	**0,1**	**24,0**	**15,0**	**0,1**	**0,5**	**7,5**	**22,0**	**21,0**	**0,1**	**5067,3**	**1**
BB	29,5	0,1	24,0	16,0	0,1	0,5	7,5	20,0	22,5	0,1	5077,9	
[50] 1)	31,5	0,1	23,0	15,5	0,1	0,5	7,5	20,5	21,0	0,1	5045,8	

1) The solution is not feasible ($v_1 = -2{,}0163$ in).



a) b)

Figure 5.3. a) The mutual efficiency of TS, SA, GA, PSO and BB in ten-bar plane truss optimization problem. SA, GA and PSO curves represent the median of mass based on 1000 independent optimization runs. b) The lower and the upper quartiles. 50 percent of runs are between the lower and the upper quartiles.

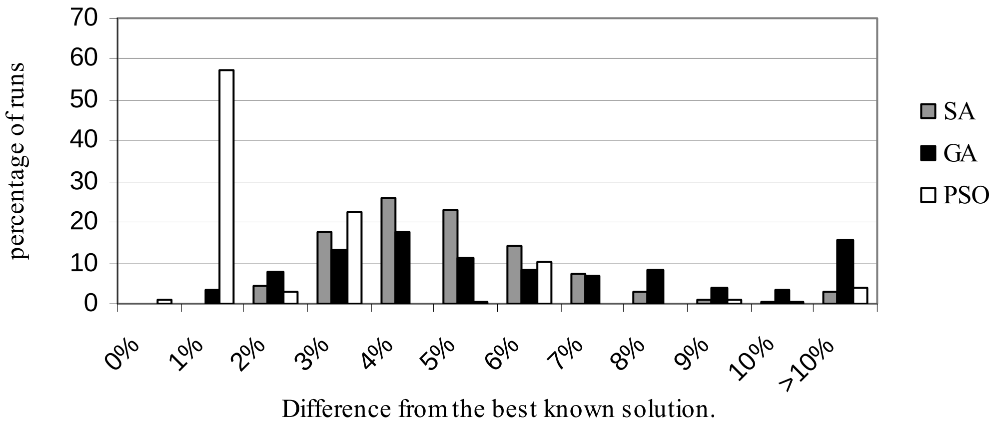


Figure 5.4. The distribution of the final results compared to the best found solution (5067,3 lbm, 2298,5 kg) in SA, GA and PSO runs.

In ten-bar truss problem without a good initial guess the same best found objective function value 5067,3 lbm (2298,5 kg) was achieved nine times using PSO in four different points in design space. In all cases the most active constraints are the stress constraint of bar five and the deflection constraint of node one. The other algorithms found single good solutions in which the value of the objective function is close to the best found value. Since BB ended up worse solution than PSO and GA some of the solutions of subproblems are local optima in BB. According to Fig. 5.4 PSO seems to be a superior algorithm compared to other stochastic methods in the sense of the reliability of the final results. 58 percent of PSO runs gave the objective function value which differs one percent or less from the best found value.

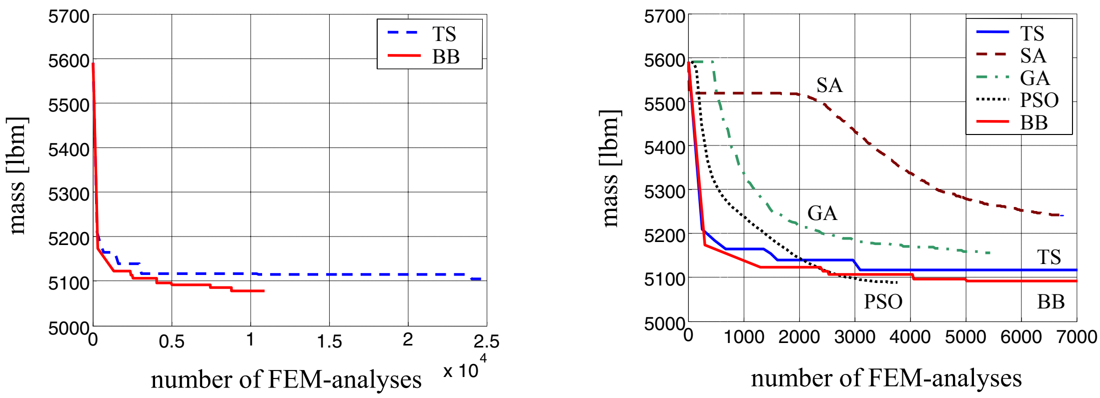
In the beginning BB improves the value of the objective function more efficiently than TS or on average SA, GA or PSO (Fig. 5.3). However after the good start BB becomes ineffective and the improvement of the objective function almost stops. According to Fig. 5.3 b) the lower quartile of PSO is quite close to the BB curve which means that several PSO runs improve the objective function basically as efficiently as BB. The drawback of stochastic PSO is that there might be many poor runs before a good one. Deterministic TS seems to improve the value of the objective function leisurely but at the end it found a better final solution than any of the 1000 SA runs.

Based on above results PSO can be considered as the most efficient algorithm among heuristic methods SA, TS, GA and PSO if good initial guess is not known. It found the best final result, the improvement of the objective function was the most rapid and its reliability was very high.

5.1.2 Comparison of algorithms using a good initial guess

In many applications a good initial design is already known or it can be easily sought. In ten-bar truss example a good initial solution is $A_1 = A_8 = 30$ in^2 = 193,5 cm^2, $A_3 = A_4 = A_9 = 20$ in^2 = 129,0 cm^2 , $A_7 = 10$ in^2 = 64,5 cm^2 and $A_2 = A_5 = A_6 = A_{10} = 0{,}1$ in^2 = 0,6 cm^2 (m = 5590,6 lbm = 2535,9 kg, max $\{|\sigma_i| \mid i = 1,\ldots,10\}$ = 21194,1 psi = 146,1 MPa and max $\{-v_j \mid j = 1,\ldots,4\}$ = 1,89 in = 48,0 mm). The good initial solution is put to the initial population/swarm and the rest of members are selected randomly in GA and PSO.

The optimization problem is solved again once using TS and BB and 1000 times using SA, GA and PSO. The parameter values remain the same as in the previous optimization runs. Figure 5.5 represents the progress of optimization and Fig. 5.6 the distribution of final results. The best found solutions can be seen from Table 5.2.



a) b)

Figure 5.5 The decrease of mass using a good initial guess in ten-bar plane truss optimization problem. a) TS and BB. b) The mutual efficiency of TS, SA, GA, PSO and BB. SA, GA and PSO curves represent the median of mass based on 1000 independent optimization runs.

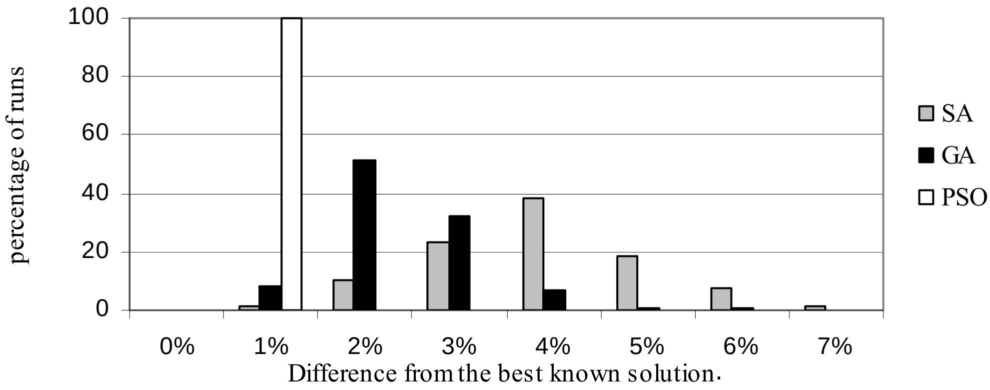


Figure 5.6. The distribution of the final results compared to the best found solution (5067,1 lbm, 2298,5 kg) in SA, GA and PSO runs using a good initial guess.

Table 5.2. The best found final results in optimization using a good initial guess.

	cross section areas [in^2]										mass	number
	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	[lbm]	of runs
SA	29,0	0,1	24,5	14,0	0,1	1,0	7,5	22,5	21,5	0,1	5100,2	3
TS	34,5	0,1	23,0	17,0	0,1	0,5	7,5	19,0	20,5	0,1	5105,1	
GA	32,0	0,1	21,5	15,5	0,1	0,5	8,0	22,5	20,0	0,1	5085,3	1
PSO	**30,0**	**0,1**	**23,5**	**15,0**	**0,1**	**0,5**	**7,5**	**21,5**	**21,5**	**0,1**	**5067,3**	**1**
BB	29,5	0,1	24,0	16,0	0,1	0,5	7,5	20,0	22,5	0,1	5077,9	

By comparing Tables 5.1 and 5.2 it can be noticed that the good initial guess worsens the best found final results in the cases of TS and GA. PSO found once a new solution which has the same best known objective function value 5067,3 lbm (2298,5 kg) and SA found three times a new better solution than previously. BB ended up with the same solution as from average initial guess. According to Fig. 5.6 PSO is again superior stochastic algorithm in the sense of the reliability of final results.

Figure 5.5 shows that the good initial guess improves the efficiency of TS clearly and it is almost as good as BB during early iteration rounds. PSO works still well and on average it found a better solution (5088,9 lbm, 2308,3 kg) than TS using less FEM-analysis. It takes some time before the objective function starts to improve in GA and SA and especially SA seems to be generally ineffective. To improve the performance of SA the initial temperature could be decreased.

If a good initial guess is known PSO and TS are the two best heuristic methods in the light of above results. The final optimization results are not necessary as good as from an average initial guess but the improvement of the objective function is fast already in the beginning of optimization.

5.2 Tubular plane truss

5.2.1 Comparison of algorithms in sizing optimization

In this example the mutual efficiency of TS, SA, GA and PSO has been studied using a sizing optimization problem connected to a simple plane truss (Fig. 5.7). The topology of the truss is fixed and the height is $H = 1,5$ m. The task is to select the values of the size design variables $x_i^s \in \{1, 2, \ldots, 112\}$ ($i = 1,2,\ldots,7$) so that the cost of the truss is minimized and the strength and the buckling strength requirements for all members and the strength requirements for welded joints (except joints 1 and 11) are fulfilled and the maximum deflection is less than $\bar{v}^{\max} = L/200 = 100\,\text{mm}$ ([63], roofs in general). The size design variable x_1^s is connected to the upper chord, x_2^s to the lower chord and $x_3^s, \ldots, x_7^s$ to the bracing members. The set of available profiles includes 112 RHS and SHS profiles given in the ascending order according to the cross-section area in Appendix D.

The sizes of upper and lower flanges are constant and they both consist of three pieces welded consecutively. This means that the number of structural elements κ is 3+3+10=16. The compressed upper flange is supported in out of plane direction in joints 1, 3, 5, 7, 9 and 11. In all joints the eccentricity is zero. The alignment of profiles is such that the height of the profile cross section h is parallel to the xy-plane. There is only one load case and FEM-model includes five beam elements in the upper chord, four in the lower chord and one bar element per each bracing member.

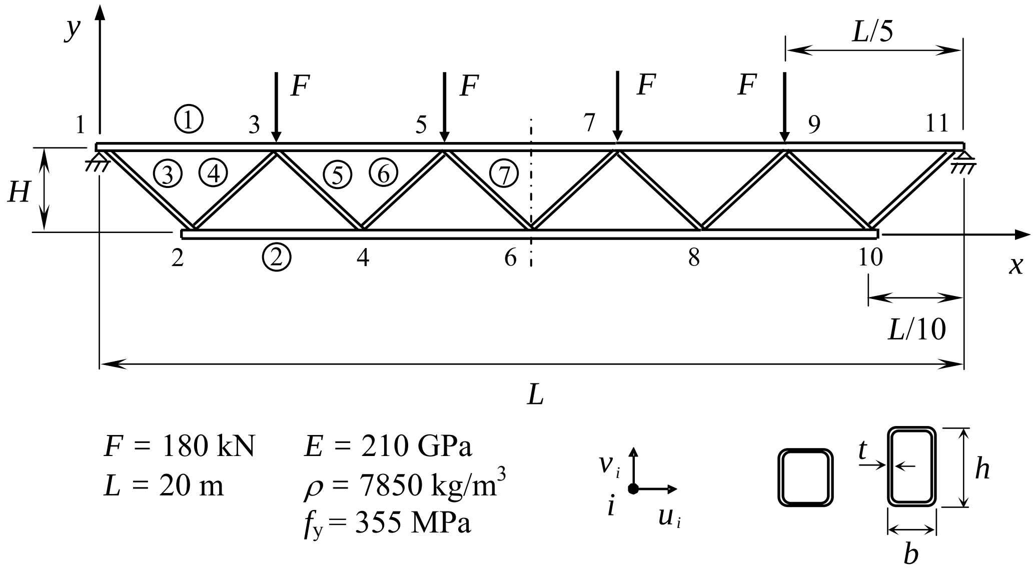


Figure 5.7. Symmetrical tubular plane K-truss made of SHS- or RHS-profiles, cross-sections (1, ..., 7), joints (1, ..., 11) and displacements u_i and v_i connected to joint i.

Table 5.3. The values of parameters in the manufacturing cost calculation.

Preparation, assembly and tacking	$C_1 = 1$, $\Theta_d = 3$ and $\kappa = 16$
Welding and additional fabrication costs	$C_2 = 0{,}4$ and $\tilde{n} = 2$ (GMAW-C)
Surface preparation	$\Theta_{ds} = 2$ and $a_{sp} = 3 \cdot 10^{-6}$
Painting	$\Theta_{dp} = 2$, $a_{gc} = 3 \cdot 10^{-6}$ and $a_{tc} = 4{,}15 \cdot 10^{-6}$
Cutting	$\Theta_{dc} = 2$ or $\Theta_{dc} = 3$ (two cuts for overlapping bracing members)

The material cost factor is assumed to be $k_m = 0{,}6$ \$/kg and value $k_f = 0{,}9$ \$/min is used for the fabrication cost factor ($k_f / k_m = 1{,}5$ kg/min). In the manufacturing cost calculation the values of parameters (Table 5.3) have been chosen based on [30] and they are not connected to any certain manufacturer. According to [56] the gas-shielded arc welding is the most common welding technology in the manufacturing of tubular trusses in machine workshops. In the case of longitudinal fillet welds [30] gives for the gas metal arc welding with CO_2 (GMAW-C) values $C_2 = 0{,}3394$ and $\tilde{n} = 2$ (downhand position) or

$C_2 = 0,4930$ and $\tilde{n} = 2$ (positional welding). Due to these values $C_2 = 0,4$ and $\tilde{n} = 2$ have been chosen and it is assumed that they can be used for all welds.

In the mathematical form the optimization problem is

$$
\begin{aligned}
&\min K(\mathbf{x}) && \\
&g^{\mathrm{s}}(\mathbf{x}) \le 0 && \text{(strength)} \\
&g^{\mathrm{b}}(\mathbf{x}) \le 0 && \text{(buckling)} \\
&g^{\mathrm{j}}(\mathbf{x}) \le 0 && \text{(joints)} \\
&g^{\mathrm{u}}(\mathbf{x}) = \frac{-v_6(\mathbf{x})}{\bar{v}^{\max}} - 1 \le 0 && \text{(displacement)} \\
&\mathbf{x} = \left[\, x_1^{\mathrm{s}} \cdots x_7^{\mathrm{s}} \,\right]^{\mathrm{T}} \in \left\{ \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{n_{\mathrm{cs}}} \right\}
\end{aligned}
\qquad (5.2)
$$

The number of feasible or unfeasible candidate solutions is $n_{\mathrm{cs}} = 112^7 \approx 2,21 \cdot 10^{14}$. If 1000 FEM-analysis could be done in one second the total enumeration would take more than 7000 years.

In optimization algorithms the values of parameters are basically the same as in the ten-bar truss example problem except that in TS the depth of discrete neighborhood is $d_{\mathrm{n}} = 16$ and the number of iteration rounds is $n_{\mathrm{iter}} = 250$, in SA the number of iteration rounds in the same temperature is $n_{\mathrm{T}} = 35$, in GA the size of the population is $n_{\mathrm{pop}} = 70$ and the number of generations is $n_{\mathrm{gen}} = 180$ and in PSO the number of iteration rounds is $n_{\mathrm{iter}} = 300$ and a new inertia is multiplied using dynamic inertia reduction parameter $\delta = 0,85$. These few modifications are done based on test runs to improve the efficiency. However the number of different parameters connected to algorithms is so big that all possible tuning opportunities have not been checked.

In TS and SA the initial guess is such that for the upper and the lower chord profile number 94 and for all bracing members profile number 46 have been selected which corresponds to the minimum cost truss in the case of two independent size design variables ($x_1^{\mathrm{s}} = x_2^{\mathrm{s}} = 94$, $x_3^{\mathrm{s}} = \ldots = x_7^{\mathrm{s}} = 46$, $K = 4273,8$ \$

and $m = 2293{,}8$ kg). This structure can be easily found using enumeration. The initial population and swarm have been chosen randomly so that they both include 15 feasible individuals. The idea is to make sure that initial population and swarm are good enough. If the initial population includes only poor unfeasible individuals in GA it is unlikely that the optimization result, which is formed mostly by combining existing individuals, would be good. It is also assumed that if there are some feasible solutions in the initial swarm it is advantageous for PSO. In this problem a randomly selected individual is usually unfeasible and it is laborious to find 15 feasible solutions in the beginning of each run. That is why feasible individuals are picked from the same set of 100 feasible solutions which are selected once randomly before all optimization runs. In the initial population and swarm one individual is forced to be the same as the initial guess in TS and SA runs.

The optimization has been done once using TS and 100 times using SA, GA and PSO. The results are presented in Fig. 5.8, …, Fig. 5.11 and Table 5.4.

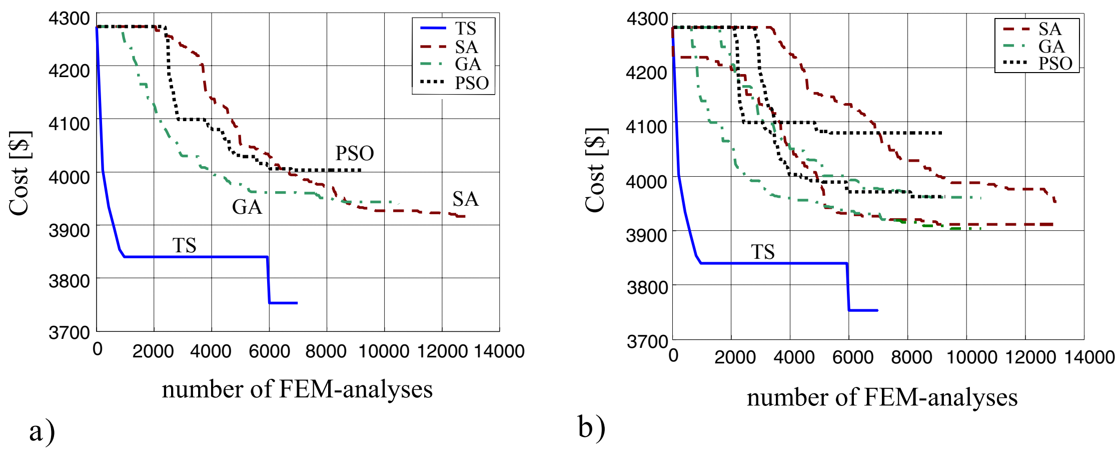


Figure 5.8. a) The efficiency of TS, SA, GA, PSO in the problem (5.2). SA, GA and PSO curves represent the median of mass based on 100 independent optimization runs. b) The lower and the upper quartiles in SA, GA and PSO runs.

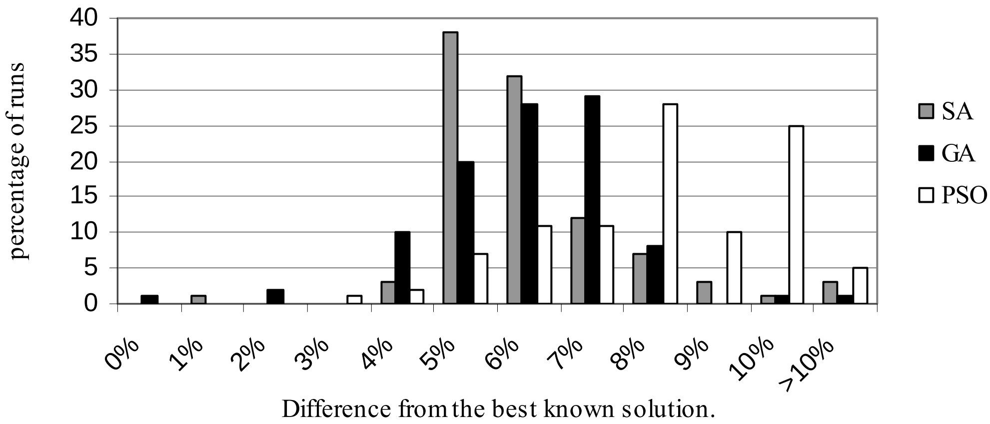


Figure 5.9. The distribution of the final results compared to the best found solution (K=3727,8 \$) in SA, GA and PSO runs in the problem (5.2).

Table 5.4. The best found final results in the problem (5.2).

	x_1^s	x_2^s	x_3^s	x_4^s	x_5^s	x_6^s	x_7^s	cost [\$]	mass [kg]	number of runs
SA	94	87	40	46	36	38	23	3745,9	2087,6	1
TS	94	87	40	46	46	32	23	3753,0	2090,8	
GA	**94**	**84**	**40**	**46**	**46**	**40**	**23**	**3727,8**	**2057,4**	**1**
PSO	94	84	40	46	46	40	40	3817,4	2092,8	1

According to Table 5.4 GA ended up once to the best found solution K=3727,8 \$ while the best results of SA and TS were slightly poorer and PSO found the weakest final solution. PSO seems to be also the weakest stochastic algorithm if the distribution of final results (Fig. 5.9) is considered. Based on Fig. 5.8 TS can be considered overall the best algorithm in the tubular plane truss sizing optimization problem (5.2). If only one or few optimization runs can be done, deterministic TS guarantees a good result which is most likely better than what SA, GA and PSO will find. The most surprising thing in the results is that this time the success of PSO is clearly poorer than what it was in the ten-bar truss example problem.

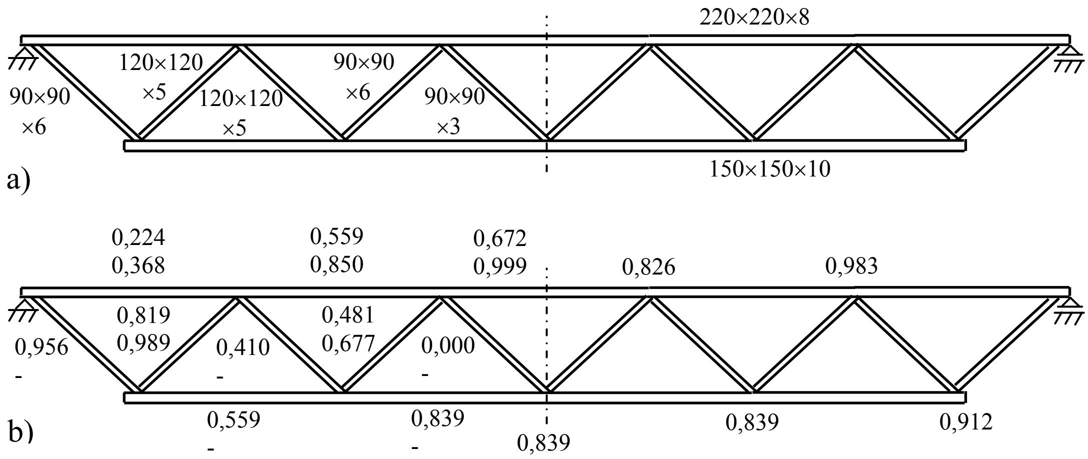


Figure 5.10. a) The best found truss (K=3727,8 \$) in the problem (5.2). b) The ratios of design values to design capacities in the best found solution. On the left side the upper values are connected to member strength and the lower values to member buckling strength. On the right side values are connected to joint strength. (The support points are not considered.)

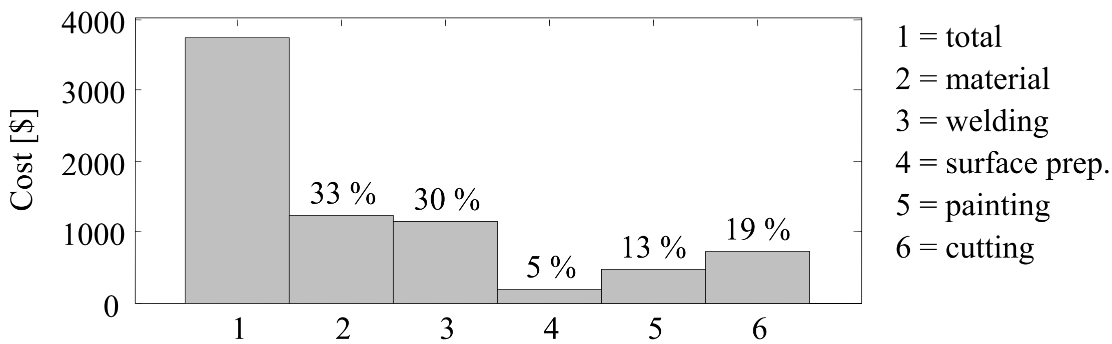


Figure 5.11. The distribution of costs for the best found truss (K=3727,8 \$) in the problem (5.2).

Figure 5.10 shows that in the best found truss the buckling strength is the most limiting phenomenon in the upper chord (Eq. (2.33) has value 0,999 in the middle member) and in the lower chord the strength demands are well fulfilled (the maximum value of Eq. (2.17) is 0,839). The most loaded bracing members are connected to design variables x_3^s and x_4^s and in these members

$N_{Sd} / N_{t.Rd} = 0{,}956$ and $N_{Sd} / N_{b.Rd} = 0{,}989$. Joints three and nine are the critical connections because in these joints $\max\{N_{i.Sd} / N_{i.Rd}\} = 0{,}983$. The displacement constraint is not close to active since the maximum deflection is 83,1 mm.

5.2.2 Comparison of algorithms in topology, shape and sizing optimization

This efficiency comparison concerns the simultaneous topology, shape and sizing optimization of a tubular plane truss. The problem is basically similar to the previous one except that there are (Fig. 5.12) four available topologies $x^{\mathrm{t}} \in \{1, 2, 3, 4\}$, one discrete shape design variable $x^{\mathrm{sh}} \in \{1{,}0\,, 1{,}1\,, \ldots, 5{,}0\}$ m and four size design variables $x_i^{\mathrm{s}} \in \{1, 2, \ldots, 112\}$ (x_1^{s} upper chord, x_2^{s} lower chord, x_3^{s} tension and x_4^{s} compression bracing members). The objective function, constraints and the set of available profiles have not been changed. Because there are only four preselected topologies available the topology of truss can change rather limited.

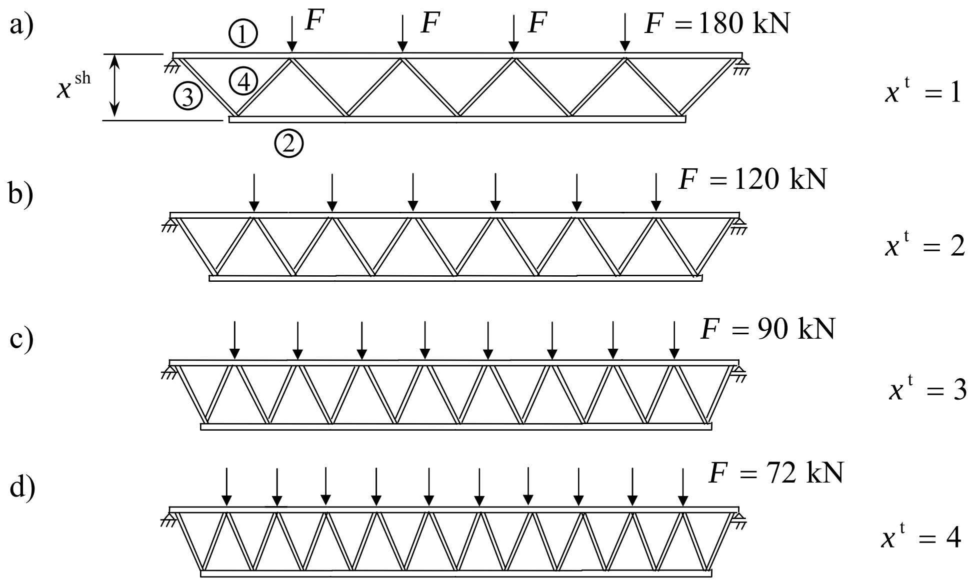


Figure 5.12. The four available topologies a), b), c) and d) ($x^{\mathrm{t}} \in \{1, 2, 3, 4\}$), the shape design variable x^{sh} and four different cross-sections in the tubular plane truss topology, shape and sizing optimization problem (5.3). The meaning of x^{sh} and cross-sections are the same in cases a), b), c) and d).

The total load 720 kN is divided into equal point forces along the upper chord. The joints at both ends of truss are not considered in the joint constraint and FEM-models include one element from joint to joint in chords and one element per each web bar.

In the mathematical form the optimization problem is

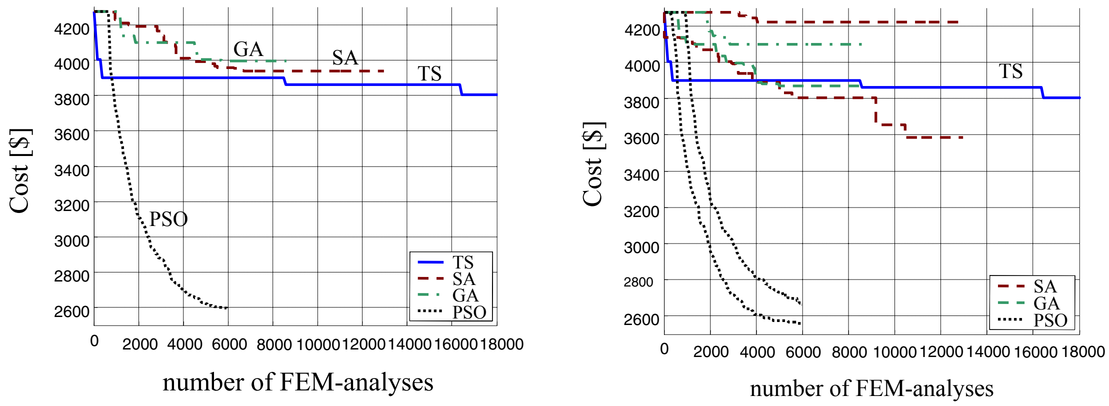
$$
\begin{aligned}
&\min K(\mathbf{x}) \\
&g^{\mathrm{s}}(\mathbf{x}) \le 0 && \text{(strength)} \\
&g^{\mathrm{b}}(\mathbf{x}) \le 0 && \text{(buckling)} \\
&g^{\mathrm{j}}(\mathbf{x}) \le 0 && \text{(joints)} \\
&g^{\mathrm{u}}(\mathbf{x}) = \frac{-v(\mathbf{x})}{\overline{v}^{\max}} - 1 \le 0 && \text{(displacement)} \\
&\mathbf{x} = \left[x^{\mathrm{t}} \; x^{\mathrm{sh}} \; x_1^{\mathrm{s}} \cdots x_4^{\mathrm{s}} \right]^{\mathrm{T}} \in \left\{ \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{n_{\mathrm{cs}}} \right\}
\end{aligned}
\tag{5.3}
$$

v means the vertical displacement in the middle and the number of candidate solutions is $n_{\mathrm{cs}} \approx 2,58 \cdot 10^{10}$. In this case the total enumeration takes 298 days if 1000 FEM-analysis are done in one second.

The initial guess ($x^{\mathrm{t}} = 1$, $x^{\mathrm{sh}} = 1,5$ m, $x_1^{\mathrm{s}} = x_2^{\mathrm{s}} = 94$ and $x_3^{\mathrm{s}} = x_4^{\mathrm{s}} = 46$, $K = 4273,8$ \$ and $m = 2293,8$ kg) and the parameter values of algorithms are the same as earlier except that in TS the last changed design variable stays fixed only two following iteration rounds ($n_{\mathrm{fix}} = 2$). The initial swarm and population are also formed in a similar way to the sizing optimization problem. The optimization has been again done one time using TS and 100 times using SA, GA and PSO. Results are presented in Table 5.5 and Figures 5.13, …, 5.16.

Table 5.5. The best found final results in the problem (5.3).

	x^{t}	x^{sh}	x_1^{s}	x_2^{s}	x_3^{s}	x_4^{s}	cost [\$]	mass [kg]	number of runs
SA	1	3,4	65	88	29	48	3131,2	1817,8	1
TS	1	1,7	90	85	43	46	3802,5	2029,7	
GA	1	3,2	65	89	32	46	3114,2	1802,7	1
PSO	**1**	**3,8**	**57**	**38**	**29**	**56**	**2500,9**	**1318,3**	**2**



a) b)

Figure 5.13. a) The mutual efficiency of TS, SA, GA, PSO in the problem (5.3). SA, GA and PSO curves represent the median of mass based on 100 independent optimization runs. b) The lower and the upper quartiles in SA, GA and PSO runs.

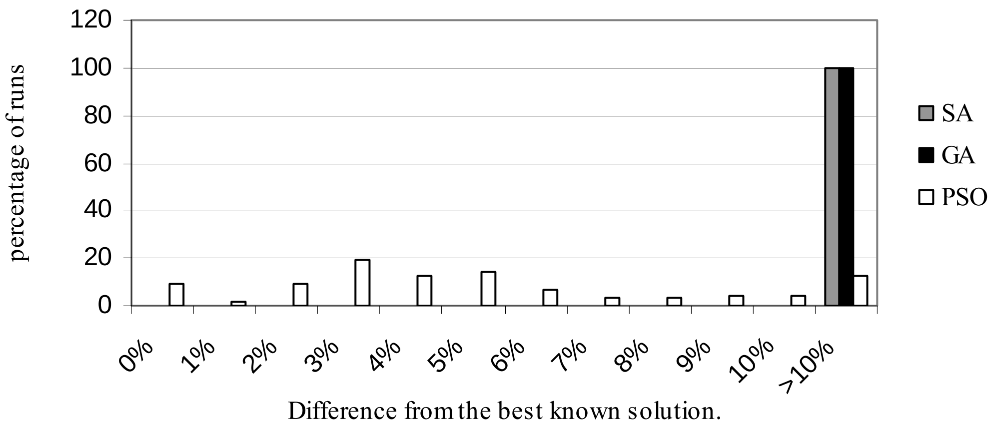


Figure 5.14. The distribution of the final results compared to the best found solution (K=2500,9 \$) in SA, GA and PSO runs in the problem (5.3).

According to Table 5.5 and Figures 5.13 and 5.14 PSO is a superior algorithm compared to SA, TS and GA in the tubular plane truss topology, shape and sizing optimization problem (5.3). By adding the topology and the shape design variables x^{t} and x^{sh} the truss sizing optimization problem becomes some how much more suitable for PSO than it was. It is still unclear what specific feature is behind this dramatic improvement. SA, TS and GA work on average the same way as in the sizing optimization except that TS needs more

FEM-analysis and the divergence of SA results is wider. In fact many of the SA runs do not improve the value of the objective function at all which was not noticed in the sizing optimization. In SA and GA runs the best found solutions are clearly better than the final result of the TS run. Despite the superiority of PSO in the beginning of optimization TS improves most efficiently the value of the objective function as can be seen from Fig, 5.13. Thus deterministic TS is the best algorithm if only a few hundred FEM-analysis can be done.

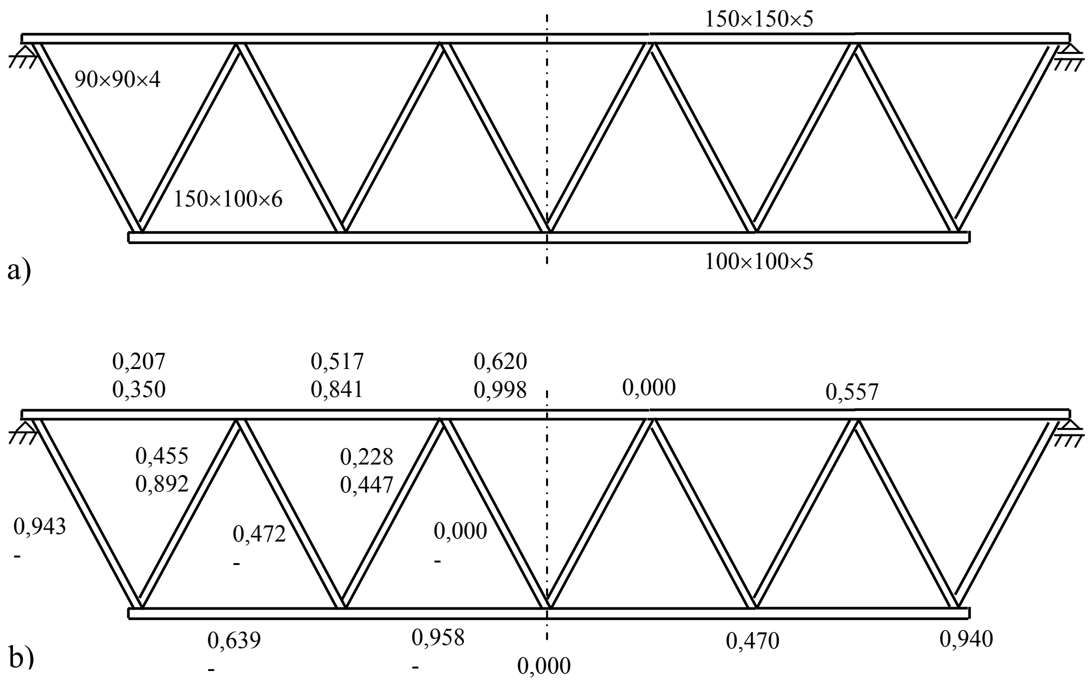


Figure 5.15. a) The best found truss (K=2500,9 $) in the problem (5.3). b) The ratios of design values to design capacities in the best found solution. On the left side the upper values are connected to member strength and the lower values to member buckling strength. On the right side values are connected to joint strength. (The support points are not considered.)

Based on Fig. 5.15 the buckling strength constraint is close to active in the upper chord (Eq. (2.33) gets value 0,998) and the strength constraint in the lower chord (Eq. (2.17) gets value 0,958). The most critical bracing members are the two outermost ($N_{Sd} / N_{t.Rd} = 0{,}943$ and $N_{Sd} / N_{b.Rd} = 0{,}892$). The connections at ends of the lower chord are critical joints ($\max\{N_{i.Sd} / N_{i.Rd}\} = 0{,}940$) and the maximum deflection of the truss is 42,9 mm.

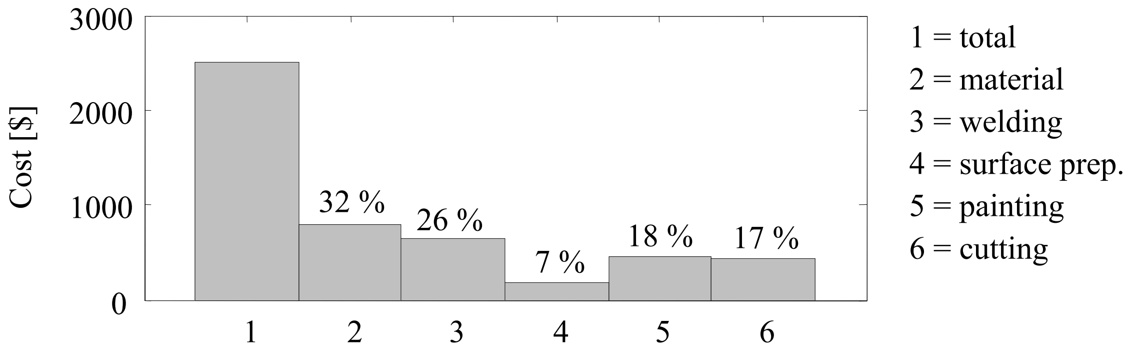


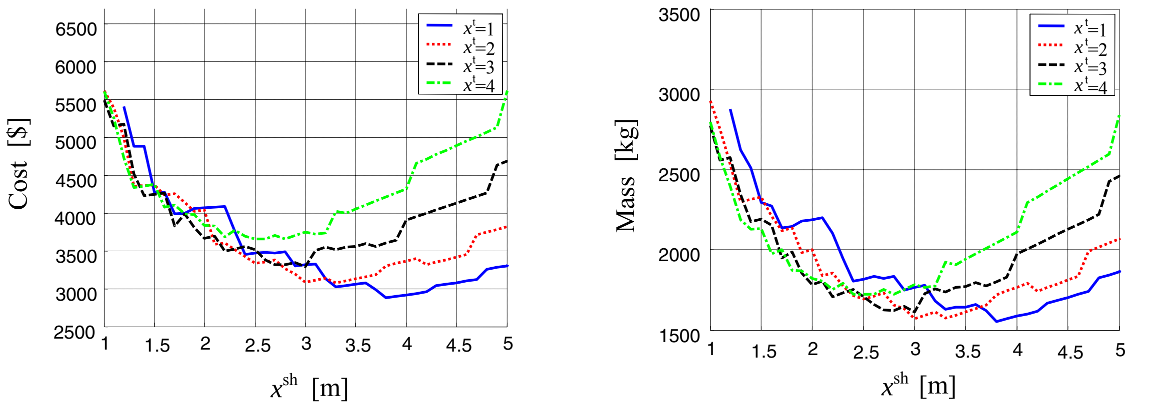
Figure 5.16. The distribution of costs for the best found truss (K=2500,9 $) in the problem (5.3).

If Figures 5.11 and 5.16 are compared it can be noticed that the cost distributions of the best found solutions are similar in the problems (5.2) and (5.3).

5.2.3 Conflict of mass and cost

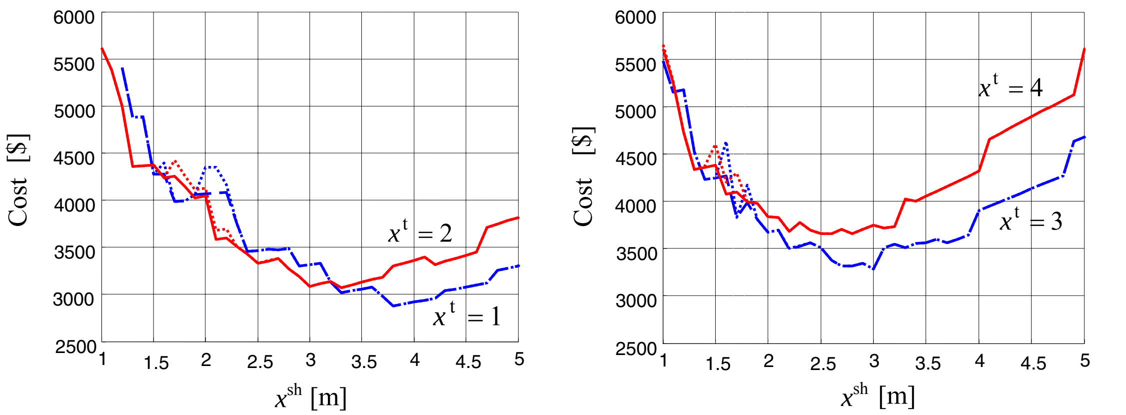
Traditionally it is assumed that the mass minimization and the cost minimization do not necessarily lead to the same final cost in steel structures. There are some results or estimates concerning the difference in the literature (Pavlovčič, Krajnc & Beg [54] 0,7% and Sarma & Adeli [60] 7%-26%) but at least in the case of tubular trusses the difference between the minimum mass and the minimum cost structures is still more or less unknown. If the conflict is small enough or it does not exist there is no need to consider mass and cost as separate criteria or to find out the values of different parameters connected to the cost function.

The conflict between mass and cost has been studied using the tubular plane truss topology, shape and sizing optimization problem (5.3) with two independent size design variables. The same profile is forced for the upper and the lower chord ($x_1^s = x_2^s$) and all bracing members have the same size ($x_3^s = x_4^s$). Due to the small number of design variables total enumeration can be used to calculate the following results.



a) b)

Figure 5.17. a) The minimum cost and b) the minimum mass in the problem (5.3) using two independent size design variables ($x_1^s = x_2^s$ and $x_3^s = x_4^s$).



a) b)

Figure 5.18. The minimum mass and the minimum cost trusses in the problem (5.3) using two independent size design variables ($x_1^s = x_2^s$ and $x_3^s = x_4^s$). a) Topologies $x^t = 1$ and $x^t = 2$ b) Topologies $x^t = 3$ and $x^t = 4$. Dotted lines present the mass minimization.

According to Fig. 5.17, which represents the minimum cost and the minimum mass as the function of height x^{sh} for different topologies, truss topology clearly affects the optimum height of the truss. In the discussed problem the minimum cost and the minimum mass trusses are the same ($x^{\mathrm{t}} = 1$, $x^{\mathrm{sh}} = 3,8$ m, $x_1^{\mathrm{s}} = x_2^{\mathrm{s}} = 57$, $x_3^{\mathrm{s}} = x_4^{\mathrm{s}} = 46$, $K = 2878,9$ \$ and $m = 1555,0$ kg). The values of the topology and the shape design variables are also the same as in the best found solution of the original problem (5.3) with four size design variables (Table 5.5). The decrease of two size design variables worsens the value of the best found objective function \$ 378 in the problem (5.3).

From Fig. 5.18 it can be seen that the mass minimization and the cost minimization lead to the same final cost unless $1,3\,\mathrm{m} \leq x^{\mathrm{sh}} \leq 2,3\,\mathrm{m}$. The biggest difference is \$ 366,4 ($x^{\mathrm{t}} = 3$ and $x^{\mathrm{sh}} = 1,6$ m: The minimum mass $x_1^{\mathrm{s}} = x_2^{\mathrm{s}} = 84$, $x_3^{\mathrm{s}} = x_4^{\mathrm{s}} = 44$, $m = 2154,3$ kg and $K = 4633,5$ \$ and the minimum cost $x_1^{\mathrm{s}} = x_2^{\mathrm{s}} = 88$, $x_3^{\mathrm{s}} = x_4^{\mathrm{s}} = 35$, $m = 2212,3$ kg and $K = 4266,9$ \$) which means that the minimum mass truss is 8,6 % more expensive than the minimum cost truss. The average of difference is 2,4 % in range $1,3\,\mathrm{m} \leq x^{\mathrm{sh}} \leq 2,3\,\mathrm{m}$.

5.2.4 Cost and displacement minimization

This example deals with the simultaneous cost and deflection minimization using previous truss optimization problems. Criteria are clearly conflicting because the most economic and at the same time very light truss can not be also the stiffest. The design variables and FEM-models are the same as in problems (5.2) and (5.3) and also the constraints are almost the same except that there is no displacement constraint. The sizing optimization problem (5.2) is also solved using two independent size design variables ($x_1^{\mathrm{s}} = x_2^{\mathrm{s}}$ and $x_3^{\mathrm{s}} = \ldots = x_7^{\mathrm{s}}$). v_6 and v are the vertical displacements in the middle of truss in problems (5.2) and (5.3).

The mathematical forms of the discussed optimization problems are

$$\min \begin{bmatrix} K(\mathbf{x}) \\ -v_6(\mathbf{x}) \end{bmatrix}$$
$$g^{\mathrm{s}}(\mathbf{x}) \le 0$$
$$g^{\mathrm{b}}(\mathbf{x}) \le 0 \qquad (5.4\mathrm{a})$$
$$g^{\mathrm{j}}(\mathbf{x}) \le 0$$
$$\mathbf{x} = \begin{bmatrix} x_1^{\mathrm{s}} \cdots x_7^{\mathrm{s}} \end{bmatrix}^{\mathrm{T}} \in \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{n_{\mathrm{cs}}}\}$$

$$\min \begin{bmatrix} K(\mathbf{x}) \\ -v(\mathbf{x}) \end{bmatrix}$$
$$g^{\mathrm{s}}(\mathbf{x}) \le 0$$
$$g^{\mathrm{b}}(\mathbf{x}) \le 0 \qquad (5.4\mathrm{b})$$
$$g^{\mathrm{j}}(\mathbf{x}) \le 0$$
$$\mathbf{x} = \begin{bmatrix} x^{\mathrm{t}} \; x^{\mathrm{sh}} \; x_1^{\mathrm{s}} \cdots x_4^{\mathrm{s}} \end{bmatrix}^{\mathrm{T}} \in \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{n_{\mathrm{cs}}}\}$$

The solution of a multicriteria optimization problem is the set of Pareto-optima. Design variable vector **x*** is Pareto-optimal if there exists no feasible vector **x** which would improve some criterion without causing a simultaneous worsening in at least one criterion. Correspondingly, vector **x*** is weakly Pareto-optimal if there exists no feasible vector **x** which would improve all the criteria simultaneously.

The bi-criteria optimization problems (5.4a) and (5.4b) have been solved using the constraint method. One criterion is chosen as the minimized objective function and the other is converted into constraint. By varying the bound of constraint weakly Pareto-optimal solutions can be generated but not necessarily Pareto-optima. Some of the solutions have been calculated by minimizing the cost with varying the deflection upper limit $\bar{v}^{\max}$ and the rest by minimizing $-v_6$ or $-v$ with varying the cost upper limit $K^{\max}$ as it is presented for the sizing optimization problem (5.4a) in (5.5a) and (5.5b).

$$\min K(\mathbf{x})$$
$$-v_6(\mathbf{x}) \le \bar{v}^{\max}$$
$$g^{\mathrm{s}}(\mathbf{x}) \le 0$$
$$g^{\mathrm{b}}(\mathbf{x}) \le 0 \qquad (5.5\mathrm{a})$$
$$g^{\mathrm{j}}(\mathbf{x}) \le 0$$
$$\mathbf{x} = \begin{bmatrix} x_1^{\mathrm{s}} \cdots x_7^{\mathrm{s}} \end{bmatrix}^{\mathrm{T}} \in \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{n_{\mathrm{cs}}}\}$$

$$\min -v_6(\mathbf{x})$$
$$K(\mathbf{x}) \le K^{\max}$$
$$g^{\mathrm{s}}(\mathbf{x}) \le 0$$
$$g^{\mathrm{b}}(\mathbf{x}) \le 0 \qquad (5.5\mathrm{b})$$
$$g^{\mathrm{j}}(\mathbf{x}) \le 0$$
$$\mathbf{x} = \begin{bmatrix} x_1^{\mathrm{s}} \cdots x_7^{\mathrm{s}} \end{bmatrix}^{\mathrm{T}} \in \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{n_{\mathrm{cs}}}\}$$

Figure 5.19 represents the best found solutions for problems (5.4a) and (5.4b) in criterion space. Each single criterion problem has been solved 20 times using PSO with the same parameter values as previously and the best solution has been picked or results have been taken from previous study [29]. Table 5.6 represents the minimum cost trusses and Table 5.7 the minimum deflection trusses.

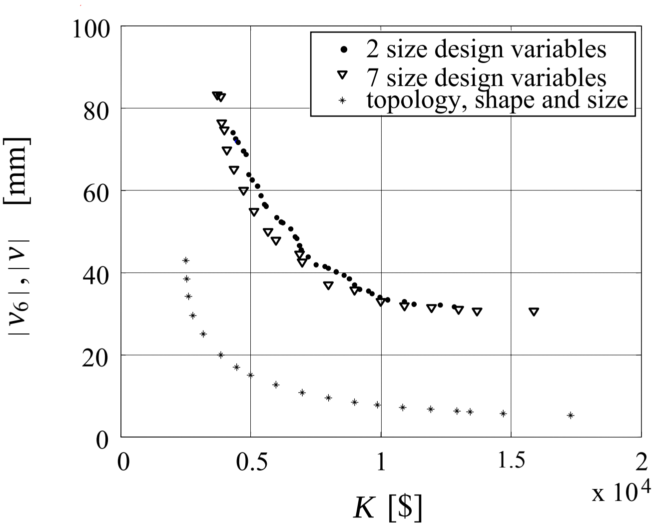


Figure 5.19. The best found solutions for the bi-criteria tubular plane truss sizing optimization problem (5.4a) (two or seven size design variables) and for the bi-criteria tubular plane truss topology, shape and sizing optimization problem (5.4b) in criterion space. In the case of two independent size design variables $x_1^s = x_2^s$ and $x_3^s = \ldots = x_7^s$.

Table 5.6. The best found trusses in the cost minimization: Sizing optimization a) two design variables and b) seven design variables. c) Topology, shape and sizing optimization.

	x^t	x^{sh}	x_1^s	x_2^s	x_3^s	x_4^s	x_5^s	x_6^s	x_7^s	K [$]	$-v_6$, $-v$ [mm]
a)	-	-	94	94	46	46	46	46	46	4273,8	74,2
b)	-	-	94	84	40	46	46	40	23	3727,8	83,1
c)	1	3,8	57	38	29	56	-	-	-	2500,9	42,9

Table 5.7. The best found trusses in the deflection minimization: Sizing optimization a) two design variables and b) seven design variables. c) Topology, shape and sizing optimization

	x^{t}	x^{sh}	x_1^{s}	x_2^{s}	x_3^{s}	x_4^{s}	x_5^{s}	x_6^{s}	x_7^{s}	K [$]	$-v_6$, $-v$ [mm]
a)	-	-	111	111	104	104	104	104	104	12797,8	31,7
b)	-	-	111	111	112	108	112	108	112	15893,1	30,6
c)	1	5,0	112	112	112	108	-	-	-	17294,7	5,1

Figure 5.19 and Tables 5.6 and 5.7 show that the shape design variable affects more the optimized truss than the size design variables. In cost minimization the increase of five size design variables improves the value of the objective function by $ 546 (13%) but the increase of one topology, one shape and two size design variables by $ 1773 (41%). Topology design variables are probably also significant although in this case the topology remains the same as in the sizing optimization. It can be concluded that if the number of design variables has to be limited it is useful to allow changes in the topology and the shape of the truss at the expense of the number of sizing design variables.

In the sizing optimization the best found trusses contain only SHS-profiles except a few very stiff in Fig. 5.19. RHS-profiles are more common in the best found trusses in the topology, shape and size optimization.

5.3 Tubular space truss

The last numerical test problem concerns the shape and sizing optimization of a 72 member space tubular truss presented in Fig. 5.20. The truss is built up from four 12 meters long N-trusses which are perpendicular to each other. The cost of the truss should be minimized so that the strength and the buckling requirements for all members and the strength requirements for all joints (except the four highest and the four lowest joints) are fulfilled, the horizontal displacements u_1 and u_2 are less than $\bar{u}^{\max} = 12\ \mathrm{m}/500 = 24$ mm ([63], on the top of a multi-storey building), the safety factor against global buckling is at least $\overline{n}_{\mathrm{gb}} = 3$ and none of the natural frequencies fall into forbidden interval of 1 … 10 Hz. In addition the height of truss has to be 12 meters.

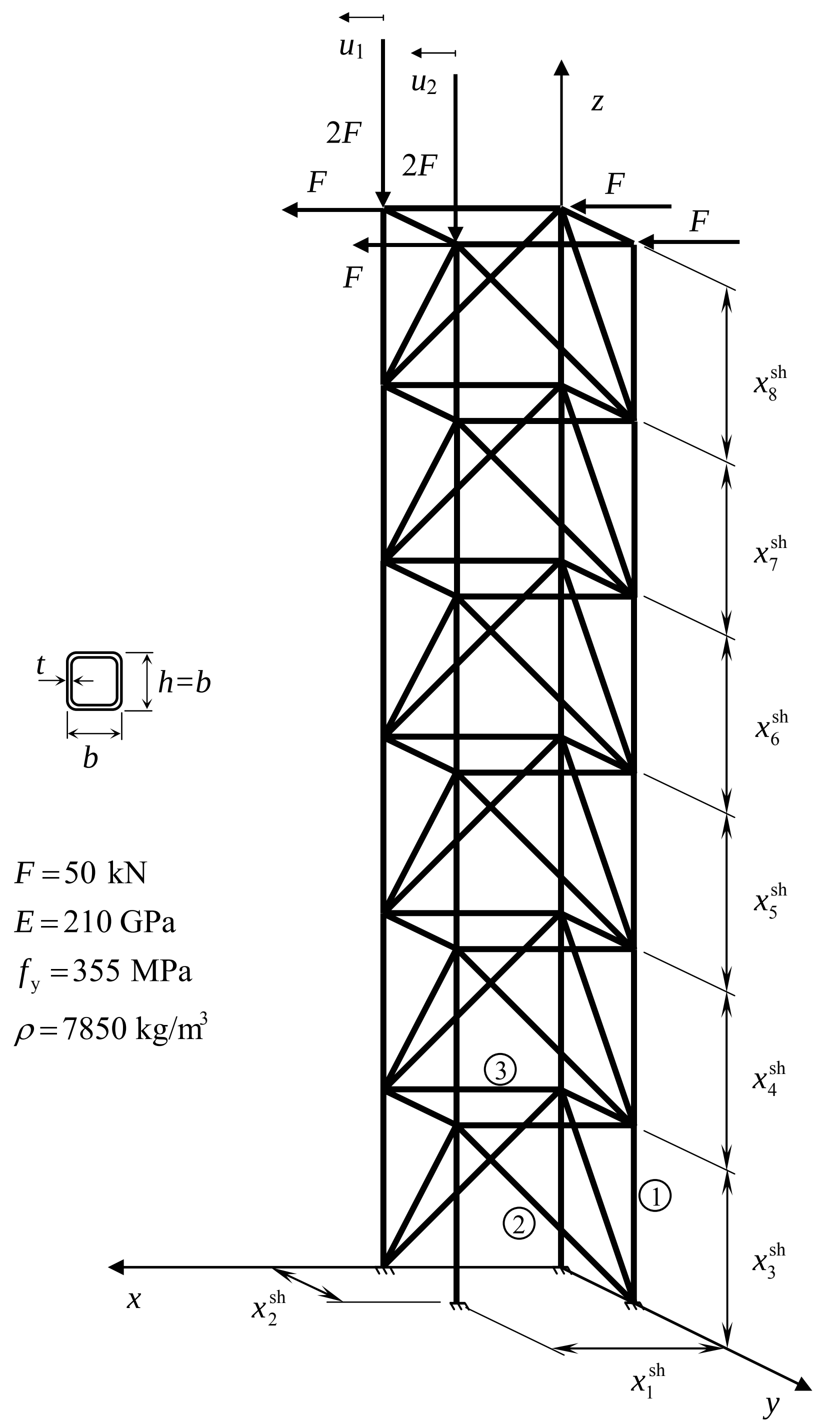


Figure 5.20. Six story 12 m high tubular space truss made of SHS-profiles.

The discrete shape design variables $x_i^{\mathrm{sh}} \in \{1{,}0\,, 1{,}1\,, \ldots,\; 5{,}0\}$ m ($i = 1,2,\ldots,8$) are connected to the width and the depth of the truss and the height of each floor (Fig. 5.20). The size design variables $x_i^{\mathrm{s}} \in \{1, 2, \ldots, 61\}$ ($i = 1,2,3$) are connected to columns (x_1^{s}), diagonals (x_2^{s}) and horizontal members (x_3^{s}). The set of available profiles includes 61 SHS profiles given in Appendix D. Columns are welded from three pieces and the eccentricity is zero in all joints. There is only one load case and the FEM-model includes six elements in each column and one element per each diagonal and horizontal member. The weight of the truss is ignored as a load and support points are pinned joints.

The material and manufacturing cost factors are the same as in previous tubular plane truss examples ($k_{\mathrm{m}} = 0{,}6$ \$/kg and $k_{\mathrm{f}} = 0{,}9$ \$/min) and the values of the cost function parameters are given in Table 5.3.

In the mathematical form the tubular space truss optimization problem is

$$
\begin{aligned}
&\min K(\mathbf{x}) && \\
&g^{\mathrm{s}}(\mathbf{x}) \le 0 && \text{(strength)} \\
&g^{\mathrm{b}}(\mathbf{x}) \le 0 && \text{(buckling)} \\
&g^{\mathrm{j}}(\mathbf{x}) \le 0 && \text{(joints)} \\
&g^{\mathrm{kkgj}}(\mathbf{x}) \le 0 && \text{(kk gap joints)} \\
&g_1^{\mathrm{u}}(\mathbf{x}) = \frac{u_1(\mathbf{x})}{\bar{u}^{\max}} - 1 \le 0 && \text{(displacement)} \\
&g_2^{\mathrm{u}}(\mathbf{x}) = \frac{u_2(\mathbf{x})}{\bar{u}^{\max}} - 1 \le 0 && \text{(displacement)} \\
&g^{\mathrm{bg}}(\mathbf{x}) \le 0 && \text{(global buckling)} \\
&g^{\mathrm{f}}(\mathbf{x}) \le 0 && \text{(frequency)} \\
&\max\left\{1 - \frac{H(\mathbf{x})}{\bar{H}}, \frac{H(\mathbf{x})}{\bar{H}} - 1\right\} \le 0 && \\
&\mathbf{x} = \left[\, x_1^{\mathrm{sh}} \cdots x_8^{\mathrm{sh}}\; x_1^{\mathrm{s}}\; x_2^{\mathrm{s}}\; x_3^{\mathrm{s}} \,\right]^{\mathrm{T}} \in \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{n_{\mathrm{cs}}}\} &&
\end{aligned}
\tag{5.6}
$$

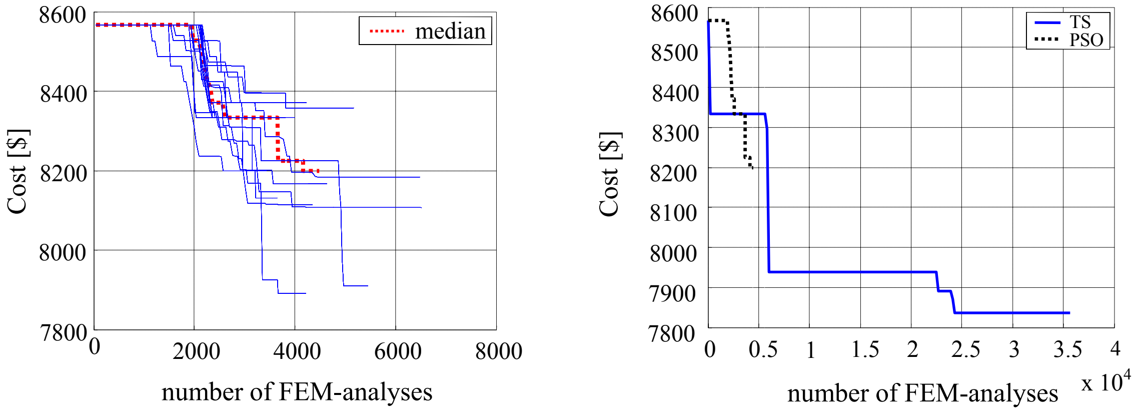
The height of the truss is $H(\mathbf{x}) = x_1^{\text{sh}} + x_2^{\text{sh}} + x_3^{\text{sh}} + x_4^{\text{sh}} + x_5^{\text{sh}} + x_6^{\text{sh}}$ and $\overline{H} = 12$ m. The number of feasible or unfeasible candidate solutions is $n_{\text{cs}} = 41^8 \cdot 61^3 \approx 1,81 \cdot 10^{18}$.

The problem (5.6) is solved once using TS and 100 times using PSO. These two algorithms have been selected because they are the most promising in the light of previous examples. In TS run the parameter values are the same as in the ten-bar truss problem (5.1) except that the depth of discrete neighborhood is $d_{\text{n}} = 8$. In PSO runs the parameters are the same as in the tubular plane truss sizing optimization problem (5.2). Duration of one FEM-analysis was approximately 0,7 – 0,8 seconds and the total time for TS run was 7,5 hours and for one PSO run 50 – 70 minutes. The initial guess is $x_1^{\text{sh}} = x_2^{\text{sh}} = 3,4$ m, $x_3^{\text{sh}} =, \ldots, = x_8^{\text{sh}} = 2$ m, $x_1^{\text{s}} = 48$ and $x_2^{\text{s}} = x_3^{\text{s}} = 18$ ($K = 8567,6$ \$ and $m = 4495,5$ kg). It corresponds to the minimum cost truss in such a case that the floor height is constant two meters and there is only one independent shape design variable ($x_1^{\text{sh}} = x_2^{\text{sh}}$) and two independent size design variables (x_1^{s} and $x_2^{\text{s}} = x_3^{\text{s}}$). This structure can be found using enumeration. The initial swarm is selected randomly so that it includes 15 feasible individuals which are picked from the same set of 100 feasible solutions. The initial guess is forced to be one of the 15 feasible individuals in the initial swarm. The optimization results are presented in Table 5.8 and Fig. 5.21.

Table 5.8. The best found final results in the problem (5.6).

	x_1^{sh}	x_2^{sh}	x_3^{sh}	x_4^{sh}	x_5^{sh}	x_6^{sh}	x_7^{sh}	x_8^{sh}	x_1^{s}	x_2^{s}	x_3^{s}	cost [\$]	number of runs
TS	3,9	2,0	2,0	2,0	2,0	2,0	2,0	2,0	42	23	17	7837,0	
PSO	4,0	2,1	2,0	2,0	2,0	2,0	2,0	2,0	43	21	18	7890,5	7

	g^{s}	g^{b}	g^{j}	g^{kkgj}	u_1 [mm]	u_2 [mm]	λ_{cr}	f_1 [Hz]	m [kg]
TS	-0,729	-0,595	-0,028	-0,930	23,94	23,95	68,10	10,04	3933,0
PSO	-0,737	-0.585	0,000	-0.933	23,97	23,95	43,08	10,20	3964,6



a) b)

Figure 5.21. a) The decrease of cost in individual PSO runs (total 100 runs) and the median of cost in the problem (5.6). b) The mutual efficiency of TS and PSO.

According to Table 5.8 TS found the better final solution than any of the 100 PSO runs but on the other hand the TS run was also much longer than the PSO runs (Fig. 5.21). However the longer duration of the single TS run is not that significant because in any case there have to be several runs if stochastic PSO is used. In addition TS improves the value of the objective function clearly more efficiently than PSO in the beginning of optimization. PSO needs on average 2000 FEM-analysis before the objective function improves. Based on these observations TS can be considered to work better than PSO in the tubular space truss shape and sizing optimization problem (5.6).

In the best found TS and PSO solutions the value of x_2^{sh} is smaller than x_1^{sh} which sounds natural if the directions of horizontal loads are considered. The heights of each floor x_3^{sh}, ..., x_8^{sh} remain the same as in the initial guess which was also forced to be one of the individuals in the initial swarm. This suggests that the shape design variables x_3^{sh}, ..., x_8^{sh} are unnecessary or at least they do not have a major role in optimization. Based on Table 5.8 the joint constraint, the displacement constraints and the frequency constraint are closest to active constraints. The global buckling constraint is far from active and the strength and the buckling strength constraints hold clearly.

6. Conclusions

The first goal of this study was to formulate an optimization problem for tubular trusses in such a way that it is not just academic but also useful in real life applications. This means that the topology, shape and sizing optimization of plane or space truss is considered, the manufacturing cost is taken into account, design constraints are based on the steel design rules and the selection of RHS and SHS profiles is not limited to few preselected sizes. The second goal was to solve the formulated problem efficiently using four heuristic multipurpose optimization algorithms simulated annealing, tabu search, genetic algorithm and particle swarm optimization. The efficiency of algorithms has been compared empirically in several example problems, the conflict of mass and cost has been studied and multicriteria cost and deflection minimization has been presented.

Numerical example problems show that heuristic algorithms are usable tools in tubular truss optimization problems. Discrete design variables and the demands of steel design rules which lead to rather awkward constraints are not obstacles for these algorithms. On the other hand heuristic algorithms from suffer high computational cost and uncertainty because they demand thousand of FEM-analysis per each run and there has to be several of these runs in the case of stochastic algorithm. In addition heuristic algorithms do not always work, they need tuning and offer no guarantee concerning the quality of the final solution.

None of the discussed heuristic algorithms was the best one in all the studied example problems. In some cases PSO was superior compared to other methods (the ten-bar truss (5.1) and the plane truss topology, shape and sizing optimization problem (5.3)) but on the other hand it works clearly less efficiently in slightly different problems (the plane truss sizing optimization problem (5.2) and the space truss optimization problem (5.6)). TS improves rapidly the value of the objective function during early iteration rounds at least if the initial guess is good. Usually a good initial guess is known based on experience or it can be found using a few design variables, enumeration and possibly the limited set of available profiles. TS is also a deterministic algorithm and thus only one optimization run is needed. In the studied example problems the efficiencies of SA and GA seem to be weaker than the efficiencies of PSO and TS. The implementation of PSO can be considered the simplest among the four heuristic algorithms and it can be used directly also in a mixed-integer problem.

As assumed the simultaneous topology, shape and sizing optimization of a tubular truss leads to better final solution than the pure sizing optimization. Generally speaking the increase of the number of the sizing design variables does not improve the optimum solution as much as the increase of the number of other design variables. The mass minimization and the cost minimization often seem to lead to the same final solution. If the minimum mass truss and the minimum cost truss were not the same the average cost difference was 2,4%.

As a conclusion it can be said that it is worthwhile to include the topology and the size design variables in a tubular truss optimization problem. In the formulated problem the number of available topologies is rather small, which emphasizes the importance of a sensitive topology selection. The mass can well be used as the minimized criterion instead of the cost if the cost function is not known in detail. Beside mass/cost it is possible to consider also other conflicting criteria in a multicriteria problem at the same time. The optimization problem can be solved using both PSO and TS. TS is a better choice if only a few hundred FEM-analysis can be done. Designer has to make time for unavoidable test runs and make several actual PSO runs.

In future studies the tubular truss optimization problem should be improved onwards by e.g. taking the eccentricities of joints and the steel grade as new design variables. The selection of criteria could be increased and new constraints added due to fatigue loading and fire safety. Different cost functions should be compared and also CHS sections studied, as well as other connections than K- and N-joints and tubular members in cross-section classes 3 and 4. The comparison of the mutual efficiency should be continued and also other heuristic algorithms than SA, TS, GA and PSO should be considered. The comparison can be developed by giving an exact definition for the good performance of a heuristic algorithm. It should also pay attention to parallel computing. In multicriteria problems the use of the special versions of heuristic algorithms should be studied instead of the constraint method. Constraint method is after all better suitable for cases where the designer's preference of criteria is already somehow fixed.

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Appendix A: 12 degrees of freedom space beam element

The stiffness matrix **k.**

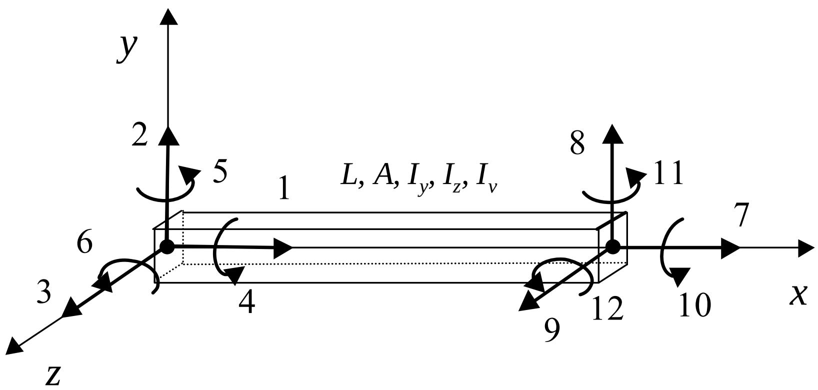
$$
\mathbf{k} = \begin{bmatrix}
\frac{EA}{L} & & & & & & & & & & & \\
0 & \frac{12EI_z}{L^3} & & & & & & & & & & \\
0 & 0 & \frac{12EI_y}{L^3} & & & & & & \text{symm.} & & & \\
0 & 0 & 0 & \frac{GI_v}{L} & & & & & & & & \\
0 & 0 & -\frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & & & & & & & \\
0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L} & & & & & & \\
-\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{L} & & & & & \\
0 & -\frac{12EI_z}{L^3} & 0 & 0 & 0 & -\frac{6EI_z}{L^2} & 0 & \frac{12EI_z}{L^3} & & & & \\
0 & 0 & -\frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 & 0 & 0 & \frac{12EI_y}{L^3} & & & \\
0 & 0 & 0 & -\frac{GI_v}{L} & 0 & 0 & 0 & 0 & 0 & \frac{GI_v}{L} & & \\
0 & 0 & -\frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & \\
0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L} & 0 & -\frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L}
\end{bmatrix}
$$

The consistent mass matrix **m**.

$$
\mathbf{m} = \rho A L
\begin{bmatrix}
\frac{1}{3} & & & & & & & & & & & \\
0 & \frac{13}{35}+\frac{6I_z}{5AL^2} & & & & & & & & & & \\
0 & 0 & \frac{13}{35}+\frac{6I_y}{5AL^2} & & & & & & \text{symm.} & & & \\
0 & 0 & 0 & \frac{I_y+I_z}{3A} & & & & & & & & \\
0 & 0 & -\frac{11L}{210}-\frac{I_y}{10AL} & 0 & \frac{L^2}{105}+\frac{2I_y}{15A} & & & & & & & \\
0 & \frac{11L}{210}+\frac{I_z}{10AL} & 0 & 0 & 0 & \frac{L^2}{105}+\frac{2I_z}{15A} & & & & & & \\
\frac{1}{6} & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & & & & & \\
0 & \frac{9}{70}-\frac{6I_z}{5AL^2} & 0 & 0 & 0 & \frac{13L}{420}-\frac{I_z}{10AL} & 0 & \frac{13}{35}+\frac{6I_z}{5AL^2} & & & & \\
0 & 0 & \frac{9}{70}-\frac{6I_y}{5AL^2} & 0 & -\frac{13L}{420}+\frac{I_y}{10AL} & 0 & 0 & 0 & \frac{13}{35}+\frac{6I_y}{5AL^2} & & & \\
0 & 0 & 0 & \frac{I_y+I_z}{6A} & 0 & 0 & 0 & 0 & 0 & \frac{I_y+I_z}{3A} & & \\
0 & 0 & \frac{13L}{420}-\frac{I_y}{10AL} & 0 & -\frac{L^2}{140}-\frac{I_y}{30A} & 0 & 0 & 0 & \frac{11L}{210}+\frac{I_y}{10AL} & 0 & \frac{L^2}{105}+\frac{2I_y}{15A} & \\
0 & -\frac{13L}{420}+\frac{I_z}{10AL} & 0 & 0 & 0 & -\frac{L^2}{140}-\frac{I_z}{30A} & 0 & -\frac{11L}{210}-\frac{I_z}{10AL} & 0 & 0 & 0 & \frac{L^2}{105}+\frac{2I_z}{15A}
\end{bmatrix}
$$

The geometric stiffness matrix $\mathbf{k}_g$.

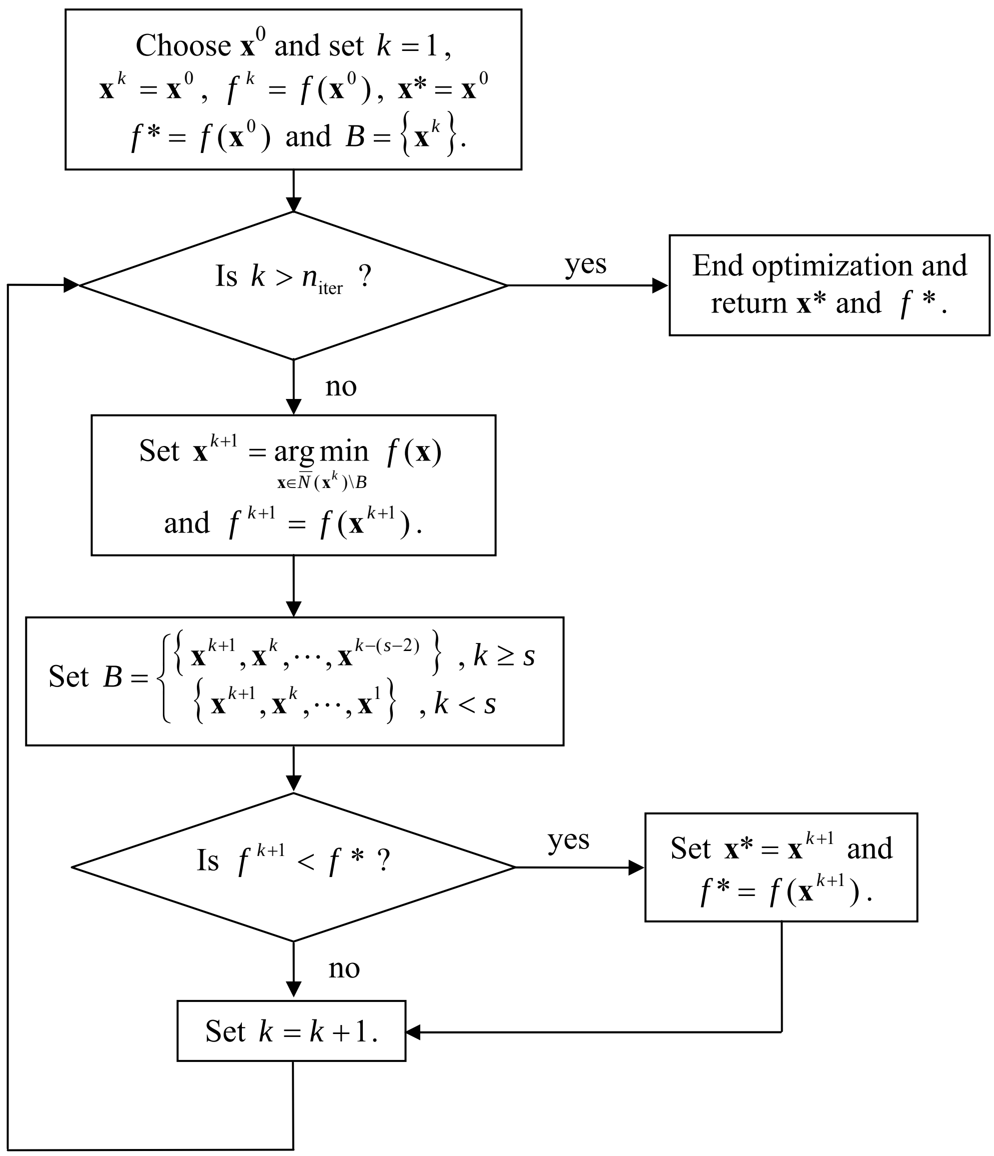
$$
\mathbf{k}_g = N \begin{bmatrix}
0 & & & & & & & & & & & \\
0 & \frac{6}{5L} & & & & & & & & & & \\
0 & 0 & \frac{6}{5L} & & & & & & & \text{symm.} & & \\
0 & 0 & 0 & \frac{I_y + I_z}{AL} & & & & & & & & \\
0 & 0 & -\frac{1}{10} & 0 & \frac{2L}{15} & & & & & & & \\
0 & \frac{1}{10} & 0 & 0 & 0 & \frac{2L}{15} & & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & \\
0 & -\frac{6}{5L} & 0 & 0 & 0 & -\frac{1}{10} & 0 & \frac{6}{5L} & & & & \\
0 & 0 & -\frac{6}{5L} & 0 & \frac{1}{10} & 0 & 0 & 0 & \frac{6}{5L} & & & \\
0 & 0 & 0 & -\frac{I_y + I_z}{AL} & 0 & 0 & 0 & 0 & 0 & \frac{I_y + I_z}{AL} & & \\
0 & 0 & -\frac{1}{10} & 0 & -\frac{L}{30} & 0 & 0 & 0 & \frac{1}{10} & 0 & \frac{2L}{15} & \\
0 & \frac{1}{10} & 0 & 0 & 0 & -\frac{L}{30} & 0 & -\frac{1}{10} & 0 & 0 & 0 & \frac{2L}{15}
\end{bmatrix}
$$



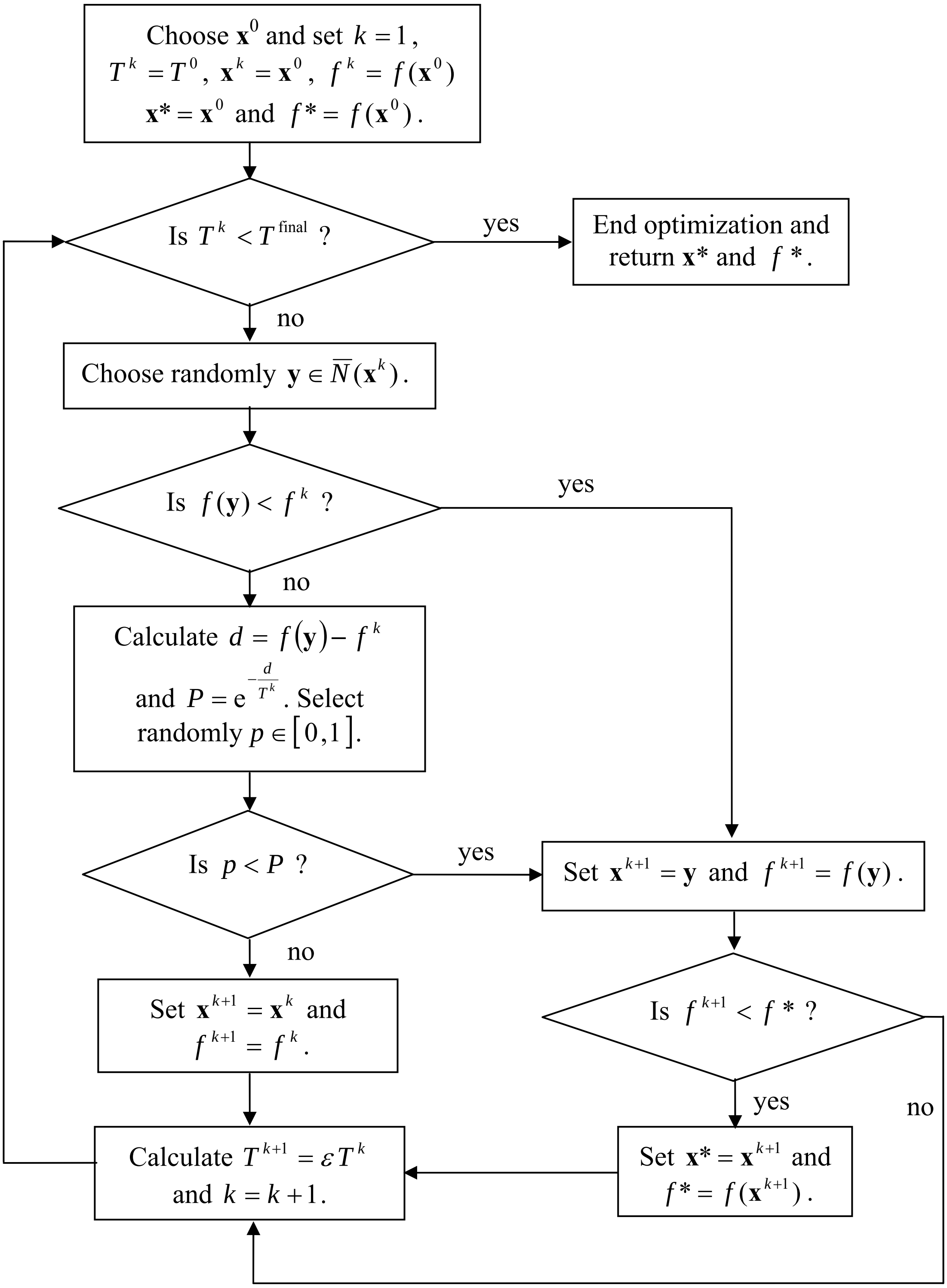
Appendix B: Flowcharts for TS, SA, GA and PSO

The flowcharts of heuristic optimization algorithms. If problem is constrained f means penalized objective function.

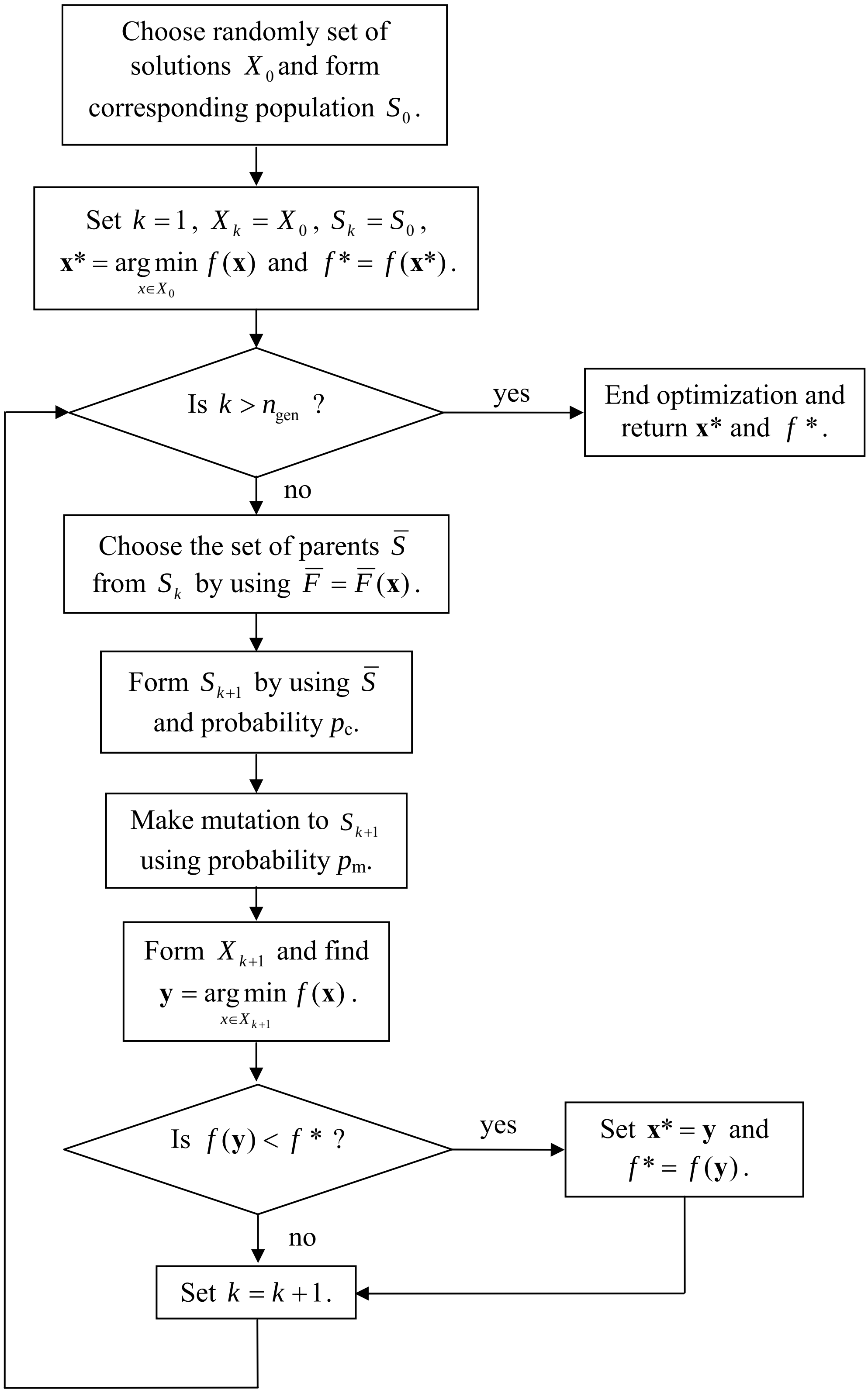
Tabu search. The static tabu list B includes previous solutions and s is the length of the tabu list.



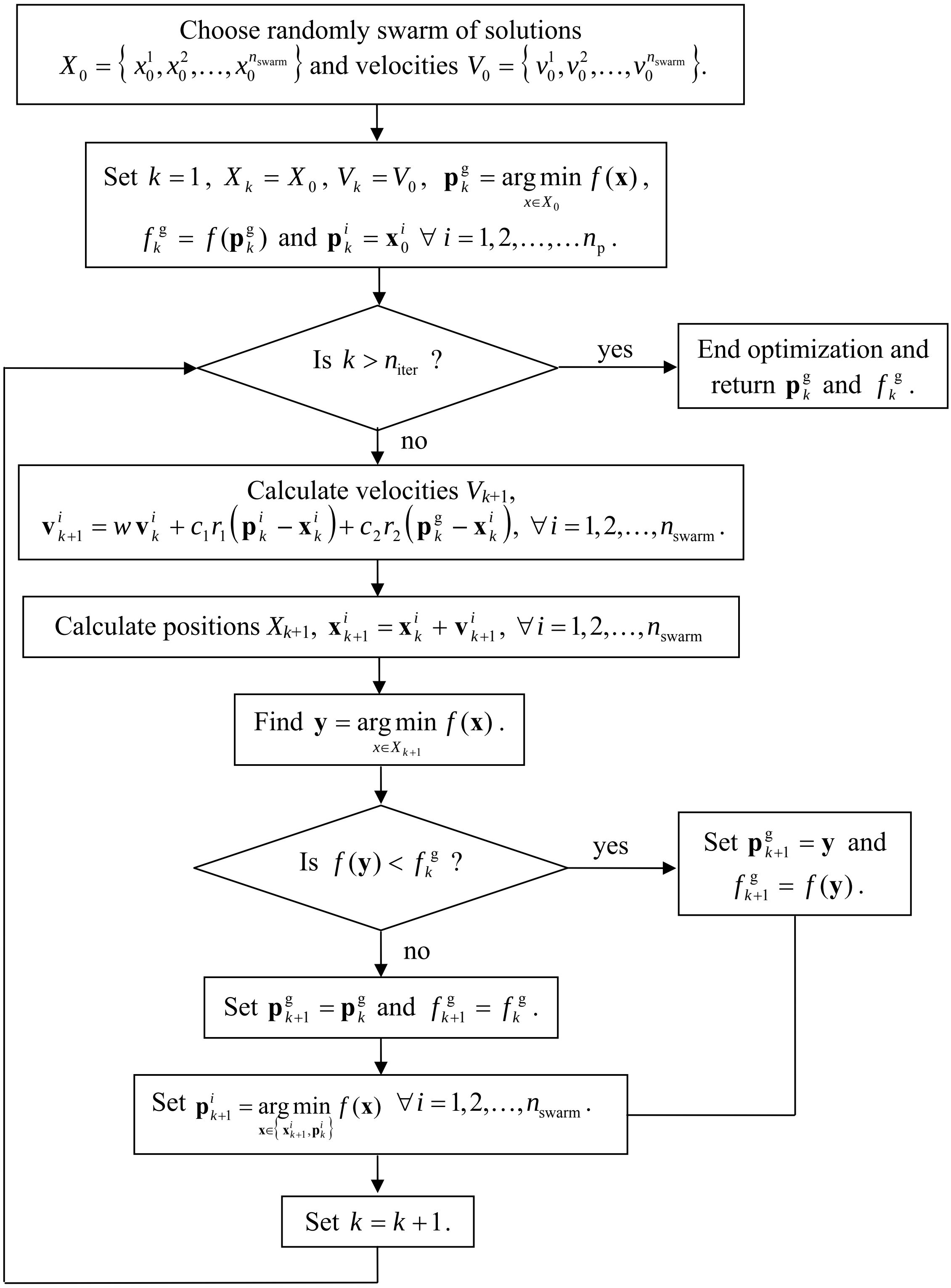
Simulated annealing. The decrease of temperature takes place every iteration round ($n_T = 1$).



Genetic algorithm.



Particle swarm optimization.



Appendix C: Analytical derivatives for displacement and stress

The derivative of displacement vec

tor **u** with respect to cross-section area A_i can be calculated differentiating equation (2.2) ($\mathbf{Ku} = \mathbf{f}$)

$$\frac{\mathrm{d}\mathbf{K}}{\mathrm{d}A_i}\mathbf{u} + \mathbf{K}\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}A_i} = \frac{\mathrm{d}\mathbf{f}}{\mathrm{d}A_i} \quad . \tag{C.1}$$

From (C.1) it can be solved

$$\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}A_i} = \mathbf{K}^{-1}\left(\frac{\mathrm{d}\mathbf{f}}{\mathrm{d}A_i} - \frac{\mathrm{d}\mathbf{K}}{\mathrm{d}A_i}\mathbf{u}\right) \quad . \tag{C.2}$$

The derivative of the global stiffness matrix **K** with respect to A_i can be formed using the derivatives of element stiffness matrixes $\mathrm{d}\mathbf{k}/\mathrm{d}A_i$. $\mathrm{d}\mathbf{f}/\mathrm{d}A_i$ is zero unless loads depend cross-section areas A_i.

In a truss the stresses of bars $\boldsymbol{\sigma}$ can be calculated multiplying the global node displacement **u** with a constant matrix **S**

$$\boldsymbol{\sigma} = \mathbf{Su} \quad . \tag{C.3}$$

Thus the derivative of $\boldsymbol{\sigma}$ with respect to A_i is

$$\frac{\mathrm{d}\boldsymbol{\sigma}}{\mathrm{d}A_i} = \mathbf{S}\mathbf{K}^{-1}\left(\frac{\mathrm{d}\mathbf{f}}{\mathrm{d}A_i} - \frac{\mathrm{d}\mathbf{K}}{\mathrm{d}A_i}\mathbf{u}\right) \quad . \tag{C.4}$$

Appendix D: RHS and SHS profiles

The set of 51 RHS-profiles and 61 SHS-profiles in the ascending order according to the cross-section area [56].

SHS RHS	*h* [mm]	*b* [mm]	*t* [mm]	SHS	SHS RHS	*h* [mm]	*b* [mm]	*t* [mm]	SHS
1.	40	40	2,5	1.	57.	150	150	5	30.
2.	60	40	2,5		58.	150	100	6,3	
3.	50	50	2,5	2.	59.	140	140	5,6	31.
4.	50	50	3	3.	60.	160	160	5	32.
5.	80	40	2,5		61.	180	100	6	
6.	60	60	2,5	4.	62.	200	80	6	
7.	70	50	2,5		63.	140	140	6	33.
8.	80	60	2,5		64.	160	90	7,1	
9.	70	70	2,5	5.	65.	150	150	6	34.
10.	60	60	3	6.	66.	200	100	6	
11.	70	50	3		67.	120	120	8	35.
12.	80	80	2,5	7.	68.	150	100	8	
13.	80	60	3		69.	180	100	7,1	
14.	90	50	3		70.	140	140	7,1	36.
15.	70	70	3	8.	71.	200	120	6	
16.	100	50	3		72.	160	160	6	37.
17.	60	60	4	9.	73.	180	100	8	
18.	100	60	3		74.	140	140	8	38.
19.	80	80	3	10.	75.	180	180	6	39.
20.	80	60	4		76.	150	150	8	40.
21.	70	70	4	11.	77.	200	100	8	
22.	100	80	3		78.	140	140	8,8	41.
23.	90	90	3	12.	79.	200	200	6	42.
24.	100	100	3	13.	80.	200	120	8	
25.	100	60	4		81.	160	160	8	43.
26.	80	80	4	14.	82.	140	140	10	44.
27.	100	80	4		83.	220	120	8	
28.	120	60	4		84.	150	150	10	45.
29.	90	90	4	15.	85.	180	180	8	46.
30.	80	80	5	16.	86.	200	120	10	
31.	120	80	4		87.	160	160	10	47.
32.	100	100	4	17.	88.	250	150	8	
33.	100	80	5		89.	260	140	8	
34.	90	90	5	18.	90.	200	200	8	48.
35.	110	110	4	19.	91.	220	120	10	

Table continues on next page.

Table continues from previous page.

36.	120	120	4	20.	92.	180	180	10	49.
37.	120	80	5		93.	260	180	8	
38.	100	100	5	21.	94.	220	220	8	50.
39.	100	80	6		95.	250	150	10	
40.	90	90	6	22.	96.	260	140	10	
41.	140	70	5		97.	200	200	10	51.
42.	140	80	5		98.	250	250	8	52.
43.	110	110	5	23.	99.	180	180	12,5	53.
44.	120	80	6		100.	260	180	10	
45.	160	80	5		101.	220	220	10	54.
46.	120	120	5	24.	102.	260	260	8,8	55.
47.	150	100	5		103.	250	150	12,5	
48.	110	110	6	25.	104.	200	200	12,5	56.
49.	140	80	6		105.	300	200	10	
50.	140	80	6,3		106.	250	250	10	57.
51.	120	120	5,6	26.	107.	260	260	11	58.
52.	140	140	5	27.	108.	300	200	12,5	
53.	160	80	6		109.	250	250	12,5	59.
54.	120	120	6	28.	110.	300	300	10	60.
55.	100	100	8	29.	111.	300	300	12,5	61.
56.	150	100	6		112.	400	200	12,5	
SHS RHS	*h* [mm]	*b* [mm]	*t* [mm]	SHS	SHS RHS	*h* [mm]	*b* [mm]	*t* [mm]	SHS

Tampereen teknillinen yliopisto
PL 527
33101 Tampere

Tampere University of Technology
P.O. Box 527
FIN-33101 Tampere, Finland