AXIOM Method for Cross-Impact Modeling and Analysis

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Abstract

This thesis describes the AXIOM method for cross-impact modeling and analysis. AXIOM is a novel cross-impact analysis technique, which combines the best features of various existing techniques in an attempt to create a practical cross-impact modeling tool with emphasis on modeling power and fitness for modeling of real systems. The thesis reviews the existing documented cross-impact analysis techniques and discusses their problems and shortcomings. The AXIOM approach improves on the existing methods in a number of ways. A software implementation of AXIOM is also presented.

Contents

1 Introduction and context

This thesis describes in detail the novel **AXIOM** method for cross-impact analysis. The AXIOM method is developed by the author. The acronym AXIOM stands for **A**dvanced **Cross-I**mpact **O**ption **M**ethod. A number of documented cross-impact analysis techniques exist, but they have problems and shortcomings that hinder their application in actual research and modeling problems. Some cross-impact techniques are also inadequate in terms of the way they facilitate drawing conclusions from the cross-impact calculation results. AXIOM aims to be a pragmatic method for cross-impact modeling and analysis, combining the best aspects of the various techniques under the banner of cross-impact analysis, to provide a practical tool for cross-impact modeling that is well suited for actual research and real modeling cases. AXIOM also facilitates easier extraction of higher added value information from the cross-impact calculation outputs on the basis of similar inputs than alternative cross-impact approaches.

This introductory chapter presents the basic ideas of cross-impact analysis and the research areas and fields in which cross-impact techniques are utilized. It also makes the case for the utility of AXIOM in those fields. Chapter 2 goes to examine the characteristics and problems of the documented cross-impact methods in detail. Chapter 3 presents the concepts and components of the AXIOM model and the computational process of extracting higher-order information from the model. Chapter 4 details what are the problems in cross-impact analysis and modeling, and how AXIOM addresses these problems and contributes to the state-of-the-art of cross-impact modeling and analysis. Finally, Chapter 5 reports the software implementation of the AXIOM method and discusses how AXIOM could be developed further.

1.1 What cross-impact analysis is

There are several documented cross-impact techniques as well as several ones that are mentioned in reviews of cross-impact analysis and in the literature on the different specific techniques, but which are not well documented. Identifying the core features that would be shared by all flavours of cross-impact analysis is challenging. For this reason, also drawing clear lines that would patently distinguish cross-impact analysis techniques from other analytical techniques with slightly similar inspirations is difficult.

According to Gordon [1994, 1] the cross-impact method was originally developed by himself and Olaf Helmer in 1966, to find out “whether forecasting could be based on perceptions about how future events may interact”. A motivation for developing the early techniques of cross-impact analysis was to enable analysis of interactions between events, which is not present in the Delphi method [Gordon, 1994; Godet *et al.*, 1994, 139]

The Delphi method is a communication and collaboration technique used with expert panels, especially in the fields of futures studies and foresight [Linstone & Turoff, 1977]. There are many variations of the Delphi method, but the basic process is as follows: People with expertise considered relevant for a studied topic answer questions and provide reasoning about their answers anonymously. The Delphi facilitator summarizes the answers and the reasoning and presents the summary to the panel, maintaining the anonymity of answers and reasoning. Discussion about the results may or may not take place. On the basis of the summary, expert participants reconsider and revise their answers. This usually leads to answers converging and the range of answers narrowing. These phases may be reiterated until some halting condition is met; the halting condition may be that a consensus is reached or that answers do not converge further or change anymore. If there is no consensus, a mean, median or mode of the answers can be used. This consensus or iterated average expert opinion is then considered to be the result of the Delphi process and to be close to the ”real” value or at least be information of higher value than the initial expert opinions.

The Delphi method is linked to the cross-impact method, as the valuations for the input data for cross-impact method can be obtained through a Delphi process. This thesis is focused on the computation aspect of the cross-impact analysis process and the details of Delphi method are not elaborated further. For more information about Delphi method, see, e.g., the manual by Linstone & Turoff [1977] and for some recent perspectives on use of technology in service of the Delphi process, see, e.g., the work of Seker [2015].

Godet *et al.* [1994, 139–140] note that cross-impact method is a generic name for a family of techniques evaluating changes in probabilities of events as the information about the interactions between the events is accounted for. With this definition, Godet et al. seem to limit the use of cross-impact analysis label to methods which operate on probability valuations and also to methods specifically dealing with events.

Gordon [1994, 1–2] appears to define the family of cross-impact approaches through more of an evolutionary approach, listing different methods that have been inspired by his and Olaf Helmer's original cross-impact method. Many of these methods try to address some particular challenge related to the original method or incorporate analytical aspects that are not present in it. Some of the methods mentioned by Gordon in his summary of the history of the cross-impact method deviate from the original ideas in such a way that it is somewhat questionable to call them variations or evolutions of the same analytical technique: for example, the KSIM approach devised by Kane [1972] cannot really be used to answer the same research questions as Gordon and Helmer's original cross-impact technique.

This thesis takes the position that the techniques that perform analysis on a system model, comprised of representations of system components expressed in discrete options and the probability-influencing interactions between them, can all be called cross-impact analysis. Having said this, the scope of reviewing the different methods and comparing them to the AXIOM method is limited to well-documented methods. As there is a market for doing cross-impact analysis in management consultancies and possibly more money revolving around these analyses than in the academia, it is completely possible that viable cross-impact techniques exist outside the documented sphere. Some of those methods might offer a similar analytical process and outputs as the AXIOM method does. As information about these techniques is not available in the literature, they cannot be reviewed and fall outside the scope of this thesis. Chapter 2 reviews the documented methods.

In general terms, cross-impact analysis could be described as a technique for studying a system modeled as a set of components, states, events and forces that are partially dependent on each other and therefore have impacts and influence on each other. Some formats of cross-impact analysis can be used to study contrafactuals, hypotheticals or the future of a system [Gordon, 1994]. Methods providing outputs like this need to valuate the probabilities of events or system states. Other formats can be used to refine the understanding about the interdependencies between the system components [Godet *et al.*, 1991, 24]. Methods like these do not necessarily need to explicitly deal with probability valuations, but they do operate with hypotheses or system descriptors that could, in theory, be assigned a probability value. This characteristic demarcates cross-impact modeling techniques from other types of simulation and modeling practices.

1.2 Aims of cross-impact analysis

Godet *et al.* [1994, 140] see that the central aim of cross-impact analysis is to test the probability valuations provided by the experts used in the exercise and iteratively correct these probabilities. This iteration results in corrected probability valuations that are consistent and abide to the laws governing probability. With the corrected probabilities, it is then possible to calculate the probabilities of different system configurations. A system configuration means a specific combination of states the different system components can have. A single system configuration can be seen as a scenario, so the cross-impact process can lead to a probability distribution for all scenarios for the system [Godet *et al.*, 1994, 139–142]. Godet *et al.* [1991, 49] also talk about the cross-impact method as a sort of preprocessing tool used in conjunction with the scenarios method (see [Godet *et al.*, 1991, 12]), used to identify the most probable, logical, consistent and interesting scenarios that warrant more detailed study and that should be the subject to the scenarios method.

Gordon [1994, 5] mentions the learning process that occurs while the cross-impact matrix values are being estimated as one of the major benefits of undertaking cross-impact analysis. Similarly to Godet et al., for Gordon the corrected *a posteriori* probabilities are an important outcome. Gordon, in contrast to Godet, specifically mentions policy testing as a way to use the analysis. It can be accomplished by defining an anticipated policy or action that would affect the events in the matrix, and adding it to the matrix. Matrix is then changed to reflect the immediate effects of the policy. This can be accounted for by changing the initial probabilities or adding a new event to the matrix. A new iteration is performed and the differences to the 'calibration run' are the effects of the newly introduced policy. Gordon quite aptly states that "The cross-impact matrix becomes a model of event interactions that is used to display the effects of complex chains of impacts caused by policy actions" [Gordon, 1994, 6].

A review of cross-impact techniques by Matic & Berry lists the uses and aims of cross-impact techniques in general. These are evaluation of possible future scenarios, impact analysis, intervention point evaluation, strategic decision-making and evaluating complex collaboration arrangements. Matic and Berry do not elaborate on the uses or provide examples. However, evaluation of possible future scenarios is definitely possible, by way of using the cross-impact method to investigate which particular states of system components typically happen together; impact analysis can be done by introducing new components to the system and comparing the outcome to a situation where the added components are not present; intervention point evaluation is possible by means of introducing policy actions as system components and testing how the system works when these policies are realized or not; strategic decision-making can be thought to benefit fairly directly from impact analysis and intervention point evaluation.

The author's summary of the aims and benefits of cross-impact analysis is as follows: Cross-impact analysis in its basic form extracts information about the indirect and total interactions between system components on the basis of the direct interactions. Direct interactions are input data for the method, so the interesting part of the analysis is the network of indirect interactions. When the system model has a lot of components, the indirect interactions can happen over a complex web and the chains of impacts can be long. Exploring these long impact chains and interaction webs can bring forward surprising and counter-intuitive analytical results. Cross-impact analysis can reveal that a system component that is seemingly unrelated to some other component of interest is actually of central importance and conversely that some other seemingly important component's effect might be cancelled or reversed by the system's web of interactions. Cross-impact analysis can identify effective policy actions and interventions in the system, or high leverage points for influencing outcomes of the system, so-called *high-leverage intervention points*, resulting in information and actionable recommendations relevant for decision-making and policy.

Other than the analytical outputs resulting from the cross-impact computations, the cross-impact modeling, preceding the application of any algorithm on the model, can be highly useful in refining the conceptual model underlying the cross-impact model and as a learning experience. This thesis focuses on extracting higher-order information from the cross-impact model through computation, and the practical modeling power that different cross-impact methods offer modelers. The other cross-impact methods are evaluated from this perspective.

1.3 Application domains of cross-impact analysis

Cross-impact analysis techniques are associated with the disciplines of futures studies and foresight. The family of techniques is often mentioned in resources listing the various futures research methods (see e.g. [Cagnin *et al.*, 2016; Wikipedia, 2016]), but normally not presented in detail. A recent guide on foresight methods by van der Duin [2016] doesn't touch cross-impact techniques at all, so they probably cannot be said to be at the methodological core of futures studies and foresight. In the author's experience the cross-impact analysis methods are not well known or understood and rarely used. This could be a result of the lack of practical software tools to perform the analysis with, the fairly opaque documentation that the existing techniques have and certain characteristics of the methods that significantly limit their applicability in research use. The AXIOM approach makes the usage of cross-impact analysis and modeling easier, more flexible, more practical and better suited for real research problems and modeling cases. (see Chapter 2 for discussion about the problems of existing techniques and Chapter 4 for notes on how AXIOM improves these aspects).

Acknowledging the association of cross-impact techniques to futures studies and foresight activities, there is no reason to perceive them as applicable and relevant only in those somewhat esoteric fields; cross-impact analysis should be seen as a general strategic planning and strategic thinking tool. The different cross-impact techniques have already been employed in the study of diverse topics including industry technological forecasting, corporate strategy, air traffic foresight and planning of energy system [Gordon, 1994; Godet *et al.*, 1994], and have potential to serve many other fields of research.

A characteristic of futures studies and foresight is that since the analytical focus is on the future and the change of the studied systems and their operating logic, there is little or no empirical data available to conduct traditional, e.g. econometric modeling based on statistical data and the relationship derived from such data; in some cases it could be said that the data relevant for traditional modeling cannot exist as the object of interest (the future or some contrafactual situation) does not exist. Futures studies and foresight activities often operate on expert insights and data collected from experts via interviews, workshops and surveys. Processing this expert-sourced data into higher-order information and providing synthesis and practical, policy-relevant results is often challenging. Cross-impact analysis provides a systematic way for extracting expert insights and dealing with the expert inputs.

Modeling of systems is normally based on empirical data. For a notable share of modeling cases and research domains the best available data for a modeling exercise will be opinion data, provided by knowledgeable people, i.e., experts of the domain in question. Cross-impact methods can be seen as tools for systems modeling and simulation using the expert-sourced data that for many domains is the best and only data available and that would be difficult or even impossible to model with traditional modeling techniques. Even when some relevant statistical data might be available for modeling, the relationships between the components of the modeled system might be difficult to derive from time-series data. In this sense, cross-impact modeling can fill a major gap in the toolkit of systems modelers. Developing pragmatic approaches and practical tools for cross-impact modeling, such as AXIOM, might prove to be of great value for adoption of the technique.

2 Cross-impact analysis techniques

This chapter reviews the well-documented cross-impact methods. To the author's knowledge, no comprehensive comparative reviews of cross-impact methods exist. In his technical paper on his cross-impact analysis technique Gordon [1994] mentions several later cross-impact methods originating from his and Olaf Helmer's method; Matic & Berry provide a very high-level overview of the different cross-impact methods; and the FOR-LEARN Online Foresight Guide [Cagnin *et al.*, 2016], which to some extent enjoys a status of methodological reference tool in the field of foresight, reviews (but does not detail) one cross-impact technique and mentions three others. The methods mentioned in all these three sources are Gordon's method, KSIM, SMIC and INTERAX and they are discussed in this thesis as well. Methods that are mentioned but of which no documentation could be obtained include the EZ-IMPACT method, which is possibly derived from KSIM [Wikipedia, 2015], and the EXPLOR-SIM method.

The most well-known and oft-mentioned cross-impact techniques are Theodore Gordon's and Olaf Helmer's cross-impact method and Michel Godet's SMIC. Gordon's and Helmer's method is often stated [Gordon, 1994; Matic & Berry; Wikipedia, 2016] to be the original cross-impact technique which has largely inspired the other, more recent approaches. In the FOR-LEARN Online Foresight Guide [Cagnin *et al.*, 2016] the template for cross-impact analysis used in describing the whole family of techniques is Godet's SMIC method.

2.1 Overview of features of different techniques

Table 1 displays the comparison of the most important characteristics of the cross-impact techniques that are relevant to the AXIOM method as reference points and alternatives. The most important differences in the various techniques lie in *a*) how the interactions between system components are expressed *b*) does the technique explicitly calculate probability values in its computations *c*) how the model components are represented and *d*) incorporation of temporal dimension in cross-impact modeling.

The way that the interactions between system components in a cross-impact model are expressed has very important consequences, for both the possibilities to derive analytical outputs and the cross-impact modeling process, and ultimately

Technique	*I)*	*II)*	*III)*	*IV)*
Gordon & Helmer	a	✓	a	
INTERAX	a	✓	a	
SMIC (Godet)	a	✓	a	
KSIM	b		c	✓
MICMAC	b		a	
EXIT	b		a/c	
Basics & JL	c	✓	b	
AXIOM	c	✓	b	✓

I) Expression of interactions
- *a* Conditional probabilities
- *b* Indices
- *c* Probability adjustment functions

II) Explicit probabilities

III) Representations of model components
- *a* Binary hypotheses/events
- *b* Multivalued statements
- *c* Descriptors having a continuous value

IV) Temporal dimension in modeling

Table 1: Comparison table of features cross-impact techniques

the modeling power of the technique. The trade-off is between the accuracy of model inputs and outputs and the ease of valuation of the model. For this issue, some techniques, AXIOM included, propose a compromise between accuracy and ease.

Calculating the probabilities explicitly enables richer analytical outputs, such as testing for effects of interventions and other changes in the system. This focus, however, requires more input information (the *a priori* probability valuations). AXIOM deals with the probabilities explicitly.

Representation of model components is important from the perspective of the modeling power of the technique. The most flexible and expressive solution for modeling real systems appears to be the multivalued statement. This is the basic component of AXIOM models.

The most widely-used techniques do not allow for easy representation of time in cross-impact models. As the most natural application for cross-impact modeling is planning and futures-oriented systems analysis, this is a significant drawback. AXIOM provides a straightforward way to take the temporal aspect into consideration in cross-impact modeling.

2.2 Gordon and Helmer's cross-impact technique

Gordon and Helmer's cross-impact model consists of binary hypotheses called *events*, and the interactions between the events described as conditional probabilities. Events, in spite of the term, need not be thought to necessarily represent only one-time occurrences, but can be thought to represent the state of the studied system in a more general and temporally longer-standing way. The modeling process starts by defining the set of included events that can be called, in a more general way, hypotheses. Once the set of included hypotheses is determined, each is assigned an initial or *a priori* probability by a group of valuators consisting of suitable experts. Considerations about use of experts and their selections are discussed in Section 3.4.

Once the a priori probabilities have been estimated, the conditional probability matrix can be estimated. The conditional probabilities should comply to the laws governing probability: as the a priori probabilities of the hypotheses are known, the conditional probabilities for each pair (A, B) of hypotheses must be in the bounds $\frac{P(A)-1+P(B)}{P(B)} \leq P(A|B) \leq \frac{P(A)}{P(B)}$, where $P(A)$ is the a priori probability of the impacted hypothesis, $P(B)$ is the a priori probability of the impacting hypothesis, and $P(A|B)$ is the conditional probability of hypothesis A if hypothesis B is true [Gordon, 1994, 6]. If the initial conditional probability valuations do not fall in the permissible bounds, the expert group providing the valuations must decide how to resolve the inconsistency. The conditional probability can be corrected into the permissible bounds, or the a priori probability valuation can be changed; it is up to the valuators to decide how the problem is resolved.

Hypothesis evaluation means assigning a value (true or false) to them according to their probability. The cross-impact model, in turn, is evaluated by placing the hypotheses in random order, evaluating them in that order and adjusting the probabilities of the hypotheses using the *odds ratio* technique described by [Gordon, 1994, 7–9]. The model evaluation is repeated "a large number of times" [Gordon, 1994, 10]. The frequencies of the hypotheses in the set of completed model evaluations are interpreted as their a posteriori probabilities.

Subsection 3.4.6 discusses the possible interpretations of the a priori probabilities in cross-impact modeling. According to the interpretation used in the valuation, different analytical outcomes can arise from the a posteriori probabilities. If the a priori probabilities are valuated in isolation (see interpretation one of a priori probability in Subsection 3.4.6) the a posteriori probabilities clearly embody the indirect impacts in the system, or the probabilities that consider the impact chains. This kind of matrix can be used for testing the sensitivity of hypotheses' probabilities to introduction of new hypotheses simulating policy or sensitivity to other changes (like adjusting probabilities) in the matrix. On the other hand, if the interpretation is that a priori probabilities are already cross-impacted (see interpretation two in Subsection 3.4.6) significant differences in the a priori and a posteriori probabilities, in Gordon's view, can reflect inconsistencies in the original valuations in addition to the effect of accounting for the higher-order interrelationships [Gordon, 1994, 10]. Regardless of the interpretation of the a priori probabilities, the model is used for illuminating the indirect interactions of the systems and policy analysis, by means of simulating policy by introducing changes to the model.

The INTERAX model [Enzer, 1980] can be understood as an application of Gordon's cross impact method. It aims to make the cross-impact technique more approachable and useful by providing a ready set of hypotheses representing anticipated technical changes, societal changes, economic trends, and various other trends and developments. The case-specific hypotheses of the modeling exercise are added by the analyst to this ready set, or the INTERAX database is modified, and the extended model is evaluated to draw conclusions about the particular strategic issue. A similar general pre-designed cross-impact models describing global, domain-specific or country-specific trends and issues and their assumed interactions could well be developed for AXIOM as well, reducing the cost of setting up a cross-impact study modeling a multitude of general societal and technological trends.

2.3 Godet's SMIC

Michel Godet et al's SMIC method is somewhat similar to Gordon and Helmer's method in terms of inputs and process. Godet et al. emphasize the role of SMIC analysis as a part of a larger framework of futures techniques that are used in conjunction with SMIC [Godet *et al.*, 1994, 143]: SMIC is used to identify the most probable scenarios (represented by the different possible combinations of hypotheses) that warrant a more detailed study with other approaches.

In contrast to Gordon's technique, the valuators provide, in addition to the a priori probabilities and conditional occurrence probabilities also conditional nonoccurrence probabilities. These initial valuations are then corrected to comply with the standard probability axioms. In SMIC, the uncorrected valuations are adjusted into the bounds of the constraints in such way that they remain as close as possible to the original estimates. The probability correction process is outlined by Godet *et al.* [1994, 144–146]. The idea is that a linear optimization function is used to find a solution for the corrections. Godet *et al.* [1994, 149] note that multiple solutions exist and that there is a degree of arbitrariness to the solution that the SMIC optimization function finds. A software implementation exists [Godet *et al.*, 1991, 51] to perform the probability correction. The various approaches to and the technical particulars of the probability correction problems in the valuation of the *a priori* and conditional probabilities of SMIC-like cross-impact matrices have aroused a lot of discussion [Sarin, 1978; Jensen, 1981; Brauers & Weber, 1988; Jackson & Lawton, 1976].

The SMIC calculation process results, in a case of n hypotheses, 2^n scenarios ordered by their probabilities. Godet recommends pulling a set of the most probable scenarios for further analysis with other futures methods. He also suggests deriving an elasticity matrix for the variables on the basis of the cross-impact model to perform sensitivity analysis. The elasticity matrix values represent the change in the probabilities of the impacted variables when the probability of the impacting variable is changed some predefined amount, such as 10% [Godet *et al.*, 1994, 147]. The row sums of the elasticity matrix reflect the total influence of the hypothesis in the system; the column sums reflect the total dependency (elasticity to changes in probability of other hypotheses) of the hypothesis. The sensitivity analysis informs the analyst about which system components represented by hypotheses should be enhanced and which inhibited to steer the system in some desired direction, interpreted as some desirable change in the probability of a target hypothesis or a set of target hypotheses.

2.4 KSIM

Julius Kane has presented an analysis technique called KSIM that is often listed as a cross-impact technique. The KSIM technique does not operate on hypotheses having a probability or that would be evaluated to some state; instead it operates on a set of semi-abstract continuous time-series variables. The values of the variables have a minimum and a maximum defined by the user (in the calculation the values of the variables range from 0 to 1 but these values can naturally be scaled to fit the needs of the analysis). The interpretations of the different values of the variables are left completely to the analyst. The variables are given an initial value and a cross-impact matrix describing the interactions between variables. With this information, the values of the variables can be forecast. [Kane, 1972].

KSIM could be described as a trend-impact analysis technique, as it doesn't really fit the definition of a cross-impact technique formulated in Section 1.1. It is discussed here for the reason that it is one of the few techniques which consider the temporal aspect of the analysis of interactions. It must be noted that the interactions remain unchanged for the entire projection period. The application could easily be amended to periodically change the cross-impact matrix to better simulate a less linear system. KSIM is computationally simple and apparently a flexible technique with wide application area, but the level of abstraction is very high and the interpretations of the variable values and interactions are very subjective; the vague, abstract nature of the variables in KSIM makes modeling systems, interpreting the results of the analysis and drawing clear recommendations somewhat difficult.

2.5 MICMAC and EXIT

Michel Godet's MICMAC® method is used as a part of a larger analytical framework Godet calls "structural analysis". Structural analysis is used to study systems consisting of interrelated elements, highlighting the structure of the relationships. Structural analysis permits analysis of the relationships and identification of the main variables [Godet *et al.*, 1994, 83]. The key variables are identified in structural analysis by using the MICMAC® method. MICMAC® is described as a classification matrix using cross-multiplication factors [Godet *et al.*, 1991, 26]. The MICMAC® classification process takes a cross-impact matrix as input.

This cross-impact matrix can have impact valuations which indicate the strength and direction of the impact, but it can also just have values 0 or 1, 0 indicating no impact from variable to another and 1 indicating an impact of some strength. The sum of the impact values on a row expresses the level of impact a variable has in the entire system. The sum of the impact values on a column tells the degree of dependence of a variable. The variables can be ordered by their general influence or dependency; in MICMAC® , this ordering is the initial ordering.

The MICMAC® approach to extracting information about the indirect impacts is based on squaring the direct impact matrix iteratively. When the cross-impact matrix describing the direct impacts is squared, the indirect impacts are revealed. In the new matrix obtained by squaring the original direct impact matrix, the variables can again be ordered according to the row or column sums like with the direct impacts. The ordering is likely to be different in the power matrix as compared to the original. This squaring of the matrix is repeated and the variable ordering is produced by calculating the row or column sums for each iteration.

As enough iterations have taken place, the ordering becomes stable, and the iteration can be stopped. This stable ordering, which no longer changes as the matrix is squared, is the MICMAC® ordering. The number of required iterations is dependent on the number of variables as well as the number of interactions in the cross-impact matrix.

The MICMAC® method produces an "a posteriori" importance (or dependency) ordering for the variables, which is based on the indirect impacts between the variables; the initial ordering is compared to the a posteriori ordering to highlight the change in the importance of variables. The analytical output is the prioritization of driving forces based on influence-dependence criteria, using the information about the indirect impacts acquired with the MICMAC® technique. [Godet *et al.*, 1994, 96–97].

EXIT (**E**xpress **C**ross **I**mpact **T**echnique) is a cross-impact analysis method developed by the author. The details of the method are described by Panula-Ontto & Piirainen [2016]. It offers a less costly (in terms of expert participation, time and other resources) way to analyse indirect interactions in systems as compared to AXIOM, with reduced analytical possibilities. The valuation phase of EXIT is faster and in many cases possible to be completed in a single-day workshop.

EXIT processes a cross-impact matrix that describes the direct impacts between hypotheses. EXIT doesn't explicitly deal with or calculate probabilities for the hypotheses; a priori probabilities are not needed as input data. The impacts are described as indices that have an interpretation relative to a maximum impact index value in the model: an impact value equal to the maximum impact value means a completely determinative direct impact from one hypothesis to another.

The computational process of EXIT calculates the total (direct and indirect) impacts between all pairs of hypotheses in the model. It starts from the direct impacts between hypotheses and recursively accounts for the effects of longer intermediary impact chains, given that they exceed the analyst-defined minimum impact threshold. This process reveals the role and significance of the system components for the other components and overall system.

EXIT is directly comparable to the MICMAC® method in terms of inputs, but provides more detailed analytical outputs. The improvements of EXIT as compared to MICMAC® are as follows: *a*) soft quantification of influence and dependency of hypotheses instead of simple ordering of hypotheses by influence or dependency, *b*) detailed information about the relationships of hypothesis pairs instead of a ranking based on general influence or dependency in the entire system, and *c*) assessment of the total impact of both direct and indirect impacts instead of providing an alternative ranking based on indirect impacts for the obvious ranking based on direct impacts. As both direct and indirect impacts are important, the cross-impact analysis technique should be able to look at both types of impacts alongside each other and under equal terms [Panula-Ontto & Piirainen, 2016].

2.6 BASICS and JL-algorithm

The BASICS method is described by Honton *et al.* [1984] (see also Huss & Honton [1987]). It processes a cross-impact model very similar to AXIOM, consisting of hypotheses and their possible values, a priori probabilities and a model of direct impacts expressed as references to probability adjustment functions. The process of model evaluation in BASICS method differs from Gordon's cross-impact method and Godet's SMIC method: it doesn't employ a Monte Carlo process and doesn't yield *a posteriori* probabilities for the hypotheses (which are called "descriptors" in the BASICS terminology). Instead, it employs a deterministic process, where the model is evaluated twice for each possible state of each individual descriptor in the cross-impact model, once assuming that the state in question is true and once that the state is false.

Each model evaluation results in all descriptors being assigned a state. The set of states is interpreted as a scenario. The motivation is to find scenarios that are ”probable and consistent” [Luukkanen, 1994, 238] in the light of the supplied *a priori* probabilities and the cross-impact matrix. The sets of descriptor states, or scenarios, that emerge from multiple different evaluations are interpreted as being probable and consistent. The output is one or more scenarios that warrant further study, possibly with other analytical techniques. This process of model evaluation is not a good fit for studying the modeled system from the perspective of the effects of interventions. Possibilities of analysis available in Gordon's method, SMIC or AXIOM are not available in BASICS-type cross-impact analysis. JL-algorithm developed by Jyrki Luukkanen proposes certain technical improvements to the model evaluation procedure of BASICS and could be seen as preferable over BASICS on the basis of the arguments made by Luukkanen [1994, 239].

3 The AXIOM method

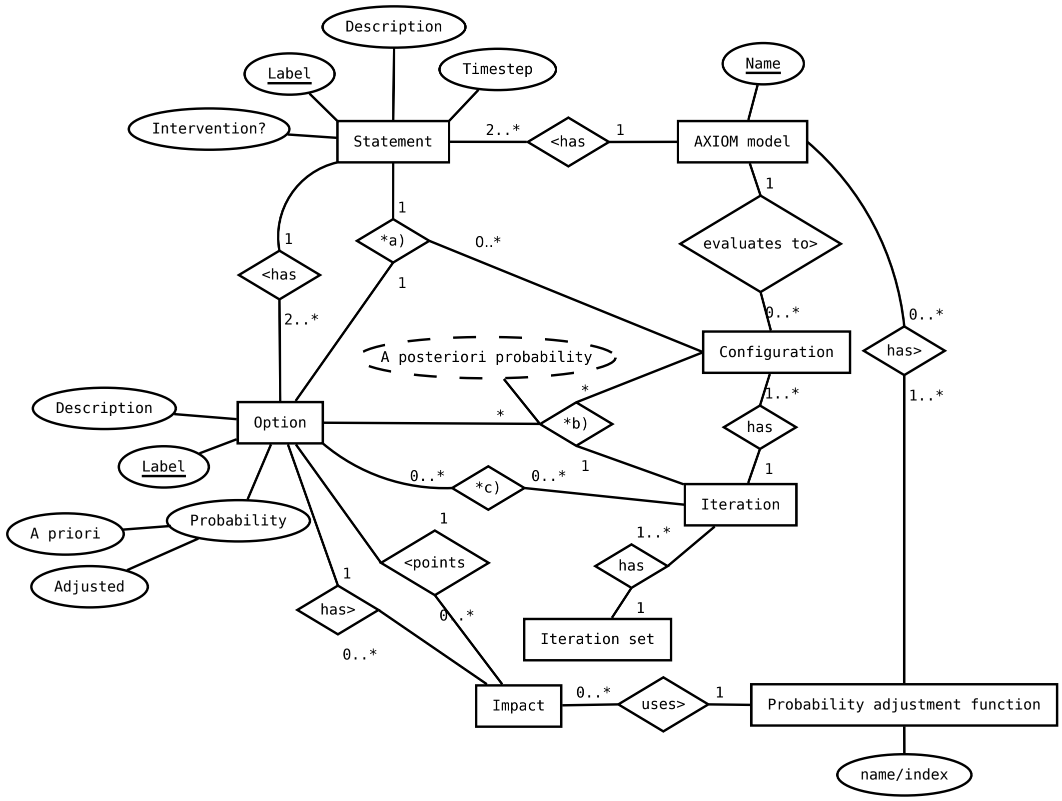
In the AXIOM method the investigated system and its objects are represented as an AXIOM model, using the components and concepts of what could be called the AXIOM modeling language. This chapter presents these components and concepts. It also presents the computational process of its evaluation, deriving AXIOM iterations and iteration sets, and extracting analytical results. After the model concepts, modeling primitives and the computational process have been explicated, Section 3.4 will discuss the process of building the AXIOM model and related considerations.

3.1 The AXIOM model

Before the computational process of AXIOM can be explained, it is necessary to define the AXIOM model and its components and concepts. Cross-impact modeling aims at representing a real-world system with the modeling primitives available in the modeling approach. AXIOM, like other cross-impact techniques, deals with hypothesis-like constructs that are used to describe the possible states of the modeled system. The hypotheses, at the time of model construction, have a yet-unknown state or value, probabilities of occurrence and a set of direct impacts that are conditional to the state of the hypotheses and the model. The modeled system is given a representation, fit for the needs of the analysis at hand, with the concepts or modeling primitives available. AXIOM provides modeling primitives that are not available in other cross-impact techniques but, from a practical point of view, can prove very useful in cross-impact modeling of real systems and research questions.

AXIOM models are comprised of multiple *statements*. Statements have two or more *options* or possible states. Options, in turn, can have *impacts* on other options. Statements also have a property called *timestep*, which reflects their temporal position in the modeled system. Other important concepts concerning the AXIOM cross-impact model are *evaluation* of statements and the model, *configuration* meaning a specific combination of states the statements of the model have, *iteration* as a set of configurations resulting from consecutive model evaluations, *interventions* as model representations of actions influencing the system, and *iteration sets* as collections of iterations with different sets of interventions. All of these concepts are explained in detail in this chapter. Figure 1 presents an

Figure 1 ER model of the AXIOM concepts



***a)** *Statement* is evaluated to an *option* in a single *configuration*
***b)** A *configuration* in an *iteration* has a single *option* for each *statement* in the model; the *a posteriori probability* of each option is the rate of occurrence of the option in configurations in the iteration.
***c)** An *iteration* can have *options* as active interventions

entity-relationship diagram of the AXIOM model concepts. For details on conceptual modeling using the entity-relationship model, see, e.g., Elmasri & Navathe [2011, 49–74].

3.1.1 Statements

Statements in an AXIOM model correspond to what Gordon [1994] calls *events* and what Godet *et al.* [1991] call *hypotheses*. However, AXIOM statements are more flexible: they represent components of the modeled system, and can have multiple possible states whereas events or hypotheses in Gordon's or Godet et al.'s approaches only have a binary state, being either true or false (or undetermined). It could be said that statements collect the different states a system component or aspect can have under one modeling primitive for convenience. Statements should have *a*) a unique, identifying label, *b*) a description detailing what they represent, *c*) a set of options (two options normally being the minimum number), *d*) a timestep value (timesteps are explained in Subsection 3.1.4) and *e*) information whether the statement is to be treated as an intervention (intervention statements are explained in Subsection 3.1.8).

As noted, statements have a set of possible states that are mutually exclusive, and that ideally should be exhaustive and in practice near-exhaustive. Exhaustiveness means here that the states a statement can have in the cross-impact model cover completely the possible states of the corresponding component of the modeled system. In practice, this is rarely possible.

In theory, a statement representing a component of a real system can have innumerable states. Modeling of any system with any modeling technique requires framing of the model to cover only some components of the system, and in the case of AXIOM, framing of the possible states the modeled components are allowed to have. There is no upper limit to the number of states a statement in AXIOM model is allowed to have, but the practical considerations of time allotted to the modeling and cognitive capacity of the experts providing the a priori probability valuations and the impact valuations set an upper limit for the number of states the system components can have. Cross-impact models must have a defined contextual frame or event horizon. This means that they only cover the pertinent parts the examined system (placing a limit on the number of statements). It also means that they leave out many marginal or irrelevant states that the statements might have (limiting the number of options the statements have).

AXIOM statements could in theory be represented by Gordon's events or Godet et al's hypotheses by sets of events or hypotheses that are set to have a cancelling-out effect on each other (as they are mutually exclusive). As many hypotheses in modeled systems have this characteristic of having other hypotheses that they cancel out, AXIOM statements provide a much more convenient modeling primitive for cross-impact modeling than Gordon's events or Godet et al's hypotheses. Conversely, an AXIOM statement can function as a Gordon-style event or Godet-style hypothesis when it is set to have two options, true and false.

Assume an AXIOM model would be built for the purpose of exploring the future of the geopolitical position and alignment of Finland. Examples of model statements representing forces and factors relevant for that alignment could be:

1. Geopolitical position of Finland after 2025.
2. Governing political parties in Finland 2019–2023.
3. Economic development in the European Union in period 2017–2023.
4. Role of the United States in international politics in 2020s.

Each statement, in turn, has a number of options, which are discussed in Subsection 3.1.2. The statements could be refined to be more detailed and accurate, to make the process of expert valuations of the probabilities and the impacts more precise and less ambiguous, but this level of detail might be sufficient for AXIOM statements. For modeling the domain of the future geopolitical alignment of Finland, this set of statements in representing the relevant factors and forces would most certainly be seen by experts of the domain as insufficient.

3.1.2 Options

Options represent the different possible states a system component modeled as an AXIOM statement can have. Every option in an AXIOM model has *a*) one statement that they fall under, *b*) identifying label, *c*) a description about what they represent, *d*) an immutable *a priori* probability, *e*) a mutable, *adjusted* probability valuation, and *f*) a (possibly empty) set of impacts directed to other options in the model.

Normally statements have at least two options. A two-option statement can represent a hypothesis with the possible states *true* and *false*. Naturally, the states of a two-option statement can also have some other interpretation. Statements can have more than two options; there is no upper bound for the number of options. A single-option statement can only be evaluated to have that single option as its state. The motivation for using this kind of single-option statement in AXIOM modeling would be to represent some event or development that is thought to be certain to take place and that the modeler wants to explicitly represent in the model.

The a priori probability is the initial, expert-sourced probability valuation of an option. The a priori probability of an option is interpreted as the probability of the option to become true, as estimated when no other information about the system is available; the a priori probability valuation is given in a context where the states of the other statements are unknown. The initial value of the *mutable* probability valuation of the option is equal to the a priori probability. The mutable probability valuation can change during model evaluation (detailed in Subsection 3.1.6) as impacts of other options directed to the option are realized.

An option can have impacts associated with it. Impacts are directed to other options in the model. When a statement is evaluated (see statement evaluation details in Subsection 3.1.5), and the evaluation results in statement evaluated to an option, the impacts associated with that particular option take effect.

As each option under a statement has both an immutable *a priori* probability and a mutable *adjusted* probability subject to change as the model is evaluated, these probabilities form two probability distributions for the statements. As the options under a statement included in the model are treated as exclusive and exhaustive, the sum of values of both sets of probabilities must be equal to 1 at all times.

The AXIOM options are flexible and can model the possibilities of the modeled system in various ways. It is possible that the different options under a statement embody a very clear and atomic value, such as a number or percentage, or a single boolean fact. The other end of the spectrum would be to load the AXIOM options with an abundance of detail that are seen by the modeler to be associated with each other closely enough to be bundled together in the AXIOM model. This approach makes the options akin to mini-scenarios.

The atomic value approach in setting the content of options results in a model with much more options (and probably more statements). The upside is that the atomic values are much easier to understand and remember for the experts providing valuations for the model. The mini-scenario approach can drastically reduce the number of options and limit the number of statements as well, making the high-level structure of the model easier to comprehend. Conversely, the downside will be the difficulty of understanding the content of each option and the danger of the experts understanding the options in a flawed way. The approaches can be combined in a single model unproblematically, using some statement options sets to express atomic values and some to express bundled values. The flexibility of the AXIOM options as modeling primitives should be used to construct a model that is cognitively inexpensive to valuate. Careful consideration of the model structure and the presentation of the options increase the chances of success of the model valuation.

Expanding the minimalistic example model consisting only of statements presented in Subsection 3.1.1 with examples of options, an AXIOM model dealing with the geopolitical position and alignment of Finland could have the following options as possible values of its statements:

1. Geopolitical position of Finland after 2025
   (a) Member state of the European Union
   (b) Member state of the Eurasian Union
   (c) A non-aligned position.
2. Governing political parties in Finland 2019–2023
   (a) True Finns as the ruling political party
   (b) SDP and Left Alliance as the ruling parties
   (c) Center Party and Coalition Party as the ruling parties
   (d) Rainbow coalition government.
3. Economic development in the European Union in period 2017–2023
   (a) Average economic growth close to zero in the EU area
   (b) Average growth in the EU area slow (close to 1% GDP growth annually)
   (c) Average growth in the EU area fast (close to 3% GDP growth annually).
4. Role of United States in international politics in 2020s
   (a) The United States polices the whole world
   (b) The United States is active in the Pacific region and passive in Europe
   (c) The United States observes an isolationist foreign policy.

The descriptions of the options of this example model are rather vague. They could be refined to be much more detailed and doing so would be very beneficial from the valuation perspective, to give the experts giving the valuations for the model the least varied and ambiguous understanding of the meaning of the options as possible. These options can, however, serve as examples of AXIOM model options as their a priori probabilities and impacts on other options could probably be estimated with some accuracy by domain experts.

3.1.3 Impacts

An AXIOM impact consists of the option the impact belongs to, the option it is directed to (and whose probability the impact changes), and a probability-adjusting function. Impacts are linked to specific probability adjustment functions, that are used in determining what the probability of the impacted option will be after the adjustment. An option in a statement can have zero or more impacts, directed to options under other statements than the one the impacting option belongs to. The experts providing the impact valuations express the impacts of options on other options as names or indices of probability adjustment functions. The probability adjustment functions will change the probabilities of the impacted options contextually; probability adjustment functions are described in detail in Subsection 3.2.1.

Expressing the interactions between model components as impact indices or names pointing to probability adjustment functions is, for the valuation-providing experts, cognitively much easier than providing exact conditional probability valuations. The contextual nature of the probability adjustment makes it possible to still calculate probabilities and provide the analytical benefits of explicitely considering probability in the cross-impact analysis.

This reduced cognitive cost of providing interaction strength valuations makes it possible, for the expert valuators, to valuate a cross-impact model with a much bigger number of components. This is essential for the fitness of the cross-impact method for actual research questions and modeling tasks. The cross-impact modeling is most valuable in cases where the modeled systems are complex and the interaction web is extensive, making the possible impact chains in the model long. In cases like this, the cross-impact analysis can reveal surprising and unintuitive characteristics of the modeled system. The cross-impact analysis method should enable the construction of fairly large-scale models, comprised of numerous components. This can be seen as a very strong selling point for AXIOM as a cross-impact modeling tool in comparison with the methods operating on exact conditional probabilities.

3.1.4 Timestep

Timestep is a property of AXIOM statements, used to represent the temporal dimension in an AXIOM model. It expresses the time category and evaluation precedence of the statement: Statements with a lower temporal category are always evaluated before statements with a higher temporal category. Details of statement evaluation order are presented in Subsection 3.1.6. If a statement represents something in the modeled system that is expected to happen only after some other events or phenomena represented by other statements have occurred or resolved, it is given a higher temporal category. Timestep values can be years, if that makes sense for the modeler, but they can also be consecutive integers; only the numeric ordering is of consequence in the model evaluation.

The possibility of modeling the temporal aspect of events and hypotheses in the modeled systems with the timestep property is a feature not present in most cross-impact techniques. This clearly increases the modeling power of the cross-impact analysis approach. This added modeling power can reasonably be expected to be very useful as the utilization area of cross-impact models is in most cases the modeling of the future of systems, where the temporal aspect is of great importance.

3.1.5 Statement evaluation

Statement evaluation gives a state to an unevaluated statement: after evaluation, the state of the statement becomes equal to one of its options, according to the *adjusted* probability distribution of the options of the statement. All statements in an AXIOM model are evaluated exactly once during a single *model evaluation* (detailed in Subsection 3.1.6).

Each option in an AXIOM model can cause zero or more impacts. When the statement's state is determined, the impacts caused by the option that is now the evaluated state of the statement take effect. The probabilities of the options that are the targets of these impacts are now adjusted contextually as detailed in Subsection 3.2.1.

A random real from the interval [0..1] can be used in a simple way to evaluate an AXIOM statement. The options of a statement have a natural ordering which can be based on the alphabetical ordering of the labels or identifiers of the options. A random number is drawn; if it is smaller than the probability of the first option, that option is assigned as the state of the statement. If the random number is equal to or greater than the first probability, the probability of the next option is added to the probability of the first statement and the same test is performed. Probabilities of consecutive options are added to the sum until the probability sum of all the included options is greater than the random number. When this condition is met, the option whose probability was added last is assigned as the state of the statement.

3.1.6 Model evaluation

Model evaluation refers to the process of iterating through all the statements of the model and performing the *statement evaluation* to them, determining the state of the statements, resulting in a single *configuration*. In a simple case where all the statements in the model belong to the same temporal category (i.e. have the same *timestep* value) all statements are placed in a random order (the random ordering is generated for each model evaluation) and then evaluated in this order. This random order of evaluation of temporally equal statements is important for eliminating the effect of input order of the statements.

In a more complex case, the statements fall into different temporal categories. The statements in each temporal category are randomly ordered within their category and evaluated. The different time categories are processed in an ascending order: the statements in temporal categories with a higher timestep value are evaluated only after the statements in temporal categories with a lower timestep value have been evaluated. This means that the statements modeled to have their state determined later cannot have effect on the probability distribution of the statements modeled to be determined earlier as those statements are guaranteed to already have been assigned a state.

3.1.7 Configuration

Model evaluation results in a *configuration*. The information content of a configuration is a set of options, one option for each statement in the model. The options in the configuration are the options evaluated to be the states of each of the statement in a single model evaluation. Configuration can be understood as a scenario for the modeled system. As the model is evaluated multiple times, the resulting sets of configurations or *iterations* can be used for deriving higher-order information from the cross-impact model. This is described in Subsection 3.1.9.

3.1.8 Intervention statements

In AXIOM model construction, certain statements can be flagged as intervention statements. These statements and their options represent the policies or actions of some actor modeled in the system. Most typically this actor will be the entity, organisation, party or stakeholder undertaking the cross-impact modeling as a part of their strategic planning. In such a case, the intervention statements might represent the strategic options this entity has in its disposal.

It is also possible to use the intervention statements to model several actors with their own strategic courses of action. Cross-impact modeling of the strategies of several actors involved with the same system could be useful in identifying a scheme of coordinated actions by the involved actors, that would result in an outcome in the system that would be desirable for all involved entities; this mode of using cross-impact technique is hinted at in the review by Matic & Berry, already mentioned in Section 1.2. Modeling the actions of several actors into a cross-impact model could naturally also be done to test strategic choices of a particular actor against the strategic choices of opposing or competing actors.

The statements flagged as intervention statements are not evaluated as normal statements. They rather have a predetermined state that is assigned when the model evaluation commences. Even when the state of the intervention statements is already assigned at the start of the model evaluation, the impacts of the assigned state are not realized immediately at the first step of the model evaluation. The intervention statements have their timestep property and they are processed in the normal order of evaluation determined by that timestep (see model evaluation details in Subsection 3.1.6).

The states of the intervention statements change only between different *iterations* (iterations are covered in Subsection 3.1.9). The idea is to run a single iteration with a specific combination of options as the states of the intervention statements. The *a posteriori* probabilities of the options are computed as their frequencies of occurence in the *configurations* of an *iteration*. They reflect the impacts of the specific combination of policy actions. Policy actions are represented by the combination of intervention statements' states in that iteration.

3.1.9 Iteration

An iteration is a set of configurations; model evaluation results in a *configuration* and several consecutive model evaluations produce an *iteration*. The utility of iterations is to be able to calculate the frequencies of different model options from a set of configurations with identical characteristics (same interventions, same model valuations etc.). The frequency of occurrence of each option in an iteration is the *a posteriori* probability of that option. The a posteriori probability is the probability of the option when the cross-impacts present in the modeled system have been factored in.

In order for an iteration to be useful for its intended purpose, the configurations it consists of must meet certain requirements. These are the following:

1. Configurations must be derived from the same model (the model structure in terms of statements and options must be identical for all configurations).
2. Configurations must have the same valuations (the a priori probabilities of the options, the impact relationships and the probability adjustment functions pointed by the impacts must be identical).
3. Configurations must have the same options as active interventions (all the statements flagged as interventions must have the same predefined state).

The purpose of these requirements is to ensure that the information extracted from an iteration is meaningful; if the iterations would comprise of configurations derived from models that have different structure and valuations, the a posteriori probabilities would not make sense. If the active interventions would be different in different configurations of the same iteration, it would be impossible to appraise the effects a certain combination of interventions has as the results would reflect the outcomes of different intervention combinations.

The number of configurations in an iteration is not defined by the AXIOM method. The more configurations iterations have, the less the randomness of the Monte Carlo process effects the option frequencies. That is why a high number of configurations is recommended. For iterations that will be used for extracting final results to be analysed, the number of configurations should be at least 10^6. As the number of statements and options in an AXIOM model grows, a greater number of configurations in an iteration to eliminate the random component in the option frequencies is required.

In addition to the rudimentary information of the a posteriori probabilities, the iterations can be used to examine the occurrence of options in more refined ways. These are discussed further in Section 3.5.

3.1.10 Iteration set

Iteration set is simply a set of iterations. The function of iteration sets is to enable comparisons between different model setups. The different model setups can mean inclusion and exclusion of different statements and options, different a priori probability or impact valuations, and using a different set of options as the intervention combination. A key mechanism offered by the AXIOM software implementation (see Section 5.1) is the automatic creation of an iteration set containing an iteration for each intervention option combination derivable from the flagged intervention statements. This facility makes it straighforward to investigate how the policy actions modeled by the intervention statements and their options affect the a posteriori probabilities of model options.

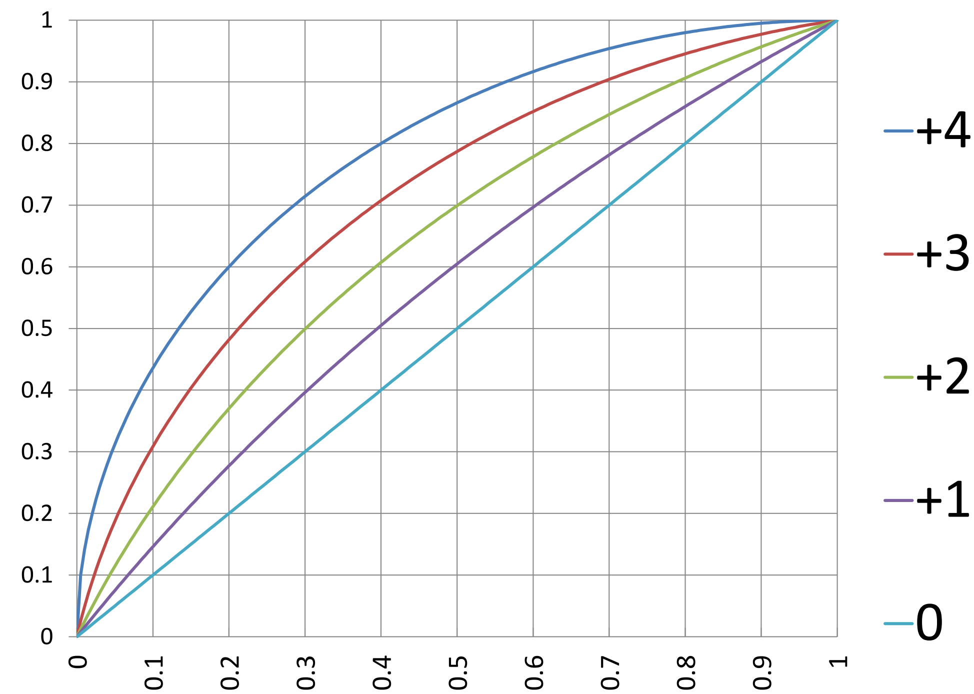
Beside comparing the effect of different intervention option sets, the analyst might be interested in examining the effect of different a priori probability valuations or impact valuations. Computing iterations with different valuations can be used, say, for sensitivity analysis. In a case where the model structure is changed by eliminating or adding options or statements, the comparisons of a posteriori probabilities of options, probabilities of specific configurations or occurrence frequencies of smaller option sets can be done only for the options present in both model setups from which the compared iterations have been obtained.

3.2 The AXIOM computational process

3.2.1 Probability adjustment and probability adjustment functions

As explained in Subsections 3.1.3 and 3.1.5, evaluation of statements results in statements being assigned a state, which is one of the options of the statement. The option might have impacts directed to other options in the model assigned to it, and those impacts are realized when the option has been assigned as the state of the evaluated statement. The impacts change the probabilities of the options they are directed to, according to the probability-adjusting function the impacts point to.

Figure 2 Examples of probability adjustment functions.



The probability adjustment functions change the probability of the targeted options in a contextual way. This normally means that, for instance, an option that already has a high probability will have its probability increased less (relative to its current adjusted probability) than a statement with low probability. This, however, completely depends on the used probability adjustment functions. Figure 2 shows the graphs of four probability adjustment functions.

The probability adjustment functions used by AXIOM models can vary, but they must have a domain of $[0, 1]$ as well as a codomain of $[0, 1]$. In addition to this, they should adjust probabilities in a way that is easy to understand and coherent from the perspective of the model valuators. For this purpose, it is probably a useful feature of a probability adjustment function to be symmetric about the line $y = -x + 1$. The probability adjustment functions in Figure 2 have this property.

When the adjusted probability of an option of an AXIOM statement is changed according to the used probability-adjusting function, the probabilities of the other options under the impacted statement must be adjusted too. We can call the probability adjustment of the option that is the target of the impact *primary probability adjustment* and the probability adjustment of all the other options under the impacted statement *secondary probability adjustment*. The secondary probability adjustment is necessary because the sum of the probabilities of the options must always be equal to 1. The adjusted probabilities of all the other options of the impacted statement are calculated as per Algorithm 3.

The basic idea is that the probability of the impacted option is changed according to the probability adjustment function pointed by the impact, and the probabilities of the other options change so that their summed probability is equal to the complement of the new adjusted probability of the impacted option and the probability share each of these other options gets out of that summed probability is equal to their share of their summed probability before the probability adjustment. When the other options under the same statement as the impacted option is have their probabilities adjusted in this way, the total sum of the probabilities of all the statement's options remains equal to 1.

3.2.2 Computing an AXIOM configurarion, iteration and iteration set

This subsection presents the pseudocode detailing the computational procedures of evaluating an AXIOM model and computing iterations and iteration sets. A step-by-step computation example is presented in Section 3.3.

Algorithm 1 AXIOM model evaluation

```
function EVALUATE MODEL(AXIOM Model m): Configuration c
    for all unique timestep values t in m from lowest to highest t do
        ss ← statements in m that have timestep t
        SHUFFLE(ss)                          ▷ Place statements in random order
        for all Statement s in ss do
            EVALUATE STATEMENT(s)
            add s.state to c
    return c
```

The model evaluation (Algorithm 1) results in a configuration. The statements are divided into temporal categories on the basis of their timestep values. The categories are processed in order from the lowest to the highest timestep value. The statements within each category are evaluated in random order. After evaluation, one of the options of the evaluated statements becomes its state, and this option is added to the configuration, which is a set of options.

Algorithm 2 AXIOM statement evaluation

```
procedure EVALUATE STATEMENT(Statement s)
    if s is an intervention statement then
        s.state ← s.model.activeIntervention(s)
    else
        r ← random real from the interval [0,1]
        sum ← 0
        for all Option o in s.options do
            sum ← sum + o.adjustedProbability
            if r ≤ sum then
                s.state ← o
    is ← SHUFFLE(s.state.impacts)
    for all Impact i in is do
        EXECUTE IMPACT(i)
```

The statement evaluation procedure (Algorithm 2) *a*) assigns a state for the statement, and *b*) for each impact the assigned state has, calls the procedure to effectuate the impact. The intervention statements have a predefined state in the model being evaluated, so they are simply assigned that predefined state; other statements are evaluated to one of their possible options according to the adjusted probability distribution of the statement's options. Impacts are placed in random order (shuffled) before being executed; this is to eliminate the effect the impact order might have on model evaluation results over the course of multiple model evaluations.

The procedure of impact execution is presented in Algorithm 3. The probability of the option targeted by the impact is adjusted according to the probability adjustment function pointed by the impact. The probabilities of the other options under the same statement as the targeted option have their probabilities adjusted as well, to ensure the sum of the probabilities of the option set remains equal to 1. The complement probability of the new adjusted probability of the target option is divided to the other options so that each option's share of the new complement probability remains equal to their share of the old complement probability.

The computation of an iteration (Algorithm 4) simply consists of performing the

Algorithm 3 AXIOM impact execution

```
procedure EXECUTE IMPACT(Impact i)
    P_new ← i.ADJUSTMENTFUNCTION(i.target.adjustedProbability)
    P_complement ← 1 − p_new
    for all Option o in i.target.statement do
        if o is i.target then
            o.adjustedProbability ← P_new
        else
            os ← i.target.statement.options where option is not i.target
            share ← o.adjustedProbability / (sum of adjusted probabilities of Options in os)
            o.adjustedProbability ← P_complement × share
```

Algorithm 4 AXIOM iteration computation

```
function COMPUTE ITERATION(Model m, iteration_count) : Iteration
    for 1 to iteration_count do
        Configuration c ← EVALUATE MODEL(m)
        add c to Iteration i
    return i
```

model evaluation multiple times and saving the resulting configurations to the iteration. The model structure, valuation and its active interventions remain the same for each model evaluation during the computation of an iteration. This is to ensure that the calculation of a posteriori probabilities for the options as the occurrence frequency of each option in the configurations of the iteration remains a meaningful operation.

Algorithm 5 AXIOM iteration set computation

```
function COMPUTE ITERATION SET(Model m, iterationCount): IterationSet
    repeat
        add COMPUTE ITERATION(m, iterationCount)
        m.NEXTINTERVENTIONCOMBINATION
    until all possible intervention combinations of m have been processed
```

One possible computation procedure for creating an iteration set is expounded in Algorithm 5. This algorithm iterates through all the possible intervention combinations, resulting in an iteration set in which each iteration and the a posteriori probabilities derivable from that iteration reflect the effects of that specific combination of interventions on the modeled system.

The iteration set could be created also by doing some other changes in the model instead of going through all the option combinations of statements flagged as interventions. Those changes could concern the probability and impact valuations, impact structure and even introducing new options or statements or removing them; the study of the effects of changes in the system could be also done in this way. The approach of studying the system with the help of intervention statements can be seen as a modeling convenience in AXIOM.

3.3 Example of model evaluation

This section details a single AXIOM model evaluation with example data resulting in a single *configuration*. The AXIOM model used in the example uses five probability adjustment functions to express the impacts between the options of the model. These functions have names +2, +1, 0, −1 and −2. Function +2 has the greatest probability-increasing effect; function −2 has the greatest probability-decreasing effect. Function 0 is used for expressing no impact between the options. Figure 3 displays the graphs of the probability-adjusting functions.

The following equations

$$P_{\text{adj}(+2)}(P) = P_{\text{adj}}(P, 1.1003) \tag{1a}$$

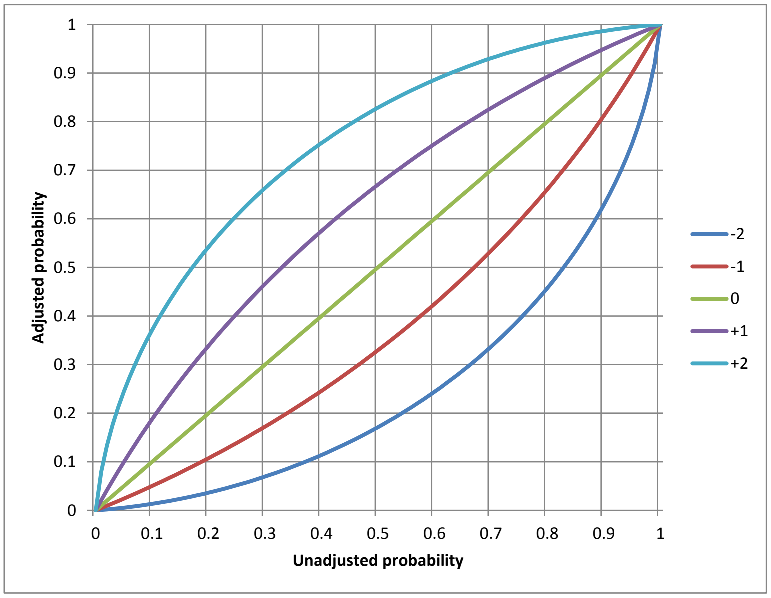
$$P_{\text{adj}(+1)}(P) = P_{\text{adj}}(P, 2.0778) \tag{1b}$$

$$P_{\text{adj}(0)}(P) = P \tag{1c}$$

$$P_{\text{adj}(-1)}(P) = 1 - P_{\text{adj}}(1 - P, 2.0778) \tag{1d}$$

$$P_{\text{adj}(-2)}(P) = 1 - P_{\text{adj}}(1 - P, 1.1003) \tag{1e}$$

Figure 3 Probability adjusting functions of the example model evaluation.



where

$$P_{\text{adj}}(P, r) = \sqrt{r^2 - (P - x_0)^2} + y_0$$

and where

$$y_0 = -(r * \cos(\beta) - 1)$$

$$x_0 = r * \sin(\beta) + 1)$$

$$\beta = \frac{\pi}{4} - \frac{\alpha}{2}$$

$$\alpha = 2 * \arcsin(\frac{1}{\sqrt{2} \times r})$$

define the five probability-adjusting functions from $+2$ to -2 used in the model of this example. Equation 1a (respectively, equations 1b, 1c, 1d, 1e) defines the function with name "+2" (respectively, "+1", "0", "−1", "−2").

Table 2 presents the statements of the example model. Statements A, B and C are in temporal category 1, and statements D and E are in temporal category 2. This means that statements A, B and C will be always evaluated and have their impacts in the model before D and E are evaluated. Statement C is flagged as an intervention statement. As the states of intervention statements are predetermined for the duration of a single model evaluation which results in a configuration, we assume that for this particular model evaluation, statement C has the predetermined state C1, meaning its first option.

Table 2: Details of statements of example model

Statement name	Timestep	Intervention?	Option count
A	1	No	2
B	1	No	4
C	1	Yes	2
D	2	No	3
E	2	No	2

Table 3 presents the cross-impact matrix of the AXIOM model from which a single configuration is derived in this example. The impact (expressed as the name of an probability-adjusting function) of each option on other options can be read from the rows of the matrix. The impact of statement A's option 1 on statement C's option 2 is the value (2) in the cell on row 1, column 8; the impact of statement B's option 4 on statement E's option 1 is the value (0) in the cell on row 6, column 12.

The adjusted probabilities of all options in the example model in different stages of the model evaluation are presented in Table 4. The second row of the table shows which impact has been processed and the probability values below what are the adjusted probabilities after processing that impact. For example, impact A1:C1(-2) refers to impact of option A1 on option C1, where the probability adjustment is made according to probability adjustment function -2. The second column of the table shows the a priori probabilities of the options; the last column

	A1	**A2**	**B1**	**B2**	**B3**	**B4**	**C1**	**C2**	**D1**	**D2**	**D3**	**E1**	**E2**
A1	∅	∅	0	0	0	0	-2	2	0	0	0	0	0
A2	∅	∅	1	1	0	0	0	0	0	0	0	0	0
B1	0	0	∅	∅	∅	∅	0	0	0	-1	0	0	0
B2	0	1	∅	∅	∅	∅	0	0	0	0	1	2	-2
B3	0	0	∅	∅	∅	∅	2	0	0	0	0	1	0
B4	2	1	∅	∅	∅	∅	0	-1	1	0	0	0	0
C1	0	0	0	0	1	0	∅	∅	0	0	-2	1	0
C2	0	1	-1	2	0	0	∅	∅	1	0	0	0	0
D1	0	0	0	-1	0	0	0	0	∅	∅	∅	0	1
D2	0	0	-1	0	2	0	0	0	∅	∅	∅	0	0
D3	2	0	0	0	0	0	0	0	∅	∅	∅	1	1
E1	-2	0	0	0	0	1	0	0	0	-1	0	∅	∅
E2	2	0	2	-2	0	1	0	0	0	-2	0	∅	∅

Table 3: Cross-impact matrix describing the direct impacts between model options.

shows the result of the computation, the *configuration*, where each statement has a state and each option a truth value (the option with the truth value *true* being the evaluated state of its statement).

		Evaluation of A		Evaluation of C		Evaluation of B			Evaluation of E	Evaluation of D
	apriori	**A1:C1(-2)**	**A1:C2(+2)**	**C1:B3(+1)**	**C1:D3(-2)**	**B2:D3(+1)**	**B2:E2(-2)**	**B2:E1(+2)**	**E1:D2(-1)**	**Configuration**
A1	0.559	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
A2	0.441	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
B1	0.297	0.297	0.297	0.296	0.296	FALSE	FALSE	FALSE	FALSE	FALSE
B2	0.328	0.328	0.328	0.327	0.327	TRUE	TRUE	TRUE	TRUE	TRUE
B3	0.003	0.003	0.003	0.007	0.007	FALSE	FALSE	FALSE	FALSE	FALSE
B4	0.372	0.372	0.372	0.371	0.371	FALSE	FALSE	FALSE	FALSE	FALSE
C1	0.727	0.366	0.097	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
C2	0.273	0.634	0.903	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
D1	0.114	0.114	0.114	0.114	0.336	0.220	0.220	0.220	0.233	FALSE
D2	0.053	0.053	0.053	0.053	0.157	0.103	0.103	0.103	0.052	TRUE
D3	0.833	0.833	0.833	0.833	0.507	0.676	0.676	0.676	0.715	FALSE
E1	0.244	0.244	0.244	0.244	0.244	0.244	0.600	0.886	TRUE	TRUE
E2	0.756	0.756	0.756	0.756	0.756	0.756	0.400	0.114	FALSE	FALSE

Table 4: The adjusted probabilities of options in the course of model evaluation

The evaluation of the example model starts with evaluating statements A, B, and C as they are in the lowest temporal category (their timestep value is 1). Statements D and E will be evaluated after statements A, B, and C have been assigned states. Statements with timestep 1 are placed in random order among themselves: assume this random ordering is A, C, B: the model evaluation starts with evaluation of statement A.

Option A1 has the a priori probability 0.559; option A2 has the a priori probability 0.441. As this is the first statement evaluation, no probability adjustments resulting from impacts have yet taken place and the mutable adjusted probabilities are equal to the a priori probabilities. A random real is used in determining the state of statement A. The use of randomness in statement evaluation is explained in Subsection 3.1.5. Assuming the random number is 0.486, statement A is evaluated to state A1.

The impacts of option A1 are now realized. A1 has two impacts: impact on option C1 by probability adjustment function -2 and impact on option C2 by probability adjustment function $+2$. At this point it must be noted that since statement C is flagged as an intervention statement, it already has a predefined state (in this example, that state is C1). Calculating the probability changes for options C1 and C2 is not necessary, as they have no effect on the resulting configuration; statement C will, upon its evaluation, be assigned its predefined state C1. The probability changes for options C1 and C2 are, however, calculated to illustrate the computation process. As option A1 has more than one impact, its impacts must be processed in random order. This is done to eliminate the effect that the input ordering of impacts might have on the results; as the impacts are processed in random order, the influence of the ordering of the impacts will become insignificant over the course of multiple model evaluations. We assume that the random ordering for impacts is C2, C1.

The probability of C2 is 0.273; after probability adjustment with function +2 the adjusted probability of C2 becomes 0.634. To preserve a valid probability distribution for the option set of statement C, the probability of C1 needs to be adjusted as well. The adjustment of the complementary options or the *secondary adjustment*, i.e., the other options of the statement than the one the impact being processed is directed to, is presented in Algorithm 3.

The adjusted probability of C1 is now, as an indirect result of the probability adjustment of C2, 0.366; after probability adjustment with function −2 the adjusted probability of C1 becomes 0.097.

Statement C is the next statement to be evaluated. As C is flagged as an intervention statement, it is not evaluated according to the adjusted probability values of this point in the model evaluation, but it has a predefined state, C1. C1 has impact +1 on option B3 and impact −2 on D3. Let the random ordering of impacts be B3, D3. B3 has probability 0.003; it is adjusted with function +1 to 0.007. The probabilities of B1, B2 and B4 are adjusted according to Algorithm 3 to 0.296, 0.327 and 0.371. D3 has the probability of 0.833; it is adjusted with function −2 to 0.507. The probabilities of D1 and D2 are adjusted to 0.336 and 0.157.

Statement B is the final statement in the first temporal category to be evaluated. Let the random number used in statement evaluation be 0.346; statement B is evaluated to B2. B2 has four impacts: A2(+1), D3(+1), E1(+2), E2(−2). Let the random ordering for the impacts be D3, A2, E2, E1. D3 has the probability of 0.507; it is adjusted with function +1 to 0.676. Probabilities of D1 and D2 are adjusted to 0.220 and 0.103, respectively. Statement A has already been evaluated and assigned a state, so the impact of B2 on A2 can be disregarded. E2 has the probability of 0.756 which is adjusted by −2 to 0.400. The probability of E1 is (as a secondary adjustment) adjusted to 0.600. Finally, the probability of E1 is (as a primary adjustment) adjusted by +2 to 0.886, and the probability of E2 (as a secondary adjustment) to 0.114.

As the statements of the first temporal category have now been evaluated, we continue with evaluating the second temporal category. Suppose the random ordering of the statements with timestap 2 is E, D. Statement E is evaluated first. Let the random number for this statement evaluation be 0.074; statement E is evaluated to option E1. Option E1 has three impacts: on A1 by−2, on B4 by +1, and on D2 by −1. Let the random ordering of the impacts be D2, B4, A1. D2 has the probability of 0.103 which is adjusted by −1 to 0.052. Probabilities of D1 and D3 are secondarily adjusted to 0.233 and 0.715. Statements B and A have already been evaluated, so it is not necessary to compute the probability changes resulting from impacts to them.

Finally, statement D is evaluated. Suppose the random number used in evaluation of statement D is 0.276; statement D is evaluated to D2. D2 has two impacts, on options B1 and B3, but since D is the last statement in this model evaluation, all the other statements already have been evaluated and been assigned a state.

The model evaluation has resulted in all of the model statements being evaluated to one of their options: statement A to A1, statement B to B2, statement C to C1, statement D to D2 and statement E to E1. This *configuration* resulting from model evaluation can be interpreted as a scenario.

3.4 Considerations for AXIOM modeling

Section 3.1 and subsection 3.2.2 have presented the concepts of AXIOM and the details of the computational process, and Section 3.3 has presented the steps of an AXIOM model evaluation with example data. While the data transformation process of the input data may be formal and exact, it is important, from the point of view of utilization of the analytical results, to understand the approximate nature of the input data (which causes the output data to have an approximate nature as well), and what can be done to improve the quality of it.

3.4.1 Data source

Experts are the source of data in cross-impact analysis. Experts of the fields relevant to the modeled domain or system should preferably be used in all stages of the modeling process preceding the actual computation, and equally preferably in the analysis of the results. Experts should be involved in *a*) building a conceptual model of the system as a basis for further modeling steps, *b*) formulating a representation of the system as AXIOM statements, c) formulating the options for the statements and consideration of the inclusion and exclusion of options, d) probability valuations, e) desing of the impact network, and f) impact valuations. The level of expert involvement in each stage is largely a question of resources and the level of commitment of the involved experts. The best practices of organizing the expert work in cross-impact modeling are not in the main focus of this thesis, but the most important and considerations are addressed in this section. Discussion related to the use of experts in cross-impact analysis in real research cases can be found e.g. in Alizadeh *et al.* [2016]; Blanning & Reinig [1999].

3.4.2 Accuracy of expert valuations

Gordon's original cross-impact method and Godet et al's method place great emphasis on extracting a consistent cross-impact matrix of conditional probablities from the used expert group. The experts are usually unable to produce a mathematically consistent matrix of cross-impacts in a single iteration. Software tools exist to assist the expert group providing valuations in adjusting the probabilities so that the changes to the initial conditional probability valuations can be made more easily and a mathematically consistent valuation can be found. It is possible that in the course of this iterative valuation, the original idea and higher abstraction level model of the experts about the interactions between the different model components becomes obfuscated; in the worst case the process might turn into an abstract number-placement puzzle whose ultimate outcome no longer reflects the experts' understanding of the interactions within the modeled system.

The perspective on the expert valuations taken in AXIOM is different from the one taken in Gordon's cross-impact analysis and Godet et al's SMIC. The basic assumption in AXIOM approach concerning the valuations is that the expert valuators are able to estimate the a priori probabilities, map out the model interactions, and valuate their strength on an approximate and rough accuracy, not with a two- or three-decimal precision. The value of the analysis is not in the ability to calculate exact a posteriori probabilities but to be able to observe the direction of change in the target statements and options. The analytical focus should, in the opinion of the author, be more on how the impact network functions and how interventions and actions translate to the system over the impact network and less on the exact individual a posteriori probabilities. The effects of the system's impact network is the area where a cross-impact computation tool provides the most added value as these effects are hard to evaluate intuitively, without a systematic computation process.

3.4.3 Selection of experts

As the experts are the sole data source, their competence and ability to contribute to model building and valuation are of utmost importance. Individual experts should have high competence in one or more aspects of the modeling and valuation effort. They preferably should also feel some ownership towards the project as the cost of the model building and valuation in time and effort is relatively high.

As the modeled systems are often big and consist of complex subsystems, the cross-impact modeling effort requires expertise in various fields. It would be unrealistic to expect a single expert to possess all the necessary expertise, as it would to expect that the combined knowledge of a group of experts of a single discipline would cover all the information requirements of the modeling. This is why ideally the model building and valuation should use panels of experts with the best possible coverage of expertise: The experts should jointly have the relevant competences and there should be more than one expert in the group with high competence for each relevant field of expertise. Some authors (see e.g. [Godet *et al.*, 1991, 49]) recommend using more than 100 experts for valuation, although it must be noted that the case for which this recommendation is made is a model of only a handful (6–10) of hypotheses to be valuated. If there is only one expert in the expert group with expertise on a specific issue relevant to the modeling, it is possible that the expert in question might have some marginal views that bias the valuations. It is beneficial to have some triangulation of expert views. The selection of statements and options and the valuations should take place after discussion in the expert group. Facilitation of this type of group work is challenging and a very important success factor to the model design phase and the whole cross-impact modeling project.

3.4.4 Extracting the AXIOM model from expert valuations

The expert group work might be organized so that it leads to valuations that can be directly used as model input. This would be the case if the expert group is able to come to consensus about all choices of the model design. The group might agree right away or reach consensus after argumentation and discussion. If a consensus is not reached, the group can take a vote to resolve which design choices or valuations should be used in the AXIOM model; the design choices used in the model can be in line with the most popular expert view or in case of valuations valuated by some measure of central tendency. Another way of making design choices in case of divergent expert views can be selecting a particular line of reasoning that some fraction of the experts might have, and building the model on that basis. In this case, a choice is made between several underlying conceptual or theoretical models.

The valuation part of the AXIOM model building can also be based on expert survey. A simple and fairly logical strategy of dealing with divergent valuations in a case where the inputs are collected by means of a survey can be averaging the valuations. If, for instance, the impact valuations have different impact directions (some experts having evaluated the effect of a hypothesis A on hypothesis B as probability-decreasing and others probability-increasing), the valuations will more or less cancel each other out and the disputed interaction will not have a lot of impact in the model. Another strategy would be to identify the valuations where the variability is high and bring those parts of the model to further scrutiny by the expert group in hope of finding a consensus on the disputed issues.

3.4.5 Contextual frame, time horizon and event horizon

It is very important that the experts used in model design and valuation understand the aims and the focus of the cross-impact modeling exercise and the information content of the statements and options clearly and as similarly as possible. Defining a clear *contextual frame*, a *time horizon* and *event horizon* for the modeling effort will help in the group work of the model design. A contextual frame explains on a general level what is the aim and focus of the model, and presents the information that is common and shared for all model statements and options. Explicating the contextual frame of the modeling can cut the information content of the statements and options, reducing the cognitive cost involved in model valuation: the common information of the model doesn't need to be loaded to the statements and options. Time horizon explains the time frame in which the model hypotheses are resolved. The valuators need to understand the time horizon precisely and in the same way as it can change the probability valuations greatly: if the time horizon is seen as near-infinite or e.g hundreds of years long, a hypothesis with very low probability in a time frame of 10 years could easily be thought to become very probable to occur at some point of that very long time frame. AXIOM models can have several temporal categories for statements; the interpretations of each category needs to be clearly defined for the valuators to have an unambiguous understanding of the valuated items. Event horizon draws boundaries on which kind of possibilities are considered in the modeling effort. In the contextual frame of a cross-impact model, some possible but unlikely or marginal developments can be seen as unimportant for the goals of the planning and decision-making that cross-impact modeling supports: in this case, they fall outside the event horizon of the modeling. Unlikely events or changes that would completely transform the studied system or the situation and goals of the actors in it should probably be placed outside the event horizon especially if there is no way to prepare to them. Event horizon is useful in the model design phase, in making judgements about which hypotheses and options are included and which excluded.

3.4.6 Nature of a priori probability

The nature of a priori probabilities of hypotheses in cross-impact analysis can be understood in two different ways. The first way is that the a priori probability of a hypothesis is the probability of the hypothesis occurring in isolation, i.e. when all other hypotheses have not occurred. The second way is that the a priori probability is the probability of the hypothesis when no information about the state of other hypotheses is available. The different interpretations of a priori probability are also discussed by Gordon [1994, 4]: he sees that the second interpretation of the a priori probability leads to using the analysis for checking the coherence of expert valuations of the initial and conditional probabilities.

The first way of understanding the a priori probability, in a cross-impact model consisting of n hypotheses, requires that the probability valuator considers the states of n hypotheses simultaneously while providing the probability valuation. In effect, the valuator gives a probability of an entire scenario. This might be feasible in a case where the number of hypotheses is low. Whether the absolute maximum number of hypotheses for a human evaluator in this type of probability valuation is thought to be 5 or 20 or even 35, it presents a serious limitation to the modeling power and possibilities of cross-impact modeling. The upside of this interpretation of the a priori probability is that it makes, in theory, the probability valuations exact (as the valuated item is known exactly). As Gordon [1994, 10] states, with this interpretation of a priori probabilities (and assuming a high accuracy of the model valuations) the cross-impact model could in theory be used for fairly precise sensitivity analysis. Understanding, however, the difficulty of providing probability valuations of future events of changing systems (and the impossibility of observing how exact those valuations were even after the fact), the use of cross-impact analysis for precise sensitivity analysis seems like a pipe dream.

The second way of understanding the a priori probability is clearly not as exact as the impacts of other hypotheses are in some way factored into the a priori probability valuation. It is, however, the more realistic interpretation considering that the valuators are humans. In AXIOM, the second way of interpreting a priori probability is natural, since AXIOM statements can have multiple possible options and since one option, by definition, has to have value *true*. This characteristic makes the first interpretation of a priori probabilities an impossibility. The second interpretation is also vastly easier for the valuators and should be preferred on that basis alone.

3.5 Interpretation of results

The analytical outputs of the AXIOM computational process are derived from the *a posteriori* probabilities of the model options and the difference between the *a priori* and *a posteriori* probabilities. The a posteriori probabilities can be seen as interesting as such: Their values reflect the effects second- and higher-order interactions that exist in the cross-impact model. These changes in the option probabilities are the detail of interest in comparing different AXIOM *iteration sets* with each other. The iteration sets normally differ in the combination of interventions used in their evaluation. Iteration sets can also differ in some other way, such as model valuations or even the structure of the evaluated model; in such a case, iteration sets can only be compared on the intersect of their option sets. The analyst can be interested in certain "target" options whose probability changes are of greatest interest for the whole analysis: The desired outcome for the analysis could be the identification of the intervention set that will maximise the probability of some desirable option, or that will minimise the probability of some undesirable option.

Other analytical outputs that can be extracted from the AXIOM iterations include at least the following:

1. Frequency of specific configurations
   - Probability of a scenario
   - Comparison of probabilities of specific scenarios
2. Frequency of co-occurrence of options (with frequent itemset mining technique)
   - Which options typically occur together
3. Frequence of option subsets
   - Probability of a "partial scenario"
   - Comparison of probabilities of partial scenarios.

It is worthwhile to consider the exact nature of the a posteriori probabilities in light of the discussion of the nature of a priori probabilities (see Subsection 3.4.6). It was noted that the a priori probabilities are already cross-impacted to some extent as they, in AXIOM, cannot be valuated in isolation even in theory. How can the difference between *a priori* and *a posteriori* probabilities be interpreted in AXIOM?

As Gordon [1994, 9] points out, the difference of *a priori* and *a posteriori* probabilities in cross-impact analysis will partially result from inconsistencies of the expert valuations of a priori probabilities. Another component of the difference will reflect the effect of higher-order interactions between the AXIOM options. As the model grows beyond the trivial size of five to ten statements, which is the maximum size recommended for a cross-impact model using the technique of defining conditional probabilities as per Gordon's and Godet's methods (see Sections 2.2 and 2.3 for details) the higher-order interactions between options typically become longer and more significant; as the cross-impact grows in size, the effect of higher-order interactions probably outweigh the valuation inconsistencies.

An open question about using the a posteriori probabilities of options under different intervention combinations is which probabilities to use as the reference point: the a priori probabilities or the a posteriori probabilities of the AXIOM iteration without any active interventions. The "normal" way established in the well-known cross-impact methods is to compare the a posteriori probabilities of model runs representing interventions to the a priori probabilities. In AXIOM (and possibly in other cross-impact techniques as well) it might be equally well or better reasoned that the a posteriori probabilities of options under a specific intervention combination are compared, instead of the a priori probabilities, to the a posteriori probabilities of options in an iteration without any active interventions. This iteration could be interpreted to have the valuation inconsistencies resolved and account for the higher-order interactions unlike the a priori probabilities. Comparison of option probabilities of iterations under specific interventions to the no-intervention combination would seem to reveal the effects of the interventions more clearly. The author will not conclusively take position on this question at this point.

4 How AXIOM improves on the other cross-impact techniques

As the AXIOM method has been outlined in Chapter 3, many of the choices made in development of the method have already been justified through comparison with other existing cross-impact methods. As stated in the introduction, AXIOM combines the strengths of several documented cross-impact techniques and for this reason the individual advantageous features are not necessarily superior to all documented techniques. The combination of the features makes AXIOM a recommendable method for use in cross-impact modeling. The advantages of AXIOM over other cross-impact techniques are as follows:

1. **Model valuation in AXIOM is relatively easy.** The impact valuation phase in AXIOM is decisively easier when compared to cross-impact methods which represent interactions as conditional probabilities. The cognitive cost of providing a large number of conditional probabilities is very high. The conditional probability valuations are needed for all ordered pairs of hypotheses in the model, even when the model valuators would conclude that there is no direct interaction between the hypotheses. For example, the conditional probability valuation $P(A|B) = P(A)$ might violate the probability axioms, so no "default" conditional probability value exists: all interactions have to be valuated. The valuations have to comply with the probability axioms, and as the number of hypotheses grows, simply finding a compliant valuation solution might become difficult (at least without a help of a computer program specifically designed for this purpose). In this difficult valuation process, the qualitative-nature understanding of the experts about the interactions in the modeled system might get distorted in the attempt to find an acceptable valuation solution, changing the focus from modeling the system in the best way possible on the basis of expert knowledge into a sudoku-like number-placement exercise.

   The interpretation of the a priori probabilities in AXIOM also eases valuation. The a priori probabilities are valuated under the assumption that the state of the rest of the system apart from the valuated option is unknown (see Subsection 3.4.6). This interpretation as such is not an advantage of AXIOM over other methods as it can be adopted in those other methods as well, but it reflects the realistic overall attitude to the model valuation process assumed in the AXIOM approach: valuations are provided by human valuators and are approximate and rough, and should not be treated as exact and completely reliable measurements.

2. **AXIOM is suited for cross-impact models with a large number of components.** Cross-impact techniques which represent the interactions as conditional probabilities are not well suited for constructing system models with a large number of components. The cognitively expensive valuation phase heavily limits the practical number of components in the model. Godet *et al.* [1994, 149] actually recommend that the number of hypotheses should not exceed 6.

   Modeling systems with such a small number of hypotheses is very limiting. In a system model with a handful of components represented by hypotheses, if those hypotheses are detailed and concrete, many relevant factors and driving forces are left outside the cross-impact model. Conversely, if the hypotheses are loaded with a lot of content so that each hypothesis represents many factors and driving forces simultaneously, the abstraction level of the hypotheses gets very high. This high abstraction level will make the model valuation difficult and ambiguous. The interpretation of results is likely to suffer from the high abstraction level and drawing concrete policy recommendations on the basis of the model might turn out difficult. Either way, practical and useful cross-impact modeling is very difficult if the nature of the cross-impact technique per se limits the number of model components.

   As the object of interest in cross-impact modeling is the impact network of the modeled system, the limitations on the number of components in cross-impact models also limit the interestingness of the analysis. In a system model of few components, the impact chains cannot be very long. If the ability to investigate higher-order interactions and long impact chains is an important motivation to do cross-impact analysis, the cross-impact modeling should definitely support this aspiration.

3. **AXIOM primitives have comparatively high modeling power.** It is easy to make the case that the multi-valued AXIOM statements are a better solution than separate boolean hypotheses for constructing useful and relevant cross-impact models. Boolean hypotheses can, to some degree, be used to model mutually exclusive system states akin to AXIOM options, but they are much less convenient and error-prone in modeling as they require the

exclusiveness to be explicitly defined through conditional probabilities. Additionally, boolean hypotheses cannot model the exhaustiveness of AXIOM options: there is no mechanism to ensure that the probability distribution of a supposedly exclusive and exhaustive set of boolean hypotheses will remain valid during the model evaluation.

Incorporating temporal aspect to cross-impact modeling is a feature of AXIOM that greatly increases its power to model real systems compared to methods that do not offer a mechanism to model time. Providing a way to model time makes it easier to construct models from the perspective of modeling interventions: today's decisions can be modeled to take their effect on the future states of the system in a very convenient and natural way instead of providing means to only model a system with a single temporal space where events happen without any temporal structure.

4. **AXIOM facilitates extracting practical and useful analytical outputs.** In Gordon's method and Godet's SMIC method, especially the process of studying the effect of interventions and policy actions on the modeled system is, compared to AXIOM, cumbersome (although this might be more dependent on the implementation than the method). Modeling interventions requires changes to the cross-impact model and possibly redefinition of the conditional probabilities. The AXIOM method offers tools to design the simulation of interventions cleanly in the model building phase, and the focus of the analytical outputs is from the start in the effects of the different intervention sets, which makes it easy to extract practical policy recommendations. In addition to this, a number of further analytical outputs can be easily extracted on the basis of the AXIOM computation. These possibilities are outlined in Section 3.5. The concepts of AXIOM bring a great deal of convenience to the cross-impact modeling. The lack of convenient tools could be the reason why cross-impact techniques have not been adopted on a larger scale.

Above-stated strengths of the AXIOM approach, the freely available implementation (see Section 5.1) and the transparent documentation of the computation details make AXIOM a strong candidate for a general cross-impact modeling approach.

5 Implementation and further development

5.1 Software implementation of AXIOM method

The author has implemented the AXIOM method as a Java program. The JAR (Java Archive) and the source code of the implementation are available for download in GitHub at address `https://github.com/jmpaon/AXIOM`. The GitHub repository contains also instructions for use and an example input and test data.

The Java program accepts input data as a text file. The input file describes the AXIOM model components in a special syntax. The program reads input data in this syntax and constructs the AXIOM model. A valid model can then be consecutively evaluated the requested number of times with all the possible intervention combinations available for the model. A single run of the AXIOM program creates an iteration set with as many iterations as there are different intervention combinations in the model. The first iteration is always generated using an "empty" combination of interventions. The a posteriori probabilities of this iteration reflect the operation of the system without any interventions. This iteration is followed by iterations computed using the different possible combinations of options of statements flagged as intervention statements in the AXIOM model. Finally, the a posteriori probabilities of model options in each iteration are output.

Listing 1 is an example input file for the AXIOM program as it is at the time of writing of this thesis. The example input file contains the input information of the example statements presented in Subsection 3.1.1 and the example options presented in Subsection 3.1.2. A command in the input file resulting in a statement addition begins with the character '`#`', in option addition with the character '`*`' and in an impact addition with character '`>`'. Options are added under the statement that is last defined before the option; impacts are added under the option that is last defined before the impact. After the symbol that defines what type of component is added to the model comes the identifying label of that component (for statements and options) and the other details about the component. For example, the first line of the example input file defines a statement addition to the model. The label of the statement is '`GEOPOLITICS`' and it is to have *timestep* value 2. The second statement (with label '`GOVERNMENT`') defined in the input data file has timestep value 1 and is also flagged as an intervention statement (as the statement definition contains the flag '`INT`'). Line 2 in the input file adds

Listing 1.AXIOM input data example

```
# GEOPOLITICS ts 2 Geopolitical position of Finland after 2025
	* EU 0.5 Member state in the European Union
		> unitedstates police -2
		> unitedstates pacific +1
	* EURASIAN 0.15 Member state in the Eurasian Union
		> unitedstates police +3
	* NEUTRAL 0.35 Non-aligned position
		> unitedstates pacific +2

# GOVERNMENT ts 1 INT Governing political parties in Finland 2019-2023
	* TRUEFINN 0.1 True Finns as the ruling political party
		> geopolitics eurasian +3
		> geopolitics neutral +4
		> geopolitics eu -3
		> economy nogrowth +1
	* SDP_LEFT 0.3 SDP and Left Alliance as the ruling parties
		> geopolitics eu -2
		> geopolitics neutral +1
		> unitedstates isolationist +1
	* CENTER_COAL 0.4 Center Party and Coalition Party as the ruling parties
		> geopolitics eu +3
		> unitedstates police +1
	* RAINBOW 0.2 Rainbow coalition government
		> geopolitics eu +1

# ECONOMY ts 1 Economic development in the European Union from 2017-2023
	* NOGROWTH 0.30 Average economic growth close to zero in EU area
		> government truefinn +3
		> government sdp_left +2
		> unitedstates pacific +2
		> unitedstates isolationist +1
	* SLOWGROWTH 0.45 Average growth in EU area slow
	(close to 1% GDP growth annually)
		> government rainbow +1
	* FASTGROWTH 0.25 Average growth fast in EU area
	(close to 3% GDP growth annually)
		> government center_coal +3

# UNITEDSTATES ts 2 Role of United States in international politics in 2020s
	* POLICE 0.15 United States polices the whole world
		> geopolitics eu +3
	* PACIFIC 0.65 United States is active in the pacific region
	and passive in Europe
		> geopolitics eurasian +1
		> geopolitics neutral +2
	* ISOLATIONIST 0.2 United States observes an isolationist foreign policy
		> geopolitics eurasian +4
```

an option with label 'EU' under statement 'GEOPOLITICS'. The option has an a priori probability of 0.5. Line 3 adds an impact to option with label 'EU'. The added impact is directed at option 'POLICE' under statement 'UNITEDSTATES'. The probability adjustment function associated with the impact is -2.

Table 5 presents the main output of the AXIOM program run. The model evaluation has been performed 10^6 times for each iteration presented in the table. The first column displays the labels of the model's statements and options. The second column displays the initial or a priori probabilities of the options. The third column displays the a posteriori probabilities of the options when no option is an active intervention. As the model has only one statement flagged as an intervention statement, and the single intervention statement has four options, the iteration set has four iterations in addition to the iteration without interventions. For these iterations, calculating the a posteriori probability for the options under the intervention statement does not make sense, as the options have a predefined state for each iteration.

The chief analytical output is the change in the probability of an option of special interest under different interventions. If the geopolitical state of Finland as an European Union member state is seen as a desirable state, according to the example cross-impact model the best intervention to accomplish this state would be to have the Center Party and the Coalition Party as the dominant parties for the next electoral term. On the other hand, if a neutral position is seen as desirable, this would be best promoted by having the True Finns party as the dominant party. The trade-off for this might be that True Finns as the ruling party appears to also maximise the probability of zero economic growth. It must be noted that the selection of the model statements and options and valuation of probabilities and impacts have been done alone by the author who claims no particular expertise in the domain of the example model.

	apriori	no intervention	center_coal	rainbow	sdp_left	truefinn
economy:fastgrowth	0.25	0.248	0.248	0.249	0.25	0.24
economy:nogrowth	0.3	0.302	0.3	0.3	0.299	0.326
economy:slowgrowth	0.45	0.448	0.45	0.45	0.45	0.432
geopolitics:eu	0.5	0.48	0.696	0.534	0.279	0.078
geopolitics:eurasian	0.15	0.135	0.094	0.144	0.189	0.132
geopolitics:neutral	0.35	0.384	0.209	0.32	0.531	0.788
government:center_coal	0.4	0.414	TRUE	FALSE	FALSE	FALSE
government:rainbow	0.2	0.189	FALSE	TRUE	FALSE	FALSE
government:sdp_left	0.3	0.294	FALSE	FALSE	TRUE	FALSE
government:truefinn	0.1	0.101	FALSE	FALSE	FALSE	TRUE
unitedstates:isolationist	0.2	0.178	0.17	0.172	0.199	0.163
unitedstates:pacific	0.65	0.694	0.693	0.706	0.68	0.713
unitedstates:police	0.15	0.127	0.136	0.12	0.12	0.122

Table 5: A posteriori probabilities of the geopolitics model under different interventions

As stated, the AXIOM program calculates the AXIOM iterations for all possible intervention combinations possible with the options of the statements flagged as interventions. This feature amounts to a comparatively high level of automation in deriving higher-order information from the cross-impact model. The analyst can focus on the model development and there is no need to manually test different intervention combinations and model setups; these tests can be planned in the model construction phase by introducing interventions and flagging other suitable statements as intervention statements in the cross-impact model, keeping the usage of the AXIOM program very simple.

The AXIOM method is rather experimental at its current state. The author believes that the right strategy to develop the software implementing the AXIOM method is to extensively test the method and program in various research and modeling cases before venturing to develop a more user-friendly interface for the program. Modeling cases to be undertaken in the near future will provide information on the real information needs of modelers and analysts as well as the most informative and useful format of outputs. These activities will likely still bring about changes and improvements to the program. When this kind of development is done, it might be called for to create a graphical user interface for the tool to lower the barrier of adoption of the AXIOM method.

5.2 Ideas for further development of the method

In this section some ideas for possible further development of the AXIOM method and software implementation are proposed. There are certain additions that could be made to the model evaluation process that might be useful for cross-impact modelers. First, the AXIOM software implementation could offer a mechanism for flagging statements and options so that the possible combinations of their inclusion and exclusion would all be automatically calculated and added to the iteration sets. This mechanism would be similar to the currently existing possibility to flag statements as intervention statements. This addition might prove useful for investigating effects of changes to the system models from a slightly different perspective than using the intervention statement method. The analytical advantages would be similar. This addition would only involve some additional computations in the highest level of the computation process; no changes in the modeling primitives or model evaluation would be necessary. This kind of analysis is completely possible now, but requires the analyst to perform it manually, by having multiple AXIOM models having the inclusion-exclusion combinations of interest and processing them separately. If the use of AXIOM method in future modeling cases reveals a need for this addition, it is likely to be added to the software implementation as doing so is fairly straightforward.

Another expansion-type improvement, which might prove to be in demand in analysis of the outputs of AXIOM method, would be to somehow provide information about the causes of the probability changes. The author cannot clearly outline a way to do this at the time of writing. Such facility could nevertheless be useful especially in analysis of big cross-impact models where surprising, perhaps counter-intuitive shifts in probability might emerge when the higher-order interactions between model components are computed.

The AXIOM method might also be developed by introducing new modeling primitives. One interesting possibility would be to add another statement type that would have a continuous probability distribution and that would, after evaluation, have a real number value as its state instead of an option. This type of statement would probably be a more natural way to represent quantities and shares than the option-based representation AXIOM currently offers. A challenge in this type of modeling primitive would be the valuation: a continuous a priori probability distribution would have to be provided by the model valuators and providing this valuation might prove to be more demanding than assigning a priori probabilities to a small set of discrete options.

Another new primitive to be introduced to AXIOM could be a statement type that would have a changing state. This kind of statement would in effect be evaluated in each timestep of the model and the different states at different time categories would be accounted. Statements like this can currently be approximated by having several statements with the same content (same options) but different timesteps. For this reason, the new statement type with a changing state would be more of a modeling convenience, rather than something that would enable analytical possibilities that are not currently available at all.

A possibly important change in the way time is modeled in AXIOM models could be to make it possible for statements to have multiple possible timesteps instead of a single definite timestep. The statement would be a candidate for evaluation at any timestep associated with it. The different possible timesteps of a statement could have a probability distribution. It is possible that modelers cannot assign a definite timestep for all statements, so this change would definitely increase the modeling power. This improvement is likely to be introduced into the AXIOM approach in the future.

As mentioned in the introduction, AXIOM and other methods of cross-impact analysis are most useful as modeling tools in domains where "hard" empirical and time-series data is not available. An interesting avenue of modeling research could be to find ways to practically combine traditional modeling and cross-impact modeling in the same framework.

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