Maximizing Detection of Target with Multiple Direction Possibilities to Support Immersive Communications in Metaverse

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ABSTRACT

The rise of immersive communication due to virtual reality (VR), augmented reality (AR) and mixed reality (MR) has imposed stringent requirements on the wireless communication systems. A basic requirement imposed by VR/AR/MR environments (in a Metaverse) on the communication system is the sensing ability. Therefore, integrated sensing and communication (ISAC) systems are considered an integral part of the Metaverse. In order to improve the sensing functionality of the ISAC system within the Metaverse, this paper proposes an iterative optimization algorithm to solve non-convex signal to clutter plus noise ratio (SCNR) maximization problem when the target direction is uncertain. Simulation results demonstrate that the effectiveness of the proposed algorithm as compared to the existing schemes which assume apriori information about the target direction.

CCS CONCEPTS

• **Networks** → *Sensor networks.*

KEYWORDS

Detection probability, integrated sensing and communication, and resource allocation.

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1 INTRODUCTION

In today's rapidly evolving digital landscape, the Metaverse stands at the forefront of a paradigm shift, promising to revolutionize the way we interact, work, and socialize in an immersive, interconnected, and mixed real-virtual world. One distinctive mark of the metaverse is creating interactive, digital experiences that blend the boundaries between the physical and virtual worlds. To achieve this feature, the entities in the real physical world will need to communicate and interact with virtual entities in the metaverse or digital counterparts of other physical objects. In this context, immersive communication becomes an essential technique underpinning the metaverse, as it facilitates seamless and realistic interactions between users, digital objects, and virtual environments for the connected virtual universe.

Immersive communications is a highly interactive and engaging form of communication that blends promising technologies such as virtual reality (VR), augmented reality (AR), and mixed reality (MR) to create rich, multi-sensory experiences that closely mimic real-world interactions. To support such technologies, the wireless system in immersive communications will not only provide high-speed, low-latency, and reliable connectivity but must also provide various sensing functions (e.g. localizing objects, position or motion state tracking and prediction) for enhancing the experience and facilitating natural interactions within digital and physical environments. Such a wireless communication system is now known as integrated sensing and communication (ISAC) system [10, 11].

With the sensing functionality integrated with the communication systems, the performance metrics also differ from those of the conventional communication systems. The most commonly used performance metrics for sensing, localization are based on Crámer-Rao bound (CRB) of the distance, angle, or speed parameters and object detection probability. Specifically, for localization purposes, the goal is to devise transmission schemes such that the performance is as close to the CRB as possible. Similarly, for detection purposes, the goal is to allocate transmission resources in such a way that the detection probability is maximized while maintaining the performance of the communications. In terms of localization performance, recent works [5–9] provided CRB minimization schemes under different system setups. In [9], a hybrid approach with known/unknown placements of multiple sensors is presented to estimate the location of multiple targets. The design of various detectors based on the minimization of the CRB is presented in [7]. The works in [9], [7] do not consider the joint operation of sensing and communication. To fill this void, [6] provides a CRB minimization scheme. Specifically, CRB is used as a performance metric of target estimation, and then a CRB minimization beamforming design is proposed which guarantees a pre-defined level of signal-to-interference-plus-noise ratio (SINR) for each communication user (CU).

In an immersive environment such as VR, AR, MR, the physical objects needs to be reconstructed virtually. However, before their reconstruction it is necessary to detect the objects. Therefore detection probability improvement is an important issue. With regard to the detection probability maximization, it is well known that the probability of detection is an increasing function of the radar received signal to clutter plus noise ratio (SCNR) [4]. Thus SCNR is an important performance metric and hence the detection probability maximization problem effectively translates to SCNR maximization problem. Therefore, several existing works have solved interesting optimization problems where the objective was to maximize the radar SCNR while satisfying the performance of the CUs. However, most of these works assume that the radar target can lie only at a specific angle from the ISAC transmitter [1], [3]. In reality, this apriori information about the target direction is not available since in reality a radar target can emerge from different directions. This practical consideration necessitates further study of radar SCNR maximization schemes. The fundamental challenge posed by this consideration is the choice of receive beamforming vector at the transmitter. Specifically, when there is only one target the receive beamforming vector can be obtained in closed-form using the minimum variance distortionless response (MVDR) beamforming technique which significantly simplifies the SCNR maximization problem. However, if there are more than one possible target directions, then we do not have a closed-form expression for receive bearmformer. Hence, new solution method is required to solve the SCNR maximization problem. In this context, this paper considers an ISAC system with multiple CUs and a target with ambiguity in arrival direction. Then, the aim is to maximize the detection probability of the radar whilst satisfying the minimum rate requirements of the CUs for all the possible target directions.

2 SYSTEM MODEL AND ASSUMPTIONS

This section presents the main system parameters, underlying assumptions and important performance metrics for the radar and communication system. In this paper, we assume an N antenna ISAC transmitter which serves K single antenna CUs. In addition to serving the CUs, we assume that a target also needs to be detected which can lie at an angle $\theta_T \in \{\theta_1, \theta_2, \cdots, \theta_M\}$ from the ISAC transmitter. This considered system model can easily reflect a VR environment. For example a VR headset mimics the ISAC transmitter and various control units within the VR system are represented by the CUs to which the VR headset must remain connected during the whole VR experience.

From the available N antennas, we assume that N_t antennas are used for transmitting information to the CUs while it uses N_r antennas for reception of reflection from the target to perform detection of the target. The beamforming vector, and information symbol for k-th CU are denoted by $\mathbf{u}_k \in \mathbb{C}^{N_t \times 1}$, s_k, respectively. In the following, we assume $E[s_k] = 0, E[|s_k|^2] = 1, E[s_j s_k] = 0$, for $j \neq k$.

Furthermore, we assume two scenarios with respect to the use of probing signal for target detection. In the first scenario, we assume no dedicated probing signal is employed. In the second scenario, we assume that a dedicated probing signal is used for for radar detection. Hence, the overall transmitted symbol for the first scenario can be written as

$$\mathbf{x}^I = \sum_{k=1}^{K} \mathbf{u}_k s_k, \tag{1}$$

while for the second scenario the transmitted symbol is given as

$$\mathbf{x}^{II} = \sum_{k=1}^{K} \mathbf{u}_k s_k + \mathbf{v} s_0, \tag{2}$$

where s_0 with $E[|s_0|^2] = 1$ is the symbol used for probing signal. The channel vector between the transmitter and k-th CU is represented by $\mathbf{h}_k \in \mathbb{C}^{N_t \times 1}$. Next, we present the performance metrics for communication and radar systems, respectively.

2.1 Communication system performance metric

With the above assumptions, the received signal at the k-th user for the first scenario can be written as

$$y_k^I = \mathbf{h}_k^H \mathbf{u}_k s_k + \sum_{i=1, i\neq k}^{K} \mathbf{h}_k^H \mathbf{u}_i s_i + \omega_k, \tag{3}$$

and for the second scenario

$$y_k^{II} = \mathbf{h}_k^H \mathbf{u}_k s_k + \sum_{i=1, i\neq k}^{K} \mathbf{h}_k^H \mathbf{u}_i s_i + \mathbf{h}_k^H \mathbf{v} s_0 + \omega_k, \tag{4}$$

where $\omega_k \in \mathbb{C}$ is the additive white Gaussian noise (AWGN) at CU k with mean zero and variance N_0.

Then, the corresponding signal to interference plus noise ratio (SINR) at the k-th CU for first scenario is given as

$$\gamma_k^I = \frac{|\mathbf{h}_k^H \mathbf{u}_k|^2}{\sum_{i=1, i\neq k}^{K} |\mathbf{h}_k^H \mathbf{u}_i|^2 + N_o}, \tag{5}$$

and for the second scenario, the SINR for k-th CU is given as

$$\gamma_k^{II} = \frac{|\mathbf{h}_k^H \mathbf{u}_k|^2}{\sum_{i=1, i\neq k}^{K} |\mathbf{h}_k^H \mathbf{u}_i|^2 + |\mathbf{h}_k^H \mathbf{v}|^2 + N_o}. \tag{6}$$

In order to have a satisfactory communication performance in both scenarios, the k-th CU requires its SINR to be at least Γ_k. Mathematically, it can be represented as

$$\gamma_k^I = \frac{|\mathbf{h}_k^H \mathbf{u}_k|^2}{\sum_{i=1, i\neq k}^{K} |\mathbf{h}_k^H \mathbf{u}_i|^2 + N_o} \geq \Gamma_k, \tag{7}$$

and

$$\gamma_k^{II} = \frac{|\mathbf{h}_k^H \mathbf{u}_k|^2}{\sum_{i=1, i\neq k}^{K} |\mathbf{h}_k^H \mathbf{u}_i|^2 + |\mathbf{h}_k^H \mathbf{v}|^2 + N_o} \geq \Gamma_k. \tag{8}$$

2.2 Radar system performance metric

In this subsection, we present the radar system performance metric for two possible scenarios. In the considered system model, assuming that the target is located at an angle θ_i^T from the transmitter, the j-th clutter component located at an angle θ_j^C, the received signal at the radar can be written as

$$\mathbf{r}^l(\theta_i^T) = \alpha_i^T \mathbf{a}_r(\theta_i^T)\mathbf{a}_t^H(\theta_i^T)\mathbf{x}^l + \sum_{j=1}^{J} \alpha_i \mathbf{a}_r(\theta_j^C)\mathbf{a}_t^H(\theta_j^C)\mathbf{x}^l + \mathbf{n},$$

which can be simplified to

$$\mathbf{r}^l(\theta_i^T) = \alpha_i^T \mathbf{A}(\theta_i^T)\mathbf{x}^l + \sum_{j=1}^{J} \alpha_j^C \mathbf{A}(\theta_j^C)\mathbf{x}^l + \mathbf{n}, \tag{9}$$

where $l \in \{I, II\}, \mathbf{A}(\theta) = \mathbf{a}_r(\theta)\mathbf{a}_t^H(\theta) \in \mathbb{C}^{N_r \times N_t}$, $\alpha_i^T, \alpha_j^C \in \mathbb{C}$ are the complex channel between target and radar, and between clutter and radar, $\mathbf{n} \in \mathbb{C}^{N_r \times 1}$ is additive white Gaussian noise (AWGN) with $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, N_r\mathbf{I})$, $\mathbf{a}_z(\theta) \in \mathbb{C}^{N_z \times 1}$ is the transmit are receive steering vectors for $z \in \{t, r\}$, respectively. The dependencies of the steering vectors $\mathbf{a}_t(\theta), \mathbf{a}_r(\theta)$ on the angle θ are given as

$$\mathbf{a}_t(\theta) = [1, e^{-j2\pi\Delta\sin(\theta)}, \cdots, e^{-j2\pi(N_t-1)\Delta\sin(\theta)}]^H, \tag{10}$$

$$\mathbf{a}_r(\theta) = [1, e^{-j2\pi\Delta\sin(\theta)}, \cdots, e^{-j2\pi(N_r-1)\Delta\sin(\theta)}]^H. \tag{11}$$

where $\Delta = \frac{\lambda}{2}$ and λ is the carrier wavelength.

We assume that α_i^T, α_j^C are independently distributed from $\mathbf{h}_k$'s. After reception, the radar performs receive beamforming (combining) with vector $\mathbf{w}$ on the received signal, then the output of the radar receiver is given as

$$y_r^l(\theta_i^T, \mathbf{w}) = \mathbf{w}^H \mathbf{r}^l(\theta_i^T). \tag{12}$$

Subsequently, the average radar signal-to-clutter-plus-noise-ratio (SCNR) can be written as

$$\gamma_r^l(\theta_i^T, \mathbf{w}) = \frac{|\alpha_i^T|^2 E[|\mathbf{w}^H \mathbf{A}(\theta_i^T)\mathbf{x}^l|^2]}{E[\mathbf{w}^H \left(\sum_{j=1}^{J} |\alpha_j^C|^2 \mathbf{A}(\theta_j^C)\mathbf{x}^l\mathbf{x}^{l^H}\mathbf{A}^H(\theta_j^C) + N_r\mathbf{I}\right)\mathbf{w}]}. \tag{13}$$

It is clear from (7), (8), (13) that the communication and radar performance depend on the beamforming vectors $\mathbf{u}_k, \mathbf{v}$. In the next subsection, we formulate an optimization problem to find the optimal values of $\mathbf{u}_k, \mathbf{v}$ while taking into consideration radar and communication performance, simultaneously.

2.3 Problem formulation

In this paper, we are interested in maximizing the detection probability of the radar. As noted above, the radar detection probability is directly proportional to the radar SCNR. Therefore, our aim is to maximize the radar SCNR whilst satisfying the communication requirements of the CUs. Overall, the mathematical formulation of the optimization problem for finding the appropriate beamforming vectors for the case when no dedicated probing signal is employed is given as

P1

$$\begin{aligned} \underset{\mathbf{u}_k, \mathbf{w}}{\text{maximize}} \quad & \min_{\theta_i^T \in \Theta^T} \gamma_r^I(\theta_i^T, \mathbf{w}) \\ \text{subject to} \quad & C1: \frac{|\mathbf{h}_k^H \mathbf{u}_k|^2}{\sum_{i=1, i\neq k}^{K} |\mathbf{h}_k^H \mathbf{u}_i|^2 + N_o} \geq \Gamma_k, \\ & C2: \sum_{k=1}^{K} \|\mathbf{u}_k\|^2 \leq P_{max}, \\ & C3: \|\mathbf{w}\|^2 = 1, \end{aligned} \tag{14}$$

where $\Theta^T = \{\theta_1^T, \theta_2^T, \cdots, \theta_M^T\}$ is the set that contains the possible angles of target with respect to ISAC transmitter. Similarly, when dedicated probing signal is employed, the SCNR maximization problem can be formulated as

P2

$$\begin{aligned} \underset{\mathbf{u}_k, \mathbf{v}, \mathbf{w}}{\text{maximize}} \quad & \min_{\theta_i^T \in \Theta^T} \gamma_r^{II}(\theta_i^T, \mathbf{w}) \\ \text{subject to} \quad & C4: \frac{|\mathbf{h}_k^H \mathbf{u}_k|^2}{\sum_{i=1, i\neq k}^{K} |\mathbf{h}_k^H \mathbf{u}_i|^2 + |\mathbf{h}_k^H \mathbf{v}|^2 + N_o} \geq \Gamma_k, \\ & C5: \sum_{k=1}^{K} \|\mathbf{u}_k\|^2 + \|\mathbf{v}\|^2 \leq P_{max}, \\ & C6: \|\mathbf{w}\|^2 = 1. \end{aligned} \tag{15}$$

In **P1**, **P2** the objective is to maximize the average SCNR of the radar system while considering the ambiguity about the angle of target. The constraints $C1, C4$ guarantee that the minimum data rate requirements of the CUs are met and $C2, C5$ makes sure that the total transmitted power is no more than the maximum allowed transmit power. Moreover, $C3, C6$ impose a constraint over the receive beamformer. It is clear that **P1**, **P2** are non-convex optimization problems. The non-convexity is caused by the non-convex objective function as well as the constraint functions. Moreover, note that any solution to **P1** is also a feasible solution to **P2** and therefore **P1** can be considered to a special case of **P2** with $\mathbf{v} = \mathbf{0}$. In the following section, we present efficient alternating optimization based algorithms for solving **P1**, **P2**.

3 OPTIMIZATION FRAMEWORK

In this section, first we present iterative optimization algorithms for the optimization problems **P1**, and **P2**. Then, we show the convergence of the proposed iterative algorithms. For ease of readability, we divide this section into two subsections. First subsection discusses the proposed solution for **P1** and the second subsection discusses the solution methodology for **P2**.

As discussed in Section III, the coupling of the optimization variables makes it difficult to solve **P1**. To handle this difficulty, we use an iterative alternating optimization approach, where in each iteration the optimization is first performed over the information beamformers $\mathbf{u}_k$'s while fixing receive combiner $\mathbf{w}$ and then the optimization is performed over receive beamformer $\mathbf{w}$ while fixing information beamformers $\mathbf{u}_k$'s.

3.1 Optimizing $\mathbf{u}_k$'s for Fixed $\mathbf{w}$

For fixed value of $\mathbf{w}$, **P1** can be written as

$$\begin{aligned} \underset{\mathbf{u}_k}{\text{maximize}} \quad & \min_{\theta_i^T \in \Theta^T} \frac{|\alpha_i^T|^2 \sum_{k=1}^K \mathbf{u}_k^H \mathbf{\Phi}_i^T \mathbf{u}_k}{\sum_{j=1}^J |\alpha_j^C|^2 \sum_{k=1}^K \mathbf{u}_k^H \mathbf{\Phi}_j^C \mathbf{u}_k + N_r \mathbf{w}^H \mathbf{w}} \\ \text{subject to} \quad & C1, C2, \end{aligned} \tag{16}$$

where we defined $\mathbf{\Phi}_i^T = \mathbf{A}^H(\theta_i^T)\mathbf{w}\mathbf{w}^H\mathbf{A}(\theta_i^T), \mathbf{\Phi}_j^C = \mathbf{A}^H(\theta_j^C)\mathbf{w}\mathbf{w}^H\mathbf{A}(\theta_j^C)$. Still the problem (16) is intractable due to the minimization of the quadratic fractional terms in the objective function. To tackle this issue, we use an iterative approach where in the m-th iteration, we replace the value of $\mathbf{u}_k$ in the denominator, $\sum_{j=1}^J |\alpha_j^C|^2 \sum_{k=1}^K \mathbf{u}_k^H \mathbf{\Phi}_j^C \mathbf{u}_k + \mathbf{w}^H\mathbf{w}$, by its optimal solution in the $m-1$-th iteration. With this approach, in the m-th iteration, (16) can be modified as

$$\begin{aligned} \underset{\mathbf{u}_k}{\text{maximize}} \quad & \min_{\theta_i^T \in \Theta^T} \frac{|\alpha_i^T|^2 \sum_{k=1}^K \mathbf{u}_k^H \mathbf{\Phi}_i^T \mathbf{u}_k}{\Omega} \\ \text{subject to} \quad & C1, C2, \end{aligned} \tag{17}$$

where $\Omega = \sum_{j=1}^J |\alpha_j^C|^2 \sum_{k=1}^K \mathbf{u}_{k,m-1}^H \mathbf{\Phi}_j^C \mathbf{u}_{k,m-1} + N_r \mathbf{w}^H\mathbf{w}$ and $\mathbf{u}_{k,m-1}$'s are the optimal solution for (17) in the $m-1$-th iteration. It can be easily shown that (17) is equivalent to

$$\begin{aligned} \underset{\mathbf{u}_k, s}{\text{maximize}} \quad & s \\ \text{subject to} \quad & \frac{|\alpha_i^T|^2 \sum_{k=1}^K \mathbf{u}_k^H \mathbf{\Phi}_i^T \mathbf{u}_k}{\Omega} \geq s, \ \forall\, i, \\ & C1, C2. \end{aligned} \tag{18}$$

Although problem (18) is much simpler than (16), it is still non-convex due to the non-convex constraints.

In order to solve (18), we resort to SDR technique. Toward this direction, first we introduce the matrices $\mathbf{U}_k = \mathbf{u}_k\mathbf{u}_k^H, \mathbf{H}_k = \mathbf{h}_k\mathbf{h}_k^H$. Then, we can equivalently write (18) as

$$\begin{aligned} \underset{\mathbf{U}_k, s}{\text{maximize}} \quad & s \\ \text{subject to} \quad & \frac{|\alpha_i^T|^2 \sum_{k=1}^K \text{Tr}(\mathbf{U}_k \mathbf{\Phi}_i^T)}{\Omega} \geq s, \forall\, i, \\ & \frac{\text{Tr}(\mathbf{U}_k\mathbf{H}_k)}{\sum_{n=1, n\neq k}^K \text{Tr}(\mathbf{U}_n\mathbf{H}_k) + N_0} \geq \Gamma_k, \ \forall\, k, \\ & \sum_{k=1}^K \text{Tr}(\mathbf{U}_k) \leq P_{max}, \\ & \text{rank}(\mathbf{U}_k) = 1, \ \forall\, k, \\ & \mathbf{U}_k \succeq 0, \ \forall\, k. \end{aligned} \tag{19}$$

In (19), the rank constraints makes the problem non-convex.

One possible approach that is widely used to solve the rank constrained optimization problem in the literature is the Gaussian randomization technique. However, such an approach is not only sub-optimal but can also lead to solutions which do not satisfy the minimum data rate constraints of CUs. Hence, in this situation we cannot use Gaussian randomization technique. Moreover, the widely known result about rank constrained separable SDP cannot be applied directly to (19) due to the fact that the number of constraints can be large due to the size of the set Θ^T.

Despite of the above issues, in the following theorem we show that the optimal solution for (19) can always admit rank one solutions for $\mathbf{U}_k$'s even if we ignore the rank one constraint.

Theorem 3.1. *The optimal solutions for $\mathbf{U}_K$ have rank one even if we relax the rank constraints in (19).*

Proof. It is clear that if we ignore the rank constraints in (19), then it becomes a convex optimization problem. Hence, we can use the well known KKT conditions to obtain the solution of rank relaxed problem. Toward this direction, we can write the Lagrangian of (19) as follows:

$$\begin{aligned} L(s, \mathbf{U}_k, \mu_i, \lambda_k, \beta) = & \sum_{k=1}^K \lambda_k \left(\Gamma_k N_0 + \Gamma_k \sum_{i=1, i\neq k}^K \text{Tr}\,(\mathbf{H}_k\mathbf{U}_i) \right) \\ & - s + \beta \left(\sum_{k=1}^K \text{Tr}(\mathbf{U}_k) - P_{max} \right) + \sum_{i=1}^I \mu_i \left(s - \sum_{k=1}^K \text{Tr}\,(\mathbf{A}_i\mathbf{U}_k) \right). \end{aligned} \tag{20}$$

Moreover, the dual problem can be written as

$$\underset{\mu_i, \lambda_k, \beta}{\text{maximize}} \quad \min_{s, \mathbf{U}_k} L(s, \mathbf{U}_k, \mu_i, \lambda_k, \beta). \tag{21}$$

After some mathematical manipulations, the dual problem (21) can be simplified as

$$\begin{aligned} \underset{\mu_i, \lambda_k, \beta}{\text{maximize}} \quad & \min_{s} -s + s \sum_{j=1}^J \mu_j + N_0 \sum_{k=1}^K \lambda_k \Gamma_k - \beta P_{max} \\ \text{subject to} \quad & \mathbf{G}_k \succeq \mathbf{0}, \end{aligned} \tag{22}$$

where

$$\mathbf{G}_k = \beta\mathbf{I} + \sum_{j=1}^K \lambda_j \mathbf{H}_j - \sum_{i=1}^I \mu_i \mathbf{A}_i - \lambda_k\,(1 + \Gamma_k)\,\mathbf{H}_k, \tag{23}$$

and the corresponding complementary slackness condition is

$$\text{Tr}\left(\mathbf{G}_k^* \mathbf{U}_k^*\right) = 0, \Rightarrow \mathbf{G}_k^* \mathbf{U}_k^* = \mathbf{0}. \tag{24}$$

From (24) it can be deduced that $\text{rank}\left(\mathbf{G}_k^*\right) \leq N_t - 1$ since $\text{rank}\,(\mathbf{U}_k) \geq 1$ due to the SINR constraint of the k-th user. Next we show, with the help of contradiction, that the matrix $\mathbf{X}_k = \beta\mathbf{I} + \sum_{j=1}^K \lambda_j \mathbf{H}_j - \sum_{i=1}^I \mu_i \mathbf{A}_i$ is always full rank in the optimal solution. To show this fact, assume that $\mathbf{X}_k$ is not full rank. Then, we can find a non-zero vector $\mathbf{y}$ such that

$$\mathbf{y}^H \mathbf{X}_k \mathbf{y} = 0, \tag{25}$$

and hence

$$\begin{aligned} \mathbf{y}^H & \left(\beta\mathbf{I} + \sum_{j=1}^K \lambda_j \mathbf{H}_j - \sum_{i=1}^I \mu_i \mathbf{A}_i - \lambda_k\,(1 + \Gamma_k)\,\mathbf{H}_k \right) \mathbf{y}, \\ & = -\mathbf{y}^H \lambda_k\,(1 + \Gamma_k)\,\mathbf{H}_k \mathbf{y} < 0, \end{aligned} \tag{26}$$

where the last strict inequality in (26) is due to the fact that $\mathbf{H}_k \succ \mathbf{0}$. However, according to the constraint in (22) $\mathbf{G}_k^*$ is a positive semi-definite matrix, hence (26) cannot be true. Thus, $\text{rank}\,(\mathbf{X}_k) = N_t$. Note also that $\text{rank}\,(\lambda_k\,(1+\Gamma_k)\,\mathbf{H}_k) = 1$. Therefore, we must have

$$\text{rank}\left(\mathbf{G}_k^*\right) + \text{rank}\,(\lambda_k\,(1+\Gamma_k)\,\mathbf{H}_k) \geq \text{rank}(\left(\mathbf{X}_k^*\right)), \tag{27}$$

which implies that

$$\text{rank}\left(\mathbf{G}_k^*\right) \geq N_t - 1. \tag{28}$$

Combining (28) with the earlier fact that rank $\left(\mathbf{G}_k^*\right) \leq N_t - 1$, we conclude that rank $\left(\mathbf{G}_k^*\right) = N_t - 1$. Thus, rank $\left(\mathbf{U}_k^*\right) \not> 1$. Furthermore, due to the minimum SINR constraints of the CUs we know that rank $\left(\mathbf{U}_k^*\right) \neq 0$. Therefore, we must have rank $\left(\mathbf{U}_k^*\right) = 1$. This completes the proof. □

Although the result in Theorem 1 is obtained for **P1**, it can also be shown that the conclusion obtained in Theorem 1 can also be obtained for problem **P2** with $\mathbf{V}^* = \mathbf{0}$. Another important property of the optimal solution for problem **P1** obtained from Theorem 1 is that all of the λ_k's are positive. Therefore, it can be concluded that in the optimal solution all of the SINR constraints for CUs are met with equality. Note that this property was also observed in a recent work which did not consider the possibility of multiple directions of target.

3.2 Optimization with respect to w

After fixing the values of $\mathbf{u}_k$'s, the optimization problem **P1** with respect to the optimization variable $\mathbf{w}$ is equivalent to the following optimization problem

$$\begin{aligned} \underset{\mathbf{w}}{\text{maximize}} \quad & \min_{\theta_i^T \in \Theta^T} \frac{|\alpha_i^T|^2 \sum_{k=1}^K \mathbf{w}^H \mathbf{B}_{i,k}^T \mathbf{w}}{\mathbf{w}^H \left[\sum_{j=1}^J |\alpha_j^C|^2 \sum_{k=1}^K \mathbf{B}_{j,k}^C + \mathbf{I}\right] \mathbf{w}} \\ \text{subject to} \quad & \|\mathbf{w}\|^2 = 1, \end{aligned} \tag{29}$$

where $\mathbf{B}_{m,k}^l = \mathbf{A}(\theta_m^l)\mathbf{u}_k\mathbf{u}_k^H\mathbf{A}^H(\theta_m^l), \ \forall\ l \in \{C, T\}, \ \forall\ m \in \{i, j\}$. Problem (29) is a non-convex generalized fractional optimization problem. However, by using a well known result for generalized fractional optimization problem with Toeplitz quadratic forms [2] we can optimally solve problem (29). In order to apply the result of [2], we note that each matrix in the summation of numerator, i.e. $\mathbf{B}_{i,k}^T$, and denominator, i.e. $\mathbf{B}_{j,k}^C$, of the objective function in (29) is a Toeplitz matrix. Hence, the result in [2] is directly applicable and it can be shown that there always exists a rank one solution for the following equivalent optimization problem

$$\begin{aligned} \underset{\mathbf{W}}{\text{maximize}} \quad & \min_{\theta_i^T \in \Theta^T} \frac{|\alpha_i^T|^2 \,\text{Tr}\left(\mathbf{W}^H \sum_{k=1}^K \mathbf{B}_{i,k}^T\right)}{\text{Tr}\left(\mathbf{W}^H \left[\sum_{j=1}^J |\alpha_j^C|^2 \sum_{k=1}^K \mathbf{B}_{j,k}^C + \mathbf{I}\right]\right)} \\ \text{subject to} \quad & \text{Tr}(\mathbf{W}) = 1, \\ & \mathbf{W} \succeq 0. \end{aligned} \tag{30}$$

Therefore, a global optimal solution can also be achieved for problem **P**. Due to the lack of space, we refer the interested reader to [] for a detailed discussion on the solution approach for solving problem (30).

3.3 Proposed Iterative Algorithm

The proposed algorithm is shown as **Algorithm 1** below. The algorithm takes initial values for $\mathbf{w}$ and $\mathbf{u}_k$'s in step 1. Then, as can be seen the algorithm iteratively solves the alternating optimization problems (19) and (29) with respect to $\mathbf{u}_k$ and $\mathbf{w}$, respectively in steps 2-10. Finally, algorithm terminates when convergence is achieved or the maximum number of iterations has been performed. [1] Initialize: $i = 0, \epsilon, i_{max}$, $\mathbf{w}$,$\mathbf{u}_k, \forall\ k \in \{1, \cdots, K\}$, $\mathbf{A}(\theta_i^T), \mathbf{A}(\theta_j^C), \forall\ i \in \{1, \cdots, M\}, \forall\ j \in \{1, \cdots, J\}$. $i \leq i_{max}$ Solve problem (19) by fixing value of $\mathbf{w}$ and denote the obtained solutions with $\mathbf{u}_k^i$. Solve problem (29) by fixing values of $\mathbf{u}_k$' and denote the obtained solution with $\mathbf{w}^i$. Store the obtained objective value as $f(i)$. Set $i = i + 1$ $|f(i) - f(i-1)| \leq \epsilon$ Break $\mathbf{u}_k^* = \mathbf{u}_k^i$, $\mathbf{w}^* = \mathbf{w}^i$.

Table 1: Simulation parameters.

Parameter	value	Parameter	value
f_c	30 GHz	Bandwidth	100 MHz
N_0	−94	Propagation	UMi
N_t	{4, 8}	N_R	{4, 8}
K	4	Γ_k	[0, 20] dB
P_{max}	30 dBm	d_1^{CU}	10 m
d_2^{CU}	15 m	d_3^{CU}	20 m
d_4^{CU}	25 m	J	2
$\lvert\alpha_1^C\rvert^2$	.001	$\lvert\alpha_2^C\rvert^2$	.00001
θ_T	$\{\pi/4, \pi/6\}$	θ_1^C	0
θ_2^C	$\frac{\pi}{2}$	α_i^T	$10^{-1.5}$

4 SIMULATION RESULTS

In this section we present the simulation results. The important simulation parameters are provided in Table 1. In the following, we illustrate two types of simulation results. First, we show the SCNR performance improvement achieved by the proposed algorithm as compared to one of the recently proposed algorithms which do not consider the possibility of uncertainty in the direction of radar target. Secondly, we show the convergence result of the proposed iterative algorithm.

Fig. 1 compares the SCNR results between the proposed algorithm and the algorithm in [1]. Note that the algorithm in [1] requires the exact information on the direction of arrival. In order to obtain the results for [1] and to demonstrate the affect of uncertainty of the target direction, we assume that the actual direction of the target is at an angle $\frac{\pi}{4}$ while receive beamform optimization is performed by assuming the target direction at an angle $\frac{\pi}{6}$. Clearly, the SCNR performance of the proposed algorithm is superior when compared to [1]. Moreover, the performance gap improves for higher SINR thresholds. This increase in the performance gap is due to the optimization performed with respect to the receive beamforming vector $\mathbf{w}$. The optimization with respect to $\mathbf{w}$ accounts for the uncertainty of the direction of target and thus improves the received power while the algorithm proposed in [1] can only improve the received power from only one direction. Hence, we observe an improvement in the performance for the proposed algorithm. Another important observation is that the performance gap is small for 8 antennas as compared to 16 antennas over smaller SINR thresholds while it increases for larger values of SINR. This is due to the fact that higher number of antennas

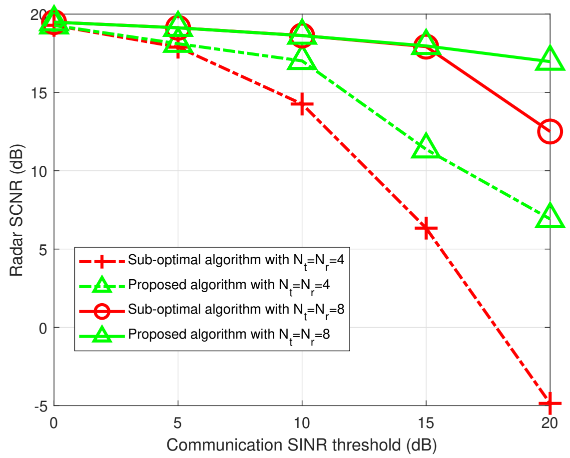

Figure 1: Radar SCNR with respect to different SINR thresholds for CUs. The algorithm proposed in [1] is denoted by Sub-optimal algorithm.

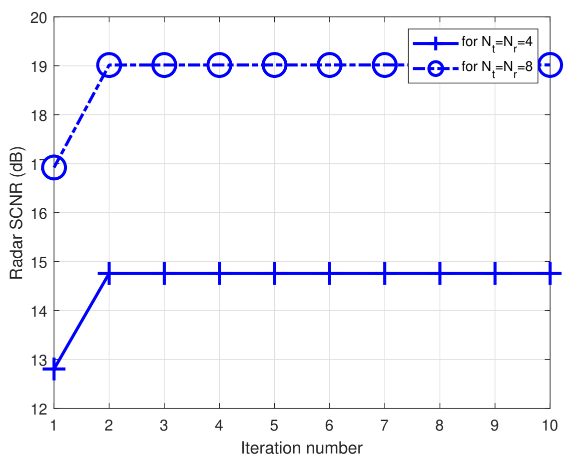

Figure 2: Convergence results of the proposed algorithm for different number of antennas.

allow for better directivity of transmit power toward more than one directions which ultimately results in larger reflected power.

It is not possible to theoretically prove the convergence of the proposed algorithm. However, in Fig. 2 we provide the convergence results of the proposed algorithm. It can be seen that the performance converges only after 2 iterations. Therefore, the proposed algorithm does not entail in high computational complexity due to the requirement of higher number of iterations for achieving convergence.

5 CONCLUSIONS

This paper proposed radar SCNR maximization algorithm in the ISAC system. The SCNR maximization is achieved by optimizing the transmit beamforming vectors for CUs as well the receive beamforming vector at the radar receiver. Contrary to the existing works on SCNR maximization in ISAC systems, this paper considered the possibility of uncertainty in the direction of target. The considered scenario of uncertainty in target direction is more appropriate for the VR environments. The performance of the proposed algorithm is shown to be much better than the existing scheme which assumes apriori information about the target direction.

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