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ASYMMETRIES: AN EMPIRICAL APPLICATION

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Implications for Aggregate Inflation of Sectoral Asymmetries: an empirical application

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Abstract

Following the theoretical identification scheme that relies on the two-sector DSGE macro model derived in a study by Koskinen and Vilmunen (2017), the focus of this study is to empirically estimate and identify the possibly divergent sector specific parameters within an economy. We analyse and compare two different sectors of the Finnish economy, manufacturing industry and building industry, each in turn to the rest of the economy during 2000:Q1- 2015Q2. It is hence assumed, that the parameters of interest within a sector reflect the divergent preferences of economic agents to the goods produced at different sectors. The relative price movements and adjustment asymmetries stemming from these kind of divergences has a central role in allocational efficiency and welfare of the economy. Then this diversity has important implications to any economic policy practised as these divergent preferences could give rise to asymmetric reaction to any shock that hits the economy. As a result, there is evidence that the economy could be characterised to be compounded of divergent groups of goods. These groups, then, has group specific parameters reflecting different behaviour of the agents and their preferences to the goods produced.

JEL Code: D52, C11, E12

1. Introduction

From the stabilization policy point of view, it is almost a necessity to understand the different characters of the particular sectors within the economy. These characters could be understood to express the nature of the preferences of the representative agent in the economy. As the preferences to the goods produced as well as market mechanism in general could diverge between the sectors, there is potential for sectoral adjustment asymmetries in an economy after hit by a shock. This is due to a fact that the adjustment to any shock that hits the economy reflect, in turn, the structural factors of that economy. In addition, as such, the economy could been characterised to be compounded of separate sectors, each of which having its own structural features. In an economy with nominal price stickiness where the frequency of individual price changes as well as price elasticities (and then the mark-ups) are sector specific, inflation distorts relative prices. This cause's economic inefficiency in terms of a loss of consumer welfare as output fluctuates around its natural level. Then, as most of the economists believe, the policy makers should respond to these deviations by actively dampening them. However, the response of the economy (output/inflation) to e.g. monetary policy shock reflect to some extent the structural factors that characterize the economy. These structural features could diverge between particular sectors in that economy, and hence the response is sector specific.

This study, then, estimates those sector specific parameters that are crucial for inflation dynamics in the economy studied. This, in turn, enables us to analyse the aggregate as well as sector specific dynamics of the economy as it is hit by various structural shocks. The main interest is on the elasticity of substitution (which relates to the magnitude of a relative cost-push shock) and autoregressive coefficient (which determines the duration of a shock) parameters. Shocks that are for our interest are a general cost-push and an interest rate shocks. These shocks are seen stemming from the preferences of the representative household (agent) as the price setting behaviour of the agent determines the magnitude of these shocks, and moreover, the price setting is subject to the elasticity of substitution parameter.

Although the idea of a multi- or two-sector model for aggregate macro modelling is a familiar one from several earlier macroeconomic studies, see e.g. Woodford (2003) and Tille (2001) for examples of the approach, here the modelling framework is extended, allowing for the underlying preference parameters to diverge between the sectors studied. Earlier studies of the inflation persistence and dynamics have concentrated to study the role of the price rigidity, allowing it to differ across the sectors studied, see e.g. Bouakez, Cardia and Ruge-Murcia (2014). Hence, this study will offer a further and deeper understanding of how the disclosed micro-level heterogeneity should affect macro-level analysis.

During the period under study (2000:1-2015:2) the overall growth of the Finnish economy has been modest. The most important aggregate shock that hit the economy, and caused a recession, was a global financial crises that took place from 2007 onward. In practise the per worker growth rate of the output e.g. for building industry was even negative for some sub periods, and the total output of manufacturing industry declined dramatically from 2008 onward. In the same respect the inflationary pressure was modest and the 3-month money market interest rate was even negative at the end of the period under investigation.

The rest of this study is organized as follows. Chapter 2 presents the underlying model used in the estimation procedure. The modelling framework in this study is an extended version of a dynamic stochastic general equilibrium (DSGE) model with two sectors, nominal rigidities and imperfect competition presented in Woodford (2003) and extended by Koskinen and Vilmunen (2017). More specifically, we allow for the price elasticities to differ between the two sectors. This feature is important and well in line with the micro-level evidence on individual as well as sectoral prices. Moreover, we allow for external habit formation (Campbell and Cochrane, 1999). Chapter 3 present the data used and the estimation procedure that is based on Bayesian methodology. Chapter 4 present the empirical results and finally chapter 5 concludes. Model estimations indicate that we can distinguish those sector specific parameters that are crucial for policy analyse.

2. A Two Sector Model

As in the previous study of Koskinen and Vilmunen (2017), the economy is composed of two sectors within which the goods are imperfectly substitutable. Hence, there is imperfect competition in the relevant markets. A representative household in this economy derives utility from a consumption bundle that is a CES aggregate of the sector specific consumption indices. These sector specific consumption indices are CES aggregates over a continuum of individual goods. Our representative household also works, thus generating disutility in the usual manner. The (flow) budget constraint determines the feasible choices for our representative household: on top of allocating income on consumption, the household can invest in one period bonds, which generates interest income. By working, the household earn wage income.

We thus assume that the representative household seeks to maximise the following intertemporal utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t,$$
(1)

where β is the discount factor and where

$$U_t = \varphi_t^B \left(\frac{1}{1-\sigma} (C_t - h_t C_{t-1})^{1-\sigma} - \frac{1}{1+\psi} (L_t)^{1+\psi} \right)$$
(2)

σ is the inverse of the inter-temporal elasticity of substitution in consumption and ψ the inverse of the Frisch elasticity. C_t is now an index of the household's consumption of the goods that are supplied, while L_t is the labor supply. Eq (2) contains also a general preference shocks φ_t^B. The external habit formation is captured by the term h.

Households maximise their objective function (1) subject to the (flow) budget constraint:

$$P_t B_t + P_t C_t = (P_t W_t L_t) + R_{t-1}^b B_{t-1}, \tag{3}$$

where P_t is the price level and B_t denotes bonds.

Total nominal income consists of two components: labour income ($P_t W_t L_t$) and the gross return on the bonds ($R_t^b B_t$). As the capital stock is assumed to be fixed in the considerations we do not include it here. This is because we will focus on the dynamic effects of cost push and interest rate shocks at the business cycle frequency. The underlying assumption here is that variations in the capital stock are not the main driver for business cycles.

Consumption behavior

The Euler condition for the optimal intertemporal allocation of consumption is derived from the maximization problem of the objective function (2) subject to budget constraint (3) with respect to consumption and (nominal) return on bonds R_t^b This yields

$$E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t^b P_t}{P_{t+1}} \right] = 1 \tag{4}$$

where $\lambda_t = \varphi_t^B (C_t - hC_{t-1})^{-\sigma}$ is the usual consumption Euler equation describing the marginal utility of consumption.

This leads to the optimal consumption dynamics of the log-linear form:

$$\hat{c}_t = \frac{h}{1+h} \hat{c}_{t-1} + \frac{1}{1+h} E_t \hat{c}_{t+1} - \frac{1-h}{\sigma(1+h)} [\hat{r}_t - E_t \hat{\pi}_{t+1}] + \hat{\varphi}_t^B. \tag{5}$$

The hatted variables represent log deviations from the steady state.

2.1 Deriving a model for sectoral asymmetries

The aggregate consumption index C_t is of the CES-form and consists of two sub-indexes

for the commodity groups n_1 and n_2 as described earlier in Koskinen and Vilmunen (2017);

$$C_t = \left[(n_{1t} \varphi_t^{n1})^{\frac{1}{\eta}} C_{1t}^{\frac{\eta-1}{\eta}} + (n_{2t} \varphi_t^{n2})^{\frac{1}{\eta}} C_{2t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{6}$$

where $\varphi_{j,t}$ is a shock to the relative weight of the commodity group in a consumption basket and η is the elasticity of substitution between the groups and the sectoral consumption index aggregates a continuum of sector-specific goods

$$C_{jt} = \left[n^{\frac{-1}{\theta_j}} \int_0^n c_t(i)^{\frac{\theta_j-1}{\theta_j}} di \right]^{\frac{\theta_j}{\theta_j-1}}, j=1, 2 \quad N_1 = [0, n_1], N_2 =]n_1, 1] \tag{7}$$

Here θ_j is the elasticity of substitution between sector j goods defining the own price elasticity of the demand for these goods. We allow the two θ:s to differ.

Sectoral price indices, which defines the minimum cost of buying a unit of the sector j good, satisfy

$$P_{j,t} = \left[n_{jt}^{\frac{-1}{\theta_j}} \int_{N_j} p_{j,t}(i)^{1-\theta_j} di \right]^{\frac{1}{1-\theta_j}}, j = 1,2 \tag{8}$$

whereas the aggregate price index corresponding to the aggregator in (6) is given by

$$P_t = \left[n_{1t}\varphi_t^{n1} P_{1,t}^{1-\eta} + n_{2t}\varphi_t^{n2} P_{2,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{9}$$

Optimal allocation for different goods in sector j = 1, 2 can be derived from the minimization problem

$$\min \int p_{j,t}(i)c_{j,t}(i)di \text{ s.t. } \left[n_j^{\frac{-1}{\theta_j}} \int_{N_j} c_{j,t}(i)^{\frac{\theta_j-1}{\theta_j}} di \right]^{\frac{\theta_j}{\theta_j-1}} \geq C_{jt} \text{ and its first order conditions.}$$

Demand for different brands c(i) within a group j is then

$$c_{jt}(i) = n_{jt}^{-1} \left[\frac{p_{jt}(i)}{P_{jt}} \right]^{-\theta_j} C_{jt}. \tag{10}$$

The sectoral market demand functions in equilibrium are then

$$C_{jt} = n_{jt}\varphi_t^{nj} \left(\frac{P_{jt}}{P_t} \right)^{-\eta} C_t. \tag{11}$$

As in the previous study the aggregators have been normalized so that under the common individual price in both sectors, $p_{jt}(i) = p_t \forall j, i,$

$$c_{jt}(i) = \varphi_{jt} C_t.$$

Disutility of labour is given by $v(h(i);\xi_{jt})$, where ξ is a vector of parameters of interest, so that sector-specific shocks to preferences regarding labour supply is allowed for, but not good-specific. Production of the good i is obtained via the production function

$$y_{jt}(i) = A_{jt} f(h_t(i)) \tag{12}$$

which thus implies that sector-specific shocks are allowed for. We assume wage-taking firms and that the (nominal) profits of firm i in sector j can be written as in the previous study together with the real total cost and real marginal cost of supplying any good i in sector j can be represented as earlier.

Note that we have used the assumption that the households are on their labour supply schedule so that the real wage equals the marginal rate of substitution between labour and consumption. The desired mark-up in sector j is now $\mu_j = \dfrac{\theta_j}{\theta_j - 1}$, a constant, but sector specific. The natural level of output in sector j, Y_{jt}^n, is defined as the common level of sector j output under flexible prices[2]. It satisfies

$$\mu_j s^j \left(Y_{jt}^n, Y_t^n ; \tilde{\xi}_t \right) = \frac{P_{jt}}{P_t} \left(\frac{Y_{jt}^n}{n_j \varphi_{jt} Y_t^n} \right)^{-\frac{1}{\eta}} \tag{13}$$

with the intended interpretation that the utmost right hand side indicates the relative price P_{jt}/P_t required to induce the relative demand Y_{jt}^n/Y_t^n. The natural rate of aggregate output Y_t^n aggregates sectoral natural outputs according to the CES-aggregator. If $\tilde{\xi}_t = 0$ and $\varphi = 1$ for all t and for both sectors, the flex price equilibrium involves a common output $\overline{Y}$ for all goods (satisfying the above equilibrium pricing equation)

$$s^j(\overline{Y}, \overline{Y}; 0) = \frac{1}{\mu_j} \left(\frac{1}{n_j} \right)^{-\frac{1}{\eta}}.$$

Log-linearizing around this equilibrium gives us the optimality conditions for the real marginal cost as in our previous study.

Assume Calvo-type (Calvo 1983) price staggering in each of the two sectors with α_j the fraction of goods prices that remain constant in any given period in sector j. A firm i in that sector that is lucky to get the change to optimize its price in period t chooses its new price $p_t(i)$ to maximize the expected present value of its profits

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[2] For derivation, see e.g. Walsh (2010) Ch. 8.2.

$$E_t \left\{ \sum_{T=t}^{\infty} \alpha_j^{T-t} Q_{t,T} \left[\prod_T^{ji} (p_t(i)) \right] \right\} \tag{14}$$

The F.O.C for this programme is, after log-linearizing, given by

$$E_t \sum_{T=t}^{\infty} (a_j \beta)^{T-t} \left\{ \hat{p}_{jt}^* - \left[\hat{s}_{t,T}^j - \hat{p}_{jT} + \sum_{\tau=t+1}^{T} \pi_{j\tau} \right] \right\} = 0, \tag{15}$$

where $\hat{p}_{jt}^* = \ln\left(\frac{p_{jt}^*}{P_{jt}}\right)$ denotes the relative price (relative to others in sector j) of the firms that get to optimize their price at date t and $\hat{p}_{jt} = \ln\left(\frac{P_{jt}}{P_t}\right)$ is the real price of sector j at date t (i.e. the relative date t price of sector j relative to "cpi" or overall price level). On the other hand, $\hat{s}_{t,T}^j$ is the real marginal cost of the firms in sector j that last change their prices at date t. We also have the following decomposition

$$\hat{s}_{t,T}^j =$$

$$\hat{s}_T^j - \omega\theta_j \left\{ \ln\left(\frac{p_{jt}^*}{P_{jT}}\right) - \ln\left(\frac{P_{jt}}{P_{j,t+1}} - ... - \frac{P_{j,T-1}}{P_{jT}}\right) \right\}$$

$$= \hat{s}_T^j - \omega\theta_j \left\{ \hat{p}_{jt}^* - \sum_{\tau=t+1}^{T} \pi_\tau \right\} \tag{16}$$

where $\hat{s}_T^j$ denotes the (deviation of the) average (i.e. real marginal cost corresponding to average sectoral output $\bar{y}_{jt}$) real marginal cost in sector j and ω denotes the elasticity of the real marginal cost function with respect to $\hat{y}_{jt}$ (and $\hat{y}_{jt}^n$). Sectoral price indexes are given in (8) and repeated here for convenience

$$P_{jt}^{1-\theta_j} = n_j^{-1} \int_{N_j} p_{jt}(i)^{1-\theta_j} \, di, \; j= 1, 2$$

which gives us

$$= (1-\alpha_j) \left(\frac{p_{jt}^*}{P_{jt}}\right)^{1-\theta_j} + \alpha_j \left(\frac{1}{1+\pi_{j,t}}\right)^{1-\theta_j} \tag{17}$$

so a log-linear approximation (around the steady state) allows us to derive the following relationship

$$0 = \left[(1-\alpha_j)\hat{p}_{jt}^* - \alpha_j\hat{\pi}_{jt}\right](1-\theta_j) \Leftrightarrow$$

$$\hat{p}_{jt}^* = \left(\frac{\alpha_j}{1-\alpha_j}\right)\hat{\pi}_{jt} \tag{18}$$

Now, insert everything into the optimal pricing equation

$$E_t \sum_{T=t}^{\infty} (a_j \beta)^{T-t} \left\{ \hat{p}_{jt}^* - \left[\hat{s}_{t,T}^j - \hat{p}_{jT} + \sum_{\tau=t+1}^{T} \pi_{j\tau} \right] \right\} = 0$$

where from further manipulations give us

$$\hat{\pi}_{jt} = \underbrace{\left(\frac{(1-\alpha_j \beta)(1-\alpha_j)}{\alpha_j (1+\omega\theta_j)} \right)}_{\zeta_j} (\hat{s}_t^j - \hat{p}_{jt}) + \beta E_t \hat{\pi}_{j,t+1}$$

$$= \zeta_j (\hat{s}_t^j - \hat{p}_{jt}) + \beta E_t \hat{\pi}_{j,t+1} \tag{19}$$

We are almost there, as we still need to express the sector specific real marginal cost in terms of the relevant average sectoral output measure. From the expression for the demand for the sectoral composite good we obtain

$$\hat{y}_{jt} = \hat{\varphi}_t^{nj} + \hat{y}_t - \eta\hat{p}_{jt}$$

$$\hat{y}_{jt}^n = \hat{\varphi}_t^{nj} + \hat{y}_t^n - \eta\hat{p}_{jt}^n , \tag{20}$$

where $\hat{y}_t \equiv \log(Y_t / \overline{Y})$ and $\hat{y}_t^n \equiv \log(Y_t^n / \overline{Y})$ are log deviations from the steady state. Substituting this into the average real marginal cost of sector j gives us

$$\hat{s}_t^j - \hat{p}_{jt} = (\sigma^{-1} + \omega)(\hat{y}_t - \hat{y}_t^n) - (1-\omega\eta)(\hat{p}_{jt} - \hat{p}_{jt}^n) \tag{21}$$

Inserting this into (19), we obtain

$$\hat{\pi}_{jt} = \zeta_j (\sigma^{-1} + \omega)(\hat{y}_t - \hat{y}_t^n) + \gamma_j (\hat{p}_{Rt} - \hat{p}_{Rt}^n) + \beta E_t \hat{\pi}_{j,t+1}, \tag{22}$$

where $\hat{p}_{Rt}^n \equiv \frac{1}{\eta} \left[(\hat{\varphi}_t^{n2} - \hat{\varphi}_t^{n1}) - (\hat{y}_{2t}^n - \hat{y}_{1t}^n) \right]$ and the sectoral relative price $\hat{p}_{Rt} = \ln\left(\frac{P_{2t}}{P_{1t}}\right)$ is obtained from the aggregate price index as follows

$$P_t = \left[n_1 \varphi_t^{n1} P_{1,t}^{1-\eta} + n_{2,t} \varphi_t^{n2} P_{2,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

$$1 = \left[n_1 \varphi_t^{n1} \left(\frac{P_{1t}}{P_t} \right)^{1-\eta} + n_{2,t} \varphi_t^{n2} \left(\frac{P_{2t}}{P_t} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}} \Rightarrow$$

$$0 = \frac{1}{1-\eta} \left[n_1 (\hat{\varphi}_t^{n1} + (1-\eta)\hat{p}_{1t}) + n_2 (\hat{\varphi}_t^{n2} + (1-\eta)\hat{p}_{2t}) \right]$$

$$= n_1 \hat{p}_{1t} + n_2 \hat{p}_{2t} = (1-n_2)\hat{p}_{1t} + n_2 \hat{p}_{2t}$$
$$= \hat{p}_{1t} + n_2 (\hat{p}_{2t} - \hat{p}_{1t}) \equiv \hat{p}_{1t} + n_2 \hat{p}_{Rt} \Rightarrow \hat{p}_{1t} = -n_2 \hat{p}_{Rt} \tag{23}$$

and similarly for $\hat{p}_{2t} = n_1 \hat{p}_{Rt}$. Now define $\kappa_j = \zeta_j(\sigma^{-1} + \omega)$ and since $\gamma_1 = n_2\zeta_1(1 + \omega\eta)$ and $\gamma_2 = -n_1\zeta_2(1 + \omega\eta)$ we can write

$$\hat{\pi}_{jt} = \kappa_j(\hat{y}_t - \hat{y}_t^n) + \gamma_j(\hat{p}_{Rt} - \hat{p}_{Rt}^n) + \beta E_t \hat{\pi}_{j,t+1}.$$

We still need to derive the dynamics for aggregate inflation. Note that

$$\hat{p}_{Rt} = \ln\left(\frac{P_{2t}}{P_{1t}}\right) = \ln\left(\frac{P_{2t}}{P_{2,t-1}} \cdot \frac{P_{1t-1}}{P_{1,t}} \cdot \frac{P_{2,t-1}}{P_{1,t-1}}\right)$$

$$= \hat{\pi}_{2t} - \hat{\pi}_{1t} + \hat{p}_{R,t-1}. \tag{24}$$

Hence, we can represent the sectoral inflation differential in terms of the lagged relative

price $\hat{p}_{R,t-1}$.

$$(1 - \beta'F)(\hat{\pi}_{1t} - \hat{\pi}_{2t}) = \kappa'(\hat{y}_t - \hat{y}_t^n) + \gamma'(\hat{p}_{Rt} - \hat{p}_{Rt}^n) \text{ or}$$

$$(\hat{\pi}_{1t} - \hat{\pi}_{2t}) = \kappa'(\hat{y}_t - \hat{y}_t^n) + \gamma'(\hat{p}_{Rt} - \hat{p}_{Rt}^n) + \beta'E_t(\hat{\pi}_{1,t+1} - \hat{\pi}_{2,t+1}) \tag{25}$$

$$Fx_t = E_t x_{t+1}$$

$$\beta' = \frac{\beta}{1 + \gamma}, \kappa' = \frac{\kappa}{1 + \gamma}, \gamma' = \frac{\gamma}{1 + \gamma}$$

$$\gamma = \gamma_1 - \gamma_2, \kappa = \kappa_1 - \kappa_2$$

Aggregate inflation rate $\hat{\pi}_t = n_1\hat{\pi}_{1t} + n_2\hat{\pi}_{2t}$ has a similar representation

$$\hat{\pi} = \bar{\kappa}(\hat{y}_t - \hat{y}_t^n) + \bar{\gamma}(\hat{p}_{Rt} - \hat{p}_{Rt}^n) + \beta E_t \hat{\pi}_{t+1} \tag{26}$$

$$\bar{\kappa} = n_1\kappa_1 + n_2\kappa_2, \quad \bar{\gamma} = n_1\gamma_1 + n_2\gamma_2,$$

where we can use equation (25) for the period t relative price to ascertain that aggregate inflation in general depends on the lagged relative price and not only on aggregate variables. Note also that identical sectoral price adjustment frequency – $\alpha_1 = \alpha_2$ – is not anymore sufficient to eliminate the dependence of aggregate inflation on the sectoral relative price. On the other hand, if both the frequency of price adjustment and price elasticity of demand are equal across sectors, then we have

$$\bar{\gamma} = n_1\gamma_1 + n_2\gamma_2 = 0$$

so that aggregate inflation is independent of the sectoral relative price.

2.2 The key equations describing the behaviour of inflation adjustment

Here we collect the equations we utilise in the log-linearized model for estimation of sectoral inflation adjustment. Output is given by a linear production function and it consist of goods to be consumed so that Y= C because in the setting below it does not make a difference to distinguish production to consumption and investment goods, we further assume that capital is firm specific and fixed at the business cycle fluctuations at least, see Woodford (2005). Hence

$$C_t = A_t L_t,$$

where we assume, that technical progress and productivity are catch by

$$\frac{A_t}{A_{t-1}} \equiv dA_t = \exp z_t, \text{ where } z_t \text{ is an AR(1) process of the form } z_t = \alpha_t + \rho z_{t-1} + \varepsilon_t, \ \varepsilon_t \sim N(0, 1).$$

The aggregate consumption Euler equation is given by (with external habit formation)

$$\hat{c}_t = \frac{h}{1+h}\hat{c}_{t-1} + \frac{1}{1+h}E_t\hat{c}_{t+1} - \frac{1-h}{\sigma\,(1+h)}[(\hat{r}_t^b + \varphi_t^r) - E_t\hat{\pi}_{t+1}],$$

and the sectoral market demand given by

$$c_{j,t} = n_j \log\left(\left(\frac{P_j}{P}\right)^{-\eta} C_t\right).$$ Then total demand of goods across the economy is given by

$$c_t = c_{1,t} + c_{2,t} + \varphi_t^{dj}$$

Inflation dynamics is given by the New Keynesian Phillips curve

$$\hat{\pi}_{j,t} = \kappa_j(\hat{c}_t - \hat{c}_t^n) + \gamma_j(\hat{p}_{R,t} - \hat{p}_{R,t}^n) + \beta E_t\hat{\pi}_{j,t+1} + \varphi_t^{pj},$$

here the natural, or potential, level of output (the level with flexible prices) is given by

$$c_t^n = \log\left((\tfrac{\theta}{\theta-1})^{-1/\sigma+\psi} A_t^{\frac{1+\psi}{\sigma+\psi}}\right)$$ in the symmetric case. In a case with asymmetries (with different

thetas θ_j, j= 1, 2) we have

$$\hat{c}_t^n = \lambda_{A1}\hat{a}_{1t} + \lambda_{A2}\hat{a}_{2t} + \lambda_{\varphi1}\hat{\varphi}_t^{nj} + \lambda_{\varphi2}\hat{\varphi}_t^{nj},$$

where $\lambda_{Aj} = \dfrac{n_j^{1/\eta}(\mu_j)^{\frac{1-\eta}{\psi\eta}}}{n_1^{1/\eta}(\mu_1)^{\frac{1-\eta}{\psi\eta}} + n_2^{1/\eta}(\mu_2)^{\frac{1-\eta}{\psi\eta}}}\dfrac{(1+\psi)}{(\sigma+\psi)}$

and
$$\lambda_{\varphi j} = \frac{n_j^{1/\eta}(\mu_j)^{\frac{1-\eta}{\psi\eta}}}{n_1^{1/\eta}(\mu_1)^{\frac{1-\eta}{\psi\eta}} + n_2^{1/\eta}(\mu_2)^{\frac{1-\eta}{\psi\eta}}} \frac{\psi}{(\eta-1)(\sigma+\psi)}$$

for $j = 1, 2$. Here the productivity shocks $\hat{a}_{j,t}$ and the shocks to the relative weights of the sectors $\hat{\varphi}_t^{nj}$ causes fluctuations around the steady state. We notice, that (the inverse of) the inter-temporal elasticity of substitution in consumption together with (the inverse of) the Frisch elasticity of labour supply and the sector specific mark-up factors μ_j:s plays a crucial role in determining the level of the potential output. For derivation of these equations, see Appendix 2.

Moreover, $\hat{p}_{R,t} = \ln\left(\dfrac{P_{2,t}}{P_{1,t}}\right)$ and $\hat{p}_{R,t}^n \equiv \dfrac{1}{\eta}\left[(\hat{\varphi}_t^{n1} - \hat{\varphi}_t^{n2}) - (\hat{c}_{2,t}^n - \hat{c}_{1,t}^n)\right].$

There we have four kinds of exogenous shocks: $\varphi^{\cdot pj}$ a general cost push shock, $\varphi^{\cdot nj}$ a shock to relative weights, φ^r an interest rate shock to bond rate and $\varphi^{\cdot dj}$ a demand shock. These shock variables are assumed to follow an independent first-order autoregressive stochastic process.

$$\varphi_t^{pj} = \rho\varphi_{t-1}^{pj} + u_t^{pj},$$

$$\varphi_t^{dj} = \rho\varphi_{t-1}^{dj} + u_t^{dj},$$

$$\varphi_t^{nj} = \rho\varphi_{t-1}^{nj} + u_t^{nj}$$

$$\varphi_t^{r} = \rho\varphi_{t-1}^{r} + u_t^{r}.$$

Here we do not made any assumptions about the covariance of the shocks.

3. Data and estimation

Here we briefly introduce the estimation methodology used together with its theory and DSGE model solution strategy when implementing the estimation. Then we introduce the data used together with priors of the parameters for the estimation procedure.

3.1 Bayesian estimation

The linearized system of solved DSGE model described above is estimated by Bayesian method. As this estimation methodology is a primary tool nowadays in macroeconomics, we introduce only briefly the necessary steps involved in our estimation procedure. Estimation in this case is based on the likelihood generated by the DSGE system of equations of observed variables. The primary goal of this study is to emphasize the possibility of the underlying model to estimate and distinguish the sector specific parameters. These parameters of interest will characterize the potential differences of

the sector compared to rest of the economy. Here we do not emphasize the shock and variance decomposition behaviour of the variables, these would be an important topic in the future research of this model of course. Therefore, in this study, the shocks (and measurement errors as we didn't add the full set of shocks) are added to avoid stochastic singularity.

Priors and the observed data are used to form a density function of the parameters of the model and the likelihood function which describes the density of the observed data. The priors are a priori beliefs of the weights of the parameters on the likelihood function. Priors and likelihood function are combined together by Bayes rule to form the posterior density function. This posterior density function which describes all the posterior moments of interest, is estimated and formed with the help of Kalman filter and Metropolis-Hastings sampling-like method. To initialize the Metropolis-Hastings algorithm to obtain the posterior distribution of our parameters we utilize a Monte-Carlo based optimization routine posterior mode as a starting point.

Finally, after deriving the posterior distribution of our particular model, we may have several (with the same data set) estimated competing models. The Bayes Factor provides a particularly natural method of compare and test these models against each other given the same data set (one model does not have to be nested within the other). As in our case, where we estimated some slightly different specifications of our model, we simply compare the ratio of posterior odds.

3.2. Data and priors

The empirical estimation of the parameters of interest is done using Finnish macroeconomic seasonally adjusted quarterly data over the period 2000:1 to 2015:2. The data are drawn from the Statistics Finland data base on (aggregate) gross domestic production, producer price index and the total number of employees. For the data on building industry and manufacturing industry we have the same sectoral variables. The data for interest rate is 3 month money market interest rate (Euribor). All the variables in the analysis and listed below are constructed from these as follows: observation data series for the output gap $c_t^{obs} = \tilde{c}_t$ are demeaned log first differences of real per capita output series

$$ c_t^{obs} = \tilde{c}_t = \hat{c}_t - \hat{c}_t^n = (c_t - c_t^n) - \bar{c} . $$

In order to get the parameters defining the natural level of output, we estimated the full set of the model in two steps. First we took a prior (a ques?) for the output gap measure an estimated the parameters conditional to that prior value. Then, after deriving the full set of the parameters which could be identified within this model specification, we counted the new output gap measure and then, in the second step, estimated the parameters conditional to that new value.

Demeaning of all the other variables data gives us the observation time series used, for details see e.g. Pfeifer (2015).

What is the role for demeaning? When estimating the model, in order to avoid a situation that the shocks are forced to account for a positive mean in the observed series these series have to be demeaned. In other words, we match an observed variable (which have mean 0) to a model variable which also has mean 0 (because it is a deviation from steady state).

The focus of this paper is on aggregate and sectoral inflation adjustment block of the economy π, π_j where we have aggregate and sectoral output c, c_j and the market rate of interest as other variables. Then we have the following linearized system of equations for the inflation adjustment

$$\tilde{\pi}_{j,t} = \kappa_j(\tilde{c}_t) + \gamma_j(\tilde{p}_{R,t}) + \beta E_t \tilde{\pi}_{j,t+1} + \varphi_{j,t}^{pj}$$

$$\tilde{\pi}_t = n_1 \varphi_t^{nj} \tilde{\pi}_{1,t} + n_2 \varphi_t^{nj} \tilde{\pi}_{2,t}$$

$$\tilde{c}_t = \frac{h}{1+h} \tilde{c}_{t-1} + \frac{1}{1+h} E_t \tilde{c}_{t+1} - \frac{1-h}{\sigma(1+h)} [\tilde{r}_t^b - E_t \tilde{\pi}_{t+1}], \ \tilde{c}_t = \tilde{c}_{1,t} + \tilde{c}_{2,t} \ \text{and}$$

$$\tilde{c}_{j,t} = n_j \left(\left(\tilde{p}_t^{dj} \right)^{-\eta} \tilde{c}_t \right), \text{where } \tilde{p}_t^{dj} = \left[\ln\left(\frac{P_{j,t}}{P_t}\right) - \ln\left(\frac{P_{j,t-1}}{P_{t-1}}\right) \right] - \bar{p} \ \text{and} \ \tilde{c}_t = \tilde{c}_{1,t} + \tilde{c}_{2,t} + \varphi_t^{dj}$$

The demeaned market rate of interest evolves as

$$\tilde{r}_t^b = \tilde{r}_{t-1}^b + \varphi_t^r.$$

- Observation data series for inflation $\pi_t^{obs} = \tilde{\pi}_t$ and $\pi_{j,t}^{obs} = \tilde{\pi}_{j,t}$ are demeaned log first differences of respective price level indexes.

- Relative prices $p_{Rj,t}^{obs} = \tilde{p}_{Rj,t}$ for inflation adjustment equation are demeaned log first differences of relative prices for respective sectors.

- Relative prices in demand equations $p_{Dj,t}^{obs} = \tilde{p}_t^{dj}$ in turn are constructed from demeaned log first differences of sectoral prices relative to aggregate economy.

- Interest rate $r_t^{obs} = \tilde{r}_t$ is three months Euribor rate which itself is stationary but in order to be used in this context it is logarithmic and demeaned quarterly gross rate of return.

Regarding the choice of calibrated parameters and the prior distributions we made the following choices which ably in every sectoral comparison (i.e. we compare manufacturing industry to the rest of the economy and building industry to the rest of the economy each in turn). The model specification is the same in every case.

Calibrated values:

- β-discount factor (0.99)
- ρ – autoregressive coefficient for exogenous shocks (0.85)
- n_i – the relative steady-state share of manufacturing industry (0.2246)
- n_b – the relative steady-state share of building (0.068)

More over the priors for the weighting values of parameters kappa (κ) and gamma (γ) in inflation adjustment equation have been set according:

$$\kappa_j = \zeta_j(\sigma^{-1} + \omega) \ , \ \zeta_j = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha(1+\omega\theta_j)} \text{ and } \gamma_j = \zeta_j(1+\omega\eta), \text{ where } j = a, b \text{ and}$$

- Calvo parameter $\alpha = 0.75$,

- β-discount factor (0.99)

- intertemporal elasticity of substitution $\sigma = 1.2$,

- $\omega = 1$

- $\theta = 6$ for within category a goods (the sector we compare to the rest of the economy)

- $\theta = 15$ for within category b goods (the rest of the economy).

- $\psi = 2$ the inverse of the Frisch-elasticity.

Table 1: The priors:

parameter	prior distribution	mean	std.err
habit formation (h_a)	beta	0.7	0.14
habit formation (h)	beta	0.7	0.14
weight (κ_a)	beta	0.0225	0.009
weight (κ_b)	beta	0.0095	0.005
weight (γ_a)	beta	0.135	0.08
weight (γ_b)	beta	0.06	0.02
Inter temp. elasticity of substitution (σ)	normal	1.2	0.35
elasticity of subst. between groups (η)	Inv. Gamma	10.0	2
auregressive coeff. (μ_a)	beta	0.8	0.10

auregressive coeff.		0.8	0.10
(μ)	beta		

Industry shocks;

- u_t^{p1} , stderr 0.0154,
- u_t^{p2} , stderr 0.01614,
- u_t^{r} , stderr 0.004,
- u_t^{n}, stderr 0.024,
- u_t^{d1}, stderr 0.00643,
- u_t^{d2}, stderr 0.002.

Building shocks;

- u_t^{p1}, stderr 0.008
- u_t^{p2}, stderr 0.0163,
- u_t^{r}, stderr 0.004,
- u_t^{n}, stderr 0.0187,
- u_t^{d1}, stderr 0.013837,
- u_t^{d2}, stderr 0.000901.

4. Empirical results

As mentioned, we derived parameters for the output gap measure at the first step of estimation procedure and then evaluated this gap in the case of manufacturing industry to be $\bar{c}_{Ind}^{n} = 1,054$, this is the ratio of the natural level of output to the realised output level. Then we estimated the model parameters conditional to the output gap defined by this level of natural output.

Table 2. Estimated parameters for Industry

parameter	First differenced posterior distribution				
	mode	std. dev (Hessian)	mean	10%	90%
habit (h_a)	0.7645	0.1240	0.7407	0.5366	0.9388
habit (h_b)	0.8233	0.0918	0.8117	0.6658	0.9573
weight (κ_a)	0.0179	0.0089	0.0224	0.0091	0.0368
weight (κ_b)	0.0062	0.0047	0.0081	0.0015	0.0143
weight (γ_a)	0.0916	0.0795	0.1414	0.0141	0.2580
weight (γ_b)	0.0558	0.0198	0.0625	0.0309	0.0960
Inter temp. elast. subst. (σ)	1.2415	0.3209	1.3212	0.7858	1.8410
elasticity of subst. between groups (η)	5.7032	0.5612	5.7510	4.8205	6.6279
auregr. coeff. (μ_a)	0.8409	0.0988	0.7896	0.6309	0.9611
auregr. coeff. (μ_b)	0.6402	0.1135	0.6239	0.4344	0.8193

In case of building industry the natural level of output is $\bar{c}^n_{Ind} = 1,0029$ so it is approximately 3% higher than the existing level of output.

Table 3. Estimated parameters for Building

parameter	posterior distribution (first differenced model)				
	mode	std. dev (Hessian)	mean	10%	90%
habit (h_a)	0.8554	0.0740	0.8406	0.7336	0.9611
habit (h_b)	0.7677	0.1230	0.7301	0.5408	0.9469
weight (κ_a)	0.0185	0.0091	0.0219	0.0075	0.0348
weight (κ_b)	0.0082	0.0028	0.0090	0.0042	0.0131
weight (γ_a)	0.0871	0.07	0.1328	0.0157	0.2468
weight (γ_b)	0.0525	0.0206	0.0602	0.0281	0.0914
Inter temp. elast. subst. (σ)	1.2681	0.3175	1.3094	0.7774	1.8398
elasticity of subst. between groups (η)	6.4756	0.7287	6.6932	5.4510	7.8638
auregr. coeff. (μ_a)	0.5343	0.1019	0.5354	0.3802	0.6941
auregr. coeff. (μ_b)	0.8460	0.1046	0.7916	0.6280	0.9460

We also slightly modified these models by incorporating different options for the Calvo parameters in the inflation adjustment equations weighting parameters kappa and gamma as it was assumed in the original study by Woodford (2003). Instead of having the same Calvo parameter $\alpha = 0.75$ in every case, we put the Calvo parameter for Industry $\alpha i = 0.6$ and for the rest of the economy $\alpha = 0.83$. In case of Building industry $\alpha b = 0.83$ and for the rest of the economy $\alpha = 0.6$. These values as priors are due to sluggish adjustment of prices of the manufacturing industry compared to the rest of the economy and for building industry the relative price index seems to have developed faster than that of the rest of the economy. We proceed as follows.

As mentioned earlier a natural way to evaluate the (prediction) performance of these models over each other is by Bayes Factor. In our case this factor is in every case approximately 1 (1 +- 0,0001) as the likelihood and the log data densities are nearly the same in every model variation. Then we can't distinguish which of the specifications is better in the sense of (backward looking) weighting parameters kappa (κ) and gamma (γ). This may be a consequence of the forward looking nature of our inflation adjustment model specification, which put more weight on the expected inflation. Hence we do not report the outcomes with these different parameter values as they seems to give no additional information about the 'true' model. However, our inflation adjustment model seems to perform well as the one-step ahead linear Kalman filter forecast figures shows.

The diagnostic figures (in case of Indusry) for convergence of the parameters, goodness of fit (smoothed and historical values of the variables) by one-step ahead linear Kalman filter forecast evaluated at the posterior mode together with smoothed one-step ahead forecast of shock variables are in Appendixes below.

5. Conclusions

As relative price movements has a crucial role in resource allocation and market mechanism in general, the performance of stabilization policy is not invariant on the underlying parameters that determine the structure of this mechanism. As the parameters and hence the structure could diverse between the sectors of the economy there is potential for adjustment asymmetries after hit by a shock as well. Therefore this study has estimated some of these sector specific parameters which determine the magnitude and duration of the sectoral response to a shock. The estimates of the underlying parameters clearly indicate, that these parameters diverge between the sectors studied and those sectors each have a specific relation to the rest of the economy. Because the identified parameters rise from the preferences of the agents, the shocks studied could be seen as structural ones as they are related to behaviour of the agents as well. These shocks are hence interpretable and consistent with microeconomic evidence as the previous study of Koskinen and Vilmunen (2017) demonstrates. This, in turn, enables us to utilise this DSGE model for a policy analysis which recognize the sector specific features.
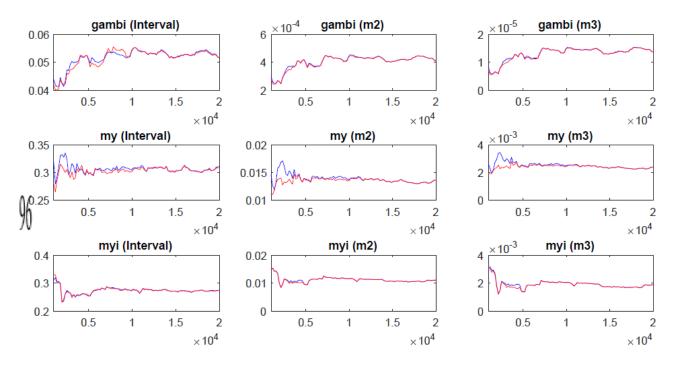
Then, concerning the policy implications of this model, we find out that the natural / potential level of output and hence output gaps varies between the sectors studied. As any stabilization as well as structural policy is conditional to the output gap, this fact should be taken into account when practising those policies. These divergences arise from the different preference parameters of the agents. The same applies when dampening the business cycle fluctuations as the magnitude and persistence of adjustment process after hit by a shock do differ between the sectors reflecting the divergence of the underlying parameters.

One of those issues that need to be solved is related to the steady state of the economy, and the other concerns the identification of the parameters θ_j and α (the own price elasticity of demand within the sector and the Calvo parameter) as so far we have used calibrated values from earlier studies. In this respect, what should be done is to estimate them (separately) with the data utilised in the respective study. The rest of the parameters of interest are estimated conditional to the steady state, and if this steady state is unstable, e.g. there has been structural breaks causing nonlinearities, the estimates could be biased as well as is our estimate of output gap. This could be the case with the Finnish time series data utilised and it is therefore a subject for further studies.
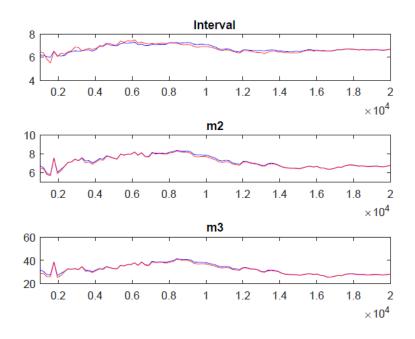
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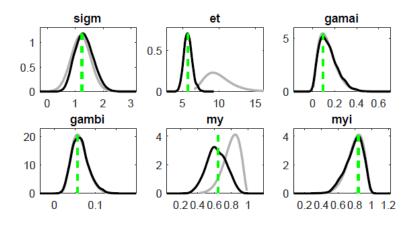   International Journal of Central Banking, September 2005, Vol. 1 No. 2
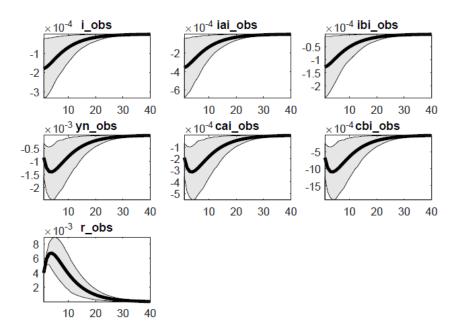
Appendixe 1.

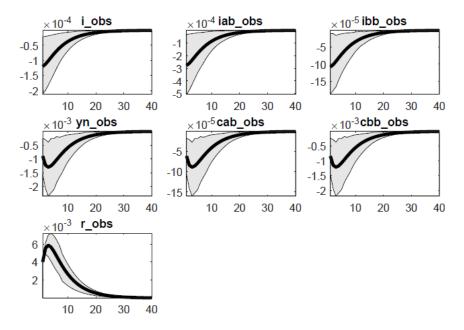Univariate convergence diagnostics above and overall below.
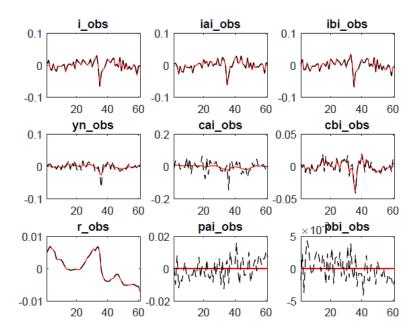
Some priors and posteriors.

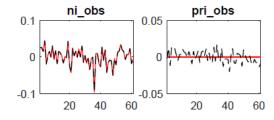An orthogonalized interest rate shock, the case of manufacturing industry, below:

An orthogonalized interest rate shock, the case of building industry, below:

Smoothed one-step ahead linear Kalman filter forecasts for the case of industry below:

Appendixe 2.

Flexible price aggregate output in a two-sector economy: a linear approximation

For the CES consumption aggregate

$$C_t = \left[(n_1 \varphi_{1t})^{\frac{1}{\eta}} C_{1t}^{\frac{\eta-1}{\eta}} + (n_2 \varphi_{2t})^{\frac{1}{\eta}} C_{2t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

with the obvious notation, the cost minimizing sector specific consumption bundles are given by

$$C_{jt} = n_{jt} \varphi_{jt} \left(\frac{P_{jt}}{P_t} \right)^{-\eta} C_t ,$$

and the aggregate price index is then

$$P_t = \left[n_1 \varphi_{1,t} P_{1,t}^{1-\eta} + n_{2,t} \varphi_{2,t} P_{2,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} .$$

From the household's optimal intratemporal trade-off between leisure and consumption we have

$$\frac{h_t^{\psi}(i)}{C_t^{-\sigma}} = w_t = \frac{A_{jt}}{\mu_j}$$

where the latter equality follows from the profit maximization condition of the monopolist.
The production function, on the other hand, gives us $h_t(i) = Y_{jt}^n / A_{jt}$ while goods market equilibrium gives us $C_t = Y_t^n$; hence

$$\left[\frac{Y_{jt}^n / A_{jt}}{(Y_t^n)^{-\phi}} \right]^{\psi} = \frac{A_{jt}}{\mu_j} ,$$

where $\phi = \sigma / \psi$. Solving for the sectoral natural rate gives us

$$Y_{jt}^n = (Y_t^n)^{-\phi} \left[\frac{A_{jt}^{1+\psi}}{\mu_j} \right]^{\frac{1}{\psi}} .$$

Finally, the CES consumption aggregator and sectoral goods market equilibrium gives us

$$Y_t^n = \left[(n_1 \varphi_{1t})^{\frac{1}{\eta}} \left(\frac{A_{1t}^{1+\psi}}{\mu_1} \right)^{\frac{1}{\psi}} + (n_2 \varphi_{2t})^{\frac{1}{\eta}} \left(\frac{A_{2t}^{1+\psi}}{\mu_2} \right)^{\frac{1}{\psi}} \right]^{\frac{\psi\eta}{(\eta-1)(\sigma+\psi)}}$$

Log-linearizing the last equation allows us to describe the local dynamics around the steady state
($\varphi_j = 1 = A_j$, $j = 1, 2$) as follows

$$\hat{y}_t^n = \lambda_{A1} \hat{a}_{1t} + \lambda_{A2} \hat{a}_{2t} + \lambda_{\varphi 1} \hat{\varphi}_{1t} + \lambda_{\varphi 2} \hat{\varphi}_{2t} ,$$

where $\lambda_{Aj} = \dfrac{n_j^{1/\eta}(\mu_j)^{\frac{1-\eta}{\psi\eta}}}{n_1^{1/\eta}(\mu_1)^{\frac{1-\eta}{\psi\eta}} + n_2^{1/\eta}(\mu_2)^{\frac{1-\eta}{\psi\eta}}} \dfrac{(1+\psi)}{(\sigma+\psi)}$

and $\lambda_{\varphi j} = \dfrac{n_j^{1/\eta}(\mu_j)^{\frac{1-\eta}{\psi\eta}}}{n_1^{1/\eta}(\mu_1)^{\frac{1-\eta}{\psi\eta}} + n_2^{1/\eta}(\mu_2)^{\frac{1-\eta}{\psi\eta}}} \dfrac{\psi}{(\eta-1)(\sigma+\psi)}$

for $j = 1, 2$.

In a symmetric case this solution reduces to $y_t^n = \log\left((\tfrac{\theta}{\theta-1})^{-1/\sigma+\psi} A_t^{\frac{1+\psi}{\sigma+\psi}} \right)$.