Rakenteiden Mekaniikka (Journal of Structural Mechanics)
Vol. 55, No. 2, 2022, pp. 42–54
http://rakenteidenmekaniikka.journal.fi
https://doi.org/10.23998/rm.111471

VERTAISARVIOITU
KOLLEGIALT GRANSKAD
PEER-REVIEWED
www.tsv.fi/tunnus

Weight optimization of truss structures by using genetic algorithms

Azad Javanmiri[1] and Jari Mäkinen

Summary Lightweight structures, especially trusses, have attracted a tremendous attention due to their extensive applications in the construction of infrastructures. Optimizing the shape and cross-sectional topology of truss members is essential since the truss systems are widely used in engineering routines. These systems form the framework of structures like bridges, steel halls for industry and trade, and towers. For the scope of this research, genetic algorithms (GAs) were used for weight optimization of truss structures. This paper aims to optimize truss structures for finding optimal cross-sectional area. To optimize the cross-sectional area, all members were selected as design variables, with the structure's weight being the objective function. The restrictions related to the change of the location of the nodes and the tension in the members were the looked-upon problems, the permissible values of which were determined under the circumstances of the problem. In addition, the resulting optimized model which masses for sizing, shape, and topology or their combinations, were compared.

Key words: truss structure, weight optimization, cross-sectional area, genetic algorithm

Received: 28 September 2021. Accepted: 30 August 2022. Published online: 15 December 2022.

Introduction

The science of optimization is rapidly evolving along with other sciences. Researchers have considered the optimal design in many structural engineering issues. The idea of reducing the structure's weight without adversely affecting its behavior has long been a concern of designers. This research aims to minimize the total mass of the structure while keeping it below the maximum allowed stress and displacement. First, the case studies were obtained from previous studies on truss optimization. Then, a comparison was made between the different results obtained. In general, the optimum design of a steel truss structure is an attempt to find the best steel profiles for its members that result in a minimum weight or cost of the design of the structure [1]. New optimization methods do not need to use linear, non-

[1]Corresponding author: azad.javanmiri@tuni.fi

linear, and derivative programming methods and are based only on computational intelligence.

A common problem in structural design is the weight minimization of structures subjected to stress and displacement constraints [3]. These methods use one or more initial designs resulting in the final designs and previously produced new optimal designs. This process is repeated several times to achieve the optimal solution. In this research, methods based on mathematical structural optimization algorithms were used to help in designing efficient structural methods for the optimal trusses design. To reach this aim, program was written in MATLAB. To verify the accuracy and efficiency of the program, two-dimensional trusses were optimized and compared with other sources.

Optimization of truss structures is popular regarding the cross-sectional area of structural optimization, and over the last decades, various algorithms have been proposed for solving these problems. Some of the most popular methods are genetic algorithms (GAs), first introduced by Holland [4] and Goldberg [5]. A genetic algorithm is defined as a general tool for optimization in the field of discrete variables, such as structural problems [2]. Most of the metaheuristic algorithms are developed based on natural phenomena, including genetic algorithms (GA) [6], colliding bodies optimization (CBO) [7], and center of mass optimization (CMO) [8].

Objectives

The objective of the present structural optimization is to minimize the total mass of the structure, while keeping the structure below the maximum allowable stress and displacement. This work aims to investigate and compare different results. A modern technique in structural optimization known as genetic algorithm (GA) was used in this research. The secondary aims of this research are:

1. observing the structural behavior of truss structure from the cross-sectional area for different design loads,
2. finding a combination variable that would minimize the structure weight by using optimization configurations since many researchers put considerable effort into solving this problem by investigating numerous optimization methods.

Genetic algorithm

The optimization method selected for this research is genetic algorithm (GA), a heuristic optimization method with operation based on imitating natural processes. This paper proposes a genetic algorithm to develop the structural configuration required for weight minimization of truss structures. An original optimization approach using a genetic algorithm was verified through comparison and used for all the optimization combinations in this research. The resulting optimized model which masses for sizing and shape or their combinations were compared. In this study, two basic steps were involved in generating a genetic algorithm relating to the mutation process for a smart genetic algorithm. Here, unlike the conventional genetic algorithm where the mutation rate is constant in all generations, a variable mutation rate was used, the value of which is higher in the early generations and gradually decreases with the mutation rate.

Design problem

Many scientists have studied a 10-bar truss structure. In this example, the objective is to find the minimal weight of structure for design optimization of the 10-bar truss. The initial model bar and node design, as shown in Figure 1, has previously been analyzed by many researchers, such as Wu [9], Rajeev [10], Ringertz [11], and Li [12]. The trial-and-cut methods are usually performed for designing trusses and frames to determine factors such as allowable stress of structural members and deflection [13]. This study only compares a few previous studies. We only examine the weight optimization section of all the reviewed studies and compare their results with our findings. The results can be seen in Table 3.

Problem formulation

The design of truss structures typically consists of a two-step linear and non-linear analyses. In optimization, the size of a truss structure, or truss cross-sectional area, are the design variables of the problem. The objective function of the problem is the weight of the truss structure. In discrete sizing optimization problems, the main task is selection. The optimal members' section consists of a list of standard sections to minimize the structure's weight, while meeting the design constraints.

Figure 1 shows a typical plane-truss element in local and global coordinate systems. In the local numbering scheme, the two nodes of the element are numbered by 1 and 2. The local coordinate system consists of the x'-axis which runs along the element from node 1 toward node 2. A prime $()'$ will denote all quantities in the local coordinate system. The global (x, y)-coordinate system is fixed and does not depend on the element's orientation. Note that x, y, and z form a right-handed coordinate system with the z-axis coming straight out of the paper.

In the global coordinate system, every node has two degrees of freedom (DoF). Here, a systematic numbering scheme is adopted: a node whose global node number is j is associated with its DoF $2j - 1$ and $2j$. Further, the global displacements are associated with node j are $Q_{2j-1}$and Q_{2j}.

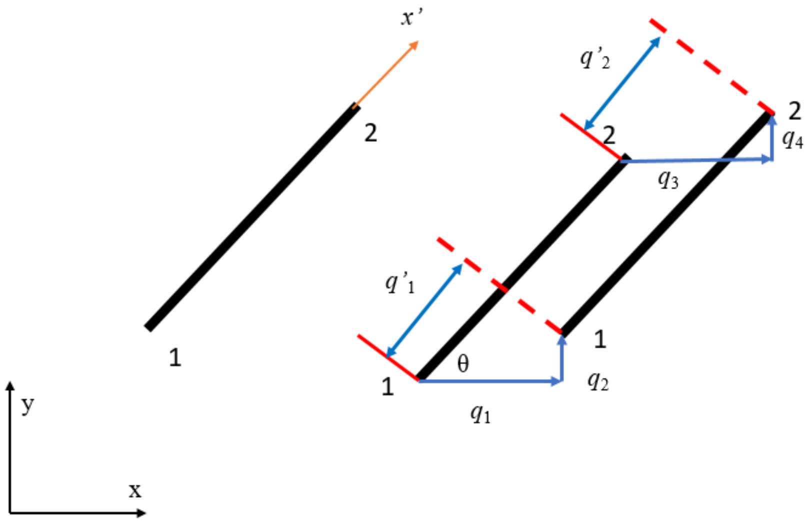

Figure 1. Truss element in local and global coordinates.

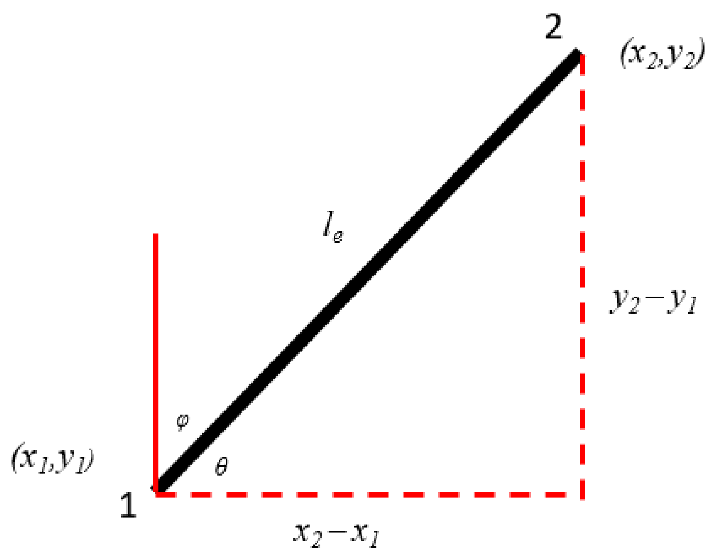

Figure 2. Truss element for calculation of direction cosines and length.

Let $q_1'$and q_2' be the displacements of nodes 1 and 2, respectively, in the local coordinate system. Thus, the element displacement vector in the local coordinate system is denoted by

$$\boldsymbol{q}' = [q_1', q_2']^T. \tag{1}$$

The element displacement vector in the global coordinate system is a (4×1) vector denoted by

$$\boldsymbol{q} = [q_1, q_2, q_3, q_4]^T. \tag{2}$$

The relationship between $\boldsymbol{q}'$ and $\boldsymbol{q}$ is developed as follows: in Figure 1, we see that q_1' equals to the sum of the projections of q_1 and q_2 onto the x'-axis. Thus

$$q_1' = q_1 \cos\theta + q_2 \sin\theta, \quad q_2' = q_3 \cos\theta + q_4 \sin\theta. \tag{3}$$

At this stage, the direction cosines l and m are introduced as $l = \cos\theta$ and $m = \sin\theta$. These direction cosines are the cosines of the angles that the local x'-axis makes with the global x- and y-axes, respectively. Therefore, one can now write in matrix form

$$\boldsymbol{q}' = \boldsymbol{L}\boldsymbol{q} \tag{4}$$

where the transformation matrix $\boldsymbol{L}$ is given by:

$$\boldsymbol{L} = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix}. \tag{5}$$

Simple formulas are now given for calculating the direction cosines l and m from nodal coordinate data. Referring to Figure 2, let (x_1, y_1) and (x_2, y_2) be the coordinates of nodes 1 and 2, respectively. We then have:

$$l = \frac{x_2 - x_1}{l_e}, m = \frac{y_2 - y_1}{l_e}, \tag{6}$$

where the length l_e is obtained as

$$l_e = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}. \tag{7}$$

The element stiffness matrix for a truss element in the local coordinate system is given as

$$\boldsymbol{k}' = \frac{E_e A_e}{l_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \tag{8}$$

where A_e is the element cross-sectional area and E_e is Young's modulus. The problem at hand is to develop an expression for the element stiffness matrix in the global coordinate system. This can be obtained by considering the strain energy in the element. Specifically, the element strain energy in local coordinates is given by

$$U_e = \frac{1}{2} \boldsymbol{q}'^T \boldsymbol{k}' \boldsymbol{q}'. \tag{9}$$

Substituting for $\boldsymbol{q}' = \boldsymbol{L}\boldsymbol{q}$, we get

$$U_e = \frac{1}{2} \boldsymbol{q}^T [\boldsymbol{L}^T \boldsymbol{k}' \boldsymbol{L}] \boldsymbol{q}. \tag{10}$$

The strain energy in global coordinates can be written as

$$U_e = \frac{1}{2} \boldsymbol{q}^T \boldsymbol{k} \boldsymbol{q}, \tag{11}$$

where $\boldsymbol{k}$ is the element stiffness matrix in the global coordinates. From the previous equation, we obtain the element stiffness matrix in global coordinates as

$$\boldsymbol{k} = \boldsymbol{L}^T \boldsymbol{k}' \boldsymbol{L}. \tag{12}$$

Substituting for $\boldsymbol{L}$ and $\boldsymbol{k}'$, we get

$$\boldsymbol{k} = \frac{E_e A_e}{l_e} \begin{bmatrix} l^2 & m^2 & -l^2 & -m^2 \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}. \tag{13}$$

The element stiffness matrices are assembled in the usual manner to obtain the structural stiffness matrix. Expressions for the element stresses can be obtained by noting that a truss element in local coordinates is a simple two-force member. Thus, stress σ in a truss element is given as

$$\sigma = E_e \varepsilon. \tag{14}$$

Since strain ε is the change in length per unit of original length,

$$\sigma = E_e \frac{q_2' - q_1'}{l_e} = \frac{E_e}{l_e} [-1 \quad 1] \begin{bmatrix} q_1' \\ q_2' \end{bmatrix}. \tag{15}$$

This equation can be written in terms of the global displacements $\boldsymbol{q}$ using the transformation $\boldsymbol{q}' = \boldsymbol{L}\boldsymbol{q}$ as

$$\sigma = \frac{E_e}{l_e} [-1 \quad 1] \boldsymbol{L}\boldsymbol{q}. \tag{16}$$

Substituting for $\boldsymbol{L}$ yields

$$\sigma = \frac{E_e}{l_e} [-l \quad -m \quad l \quad m] \boldsymbol{q}. \tag{17}$$

Once the displacements are determined by solving the finite element equations, the stresses can be recovered for each element. Note that a positive stress implies that the element is in tension and a negative stress means compression.

Finally, in finite element analysis, we have the equation

$$\boldsymbol{K}\boldsymbol{Q} = \boldsymbol{F}, \tag{18}$$

where $\boldsymbol{K}$ is the assemble of stiffness matrices in all elements, $\boldsymbol{F}$ is the load vector, and $\boldsymbol{Q}$ is the unknown global displacements vector.

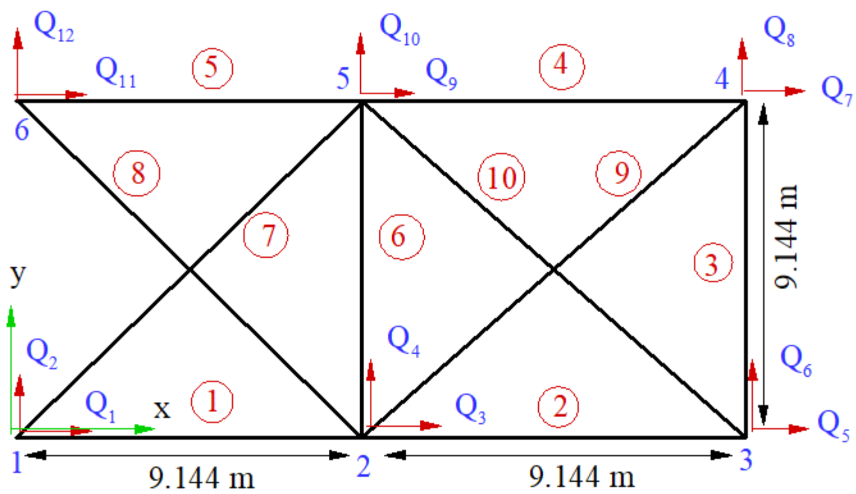

Figure 3. Configuration of truss structure problem.

The nodal coordinate data of the present problem are given in Table 1:

Table 1. Nodal coordinate.

Node	*x* (m)	*y* (m)
1	0	0
2	9.144	0
3	18.288	0
4	18.288	9.144
5	9.144	9.144
6	0	9.144

The element connectivity information is expressed in Table 2:

Table 2. Element connectivity.

Element	1	2
1	1	2
2	2	3
3	3	4
4	4	5
5	5	6
6	2	5
7	1	5
8	6	2
9	2	4
10	5	3

Moreover, using the nodal coordinate data and with element connectivity information, the direction cosines can be listed as in Table 3:

Table 3. Direction cosines.

Element	l_e	l	m
1	9.144	1	0
2	9.144	1	0
3	9.144	0	1
4	9.144	-1	0
5	9.144	-1	0
6	9.144	0	1
7	12.931	0.707	0.707
8	12.931	0.707	-0.707
9	12.931	0.707	0.707
10	12.931	0.707	-0.707

In an optimization problem, achieving the design variables that obtain an objective function with the least value, while satisfying all design constraints, is possible. The objective function in this study includes the weight of the truss structure and it is defined as follows:

$$W = \sum_{j=1}^{J} \rho \cdot L_j \cdot A_j, \tag{19}$$

where J is the number of truss members, ρ is the density of truss members, L_j is the length of the j^{th} member of the truss, and A_j is the cross-sectional area of the j^{th} member of the truss.

Optimization model constraints and constraints related to axial stress of truss members

The structure is a truss, so the only applied force in the structure is the axial force. As a result, the axial stress must be less than the allowable stress:

$$\sigma_{Ed} \leq \sigma_{Rd}, \tag{20}$$

where σ_{Ed} is the existing stress (force on the cross-sectional area of the element) and σ_{Rd} is the allowable axial stress.

Constraints on the relocation of truss nodes

In truss structures, the displacement of truss nodes is essential and should be limited. So, we have:

$$Q_{x,\max} \leq Q_{x,\text{limit}} \tag{21}$$

$$Q_{y,\max} \leq Q_{y,\text{limit}}, \tag{22}$$

where $Q_{x,\max}$, $Q_{y,\max}$, $Q_{x,\text{limit}}$ and $Q_{y,\text{limit}}$ are the displacements of the existing node i in the X-direction and Y-direction, and permissible relocation of node i in the X-direction and Y-direction.

Design examples

Several examples of scientific papers were evaluated to examine the accuracy of the present results and to show the efficiency of the proposed algorithm. The genetic algorithm optimization code was written in the MATLAB-software [1]. Configurations of the truss and constraints were created and analyzed within MATLAB. All results were obtained from the MATLAB-software. In the following sections, several examples of related studies are evaluated to examine the accuracy of the results and to show the efficiency and effectiveness of the proposed algorithm.

Six nodes and ten members

In this example, a six-node truss is evaluated. Figure 3 shows the geometric characteristics of the loading and support conditions of this truss. The material of the truss elements is Aluminum 6063-T5 with a Young modulus of elasticity E = 68.95 GPa and density ρ = 2767.99 kg/m^3.

The maximum allowable stress (σ_{Rd}) equals to 172.40 MPa, the maximum allowable nodes displacement ($Q_{\max}$) equals to 50.8 mm in both vertical and horizontal directions, and the load (P) equals to 444.82 kN. The truss is subjected to vertical loadings. The loads are considered at nodes 2 and 3. The cross-sectional areas $A1...A10$ are defined as the design variables with a minimum size limit of 1045.16 mm^2 and a maximum size limit of 21612.86 mm^2. All cross-sectional areas are selected from Table 4 to design the truss structural members.

Table 4. Cross-sectional areas.

Cross-sectional areas [mm^2]						
1045.16	1696.77	2180.64	2503.22	3206.45	8709.66	12129.01
1161.30	1858.06	2238.71	2696.77	3303.22	8967.72	12838.68
1283.87	1890.32	2290.32	2722.58	3703.22	9161.27	14193.52
1374.19	1993.54	2341.93	2896.77	4658.06	9999.98	14774.16
1535.48	1993.54	2477.41	2961.28	5141.93	10322.56	17096.74
1690.32	2019.35	2496.77	3096.77	7419.34	10903.20	19354.80
						21612.86

Tables 5 and 6 summarize the best designs presented so far, along with the results of the present study. Rajeev et al. [10], Coello et al. [1], and Camp et al. [17] used a conventional genetic algorithm, while Nanakorn et al. used an adaptive penalty function in a genetic algorithm [1].

Table 5. Comparison of optimal cross-sectional area results of six-node truss structures.

Design variables (mm^2)	Rajeev et al.	Coello et al.	Camp et al.	Nanakorn et al.	Present study
$A1$	21612.86	19354.80	19354.80	21612.86	21612.86
$A2$	1045.16	1045.16	1045.16	1045.16	1161.30
$A3$	14193.52	14774.16	17096.74	14774.16	14193.52
$A4$	9999.98	8709.66	8709.66	9999.98	9999.98
$A5$	1045.16	1045.16	1045.16	1045.16	1045.16
$A6$	1045.16	1045.16	1045.16	1045.16	1161.30
$A7$	9161.27	8967.72	4658.06	4658.06	4658.06
$A8$	12838.68	14193.52	14774.16	14774.16	14774.16
$A9$	12838.68	14193.52	14193.52	14193.52	14193.52
$A10$	1690.32	1045.16	1045.16	1045.16	1045.16

In this research, the truss structure geometry was analyzed by using the genetic algorithm built in MATLAB, with the same conditions. The optimal results of the 10-bar truss structure are reported in Table 6. The values obtained are the best compromise solutions. The corresponding estimation function value, that is the minimized weight of truss structure, was $W(A1, A2, ..., A10)$ = 2486 kg. As shown in Table 6, the value of weight obtained from the genetic algorithm is less than given by the algorithms used by Rajeev, Coello, and Nanakorn and inconsiderably greater than by Camp.

Table 6. Comparison of optimal weight results of the six-nod truss structure.

Design variables (mm2)	Rajeev et al.	Coello et al.	Camp et al.	Nanakorn et al.	Present study
$A1$	547.03	489.88	489.88	547.03	547.03
$A2$	26.45	26.45	26.45	26.45	29.39
$A3$	359.25	373.94	432.73	373.94	359.25
$A4$	253.10	220.45	220.45	253.10	253.10
$A5$	26.45	26.45	26.45	26.45	26.45
$A6$	26.45	26.45	26.45	26.45	26.45
$A7$	327.93	321.01	166.74	166.74	166.74
$A8$	459.57	508.07	528.85	528.85	528.85
$A9$	459.57	508.07	508.07	508.07	508.07
$A10$	60.51	37.41	37.41	37.41	37.41
Total weight (kg)	2546	2538	2463	2495	2486

The algorithm achieves a good solution after 200 iterations. In addition, the convergence of the genetic algorithm for the six-nodes truss is shown in Figure 4. In this figure, the process of reducing the structure's weight with the number of simulations is observed. Therefore, the displacement vector can be obtained as in Table 7. In addition, the stresses in elements are obtained as in Table 8.

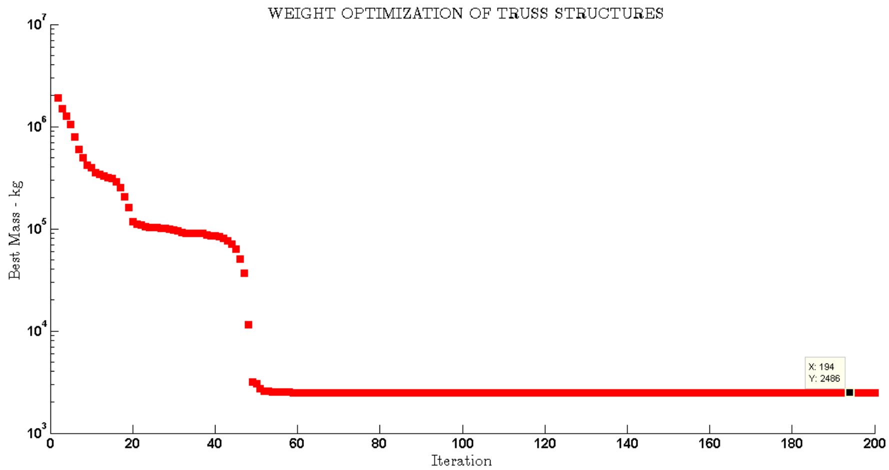

Figure 4. Convergence of the genetic algorithm for a six-node planar truss.

Table 7. The result of deflection for each node.

Degree of freedom	Displacement (mm)
Q_1	0
Q_2	0
Q_3	-7.0316
Q_4	-34.2936
Q_5	-12.8574
Q_6	-50.6817
Q_7	6.7422
Q_8	-50.0506
Q_9	6.1112
Q_{10}	-20.1036
Q_{11}	0
Q_{12}	0

Table 8. The result of stress for each element.

Element	Stress (MPa)
1	-53.0213
2	-43.9295
3	4.7584
4	4.7584
5	46.0811
6	106.9989
7	-52.7548
8	102.7548
9	-7.4772
10	43.7704

Conclusions

Genetic algorithms are frequently applied in engineering design problems. In this paper, a genetic algorithm approach was used to simultaneously solve the optimization problem of sizing and topology optimization of truss structures. After comparing the 10-bar truss structure and our calculation results, the minimum weight of $W(A1, A2, \ldots, A10) = 2486$ kg was acquired. Then, the weight of the 10-bar truss structure was improved, and the efficiency of the proposed method was confirmed. This is inferred since the improved GA is a global solution.

References

[1] Kazemzadeh Azad S, Hasançebi O and Saka MP (2014). Guided stochastic search technique for discrete sizing optimization of steel trusses: a design-driven heuristic approach, *Computers & Structures,* 134, 62–74. DOI:10.1016/j.compstruc.2014.01.005

[2] Kaveh A, Shahrouzi M (2006). Simulated annealing and adaptive dynamic variable band mutation for structural optimization by genetic algorithms, *Asian Journal of Civil Engineering (Building and Housing),* 7, 651–670. https://www.sid.ir/en/Journal/ViewPaper.aspx?ID=85223

[3] Croce ES, Ferreira EG, Lemonge AC, Fonseca LG, Barbosa HJC (2004). A genetic algorithm for structural optimization of steel truss roofs, NUMEC - Nucleo ´ de Pesquisas em Metodos ´ Computacionais em Engenharia Universidade Federal de Juiz de Fora, Faculdade de Engenharia, Departamento de Estruturas Campus Universitario, Bairro Martelos, CEP 36036-330, Juiz de Fora, MG, Brasil. artigo (567).pdf (lncc.br)

[4] Holland JH (1975). *Adaptation in Natural and Artificial Systems*. Ann Arbor: The University of Michigan Press. Genetic Algorithms and Adaptation | SpringerLink

[5] Goldberg DE (1983). *Computer-aided gas pipeline operation using genetic algorithms and rule learning*. Ph.D. dissertation, University of Michigan, Ann Arbor.

DOI:10.1007/BF01198147
[6] Holland JH, Reitman JS (1977). Cognitive systems based on adaptive algorithms, ACM *Sigart Bull*, 63, 49. DOI:10.1145/1045343.1045373
[7] Kaveh A, Mahdavi VR (2014). Colliding bodies optimization method for the optimum discrete design of truss structures, *Computers & Structures*, 139, 43–53. DOI:10.1016/j.compstruc.2014.04.006
[8] Gholizadeh S, Ebadijalal M (2018). Performance-based discrete topology optimization of steel braced frames by a new metaheuristic, *Advances in Engineering Software*, 123, 77–92. DOI:10.1016/j.advengsoft.2018.06.002
[9] Wu SJ, Chow PT (1995). Steady-state genetic algorithms for discrete optimization of trusses, *Computers & Structures*, 56, 979–991. DOI:10.1016/0045-7949(94)00551-D
[10] Rajeev S, Krishnamoorthy CS (1992). Discrete optimization of structures using genetic algorithm, *Journal of Structural Engineering*, 118, 1233–1250. DOI:10.1061/(ASCE)0733-9445(1992)118:5(1233)
[11] Ringertz UT (1988). On methods for discrete structural constraints, *Engineering Optimization*, 13, 47–64. DOI:10.1080/03052158808940946
[12] Li LJ, Huang ZB, Liu F (2009). A heuristic particle swarm optimization method for truss structures with discrete variables, *Computers & Structures*, 87, 435–44. DOI:10.1016/j.compstruc.2009.01.004
[13] Yokota T, Taguchi T, Gen M (1998). A Solution Method for Optimal Weight Design Problem of 10 Bar Truss Using Genetic Algorithms, GEN Department of Industrial and Systems Engineering Ashikaga Institute of Technology, Ashikaga 326, Japan. PII: S0360-8352(98)00096-5 (123project.ir)
[14] Crowdsourced software recommendation. website: https://www.mathworks.com/
[15] Coello CA, Rudnick R and Christiansen AD (1994). Using genetic algorithms for the optimal design of trusses, Proc of the Sixth International Conference on Tools with Artificial Intelligence. New Orleans, LA, Silver Spring MD: IEEE Computer Soc, 88–94. DOI:10.1109/TAI.1994.346509
[16] Camp C, Pezeshk S, Cao G (1998). Optimized design of two-dimensional structure using genetic algorithm, *Journal of Structural Engineering*, 124, 551–559. DOI:10.1061/(ASCE)0733-9445(1998)124:5(551)
[17] Nanakorn P, Meesomklin K (2001). An adaptive penalty function in genetic algorithms for structural design optimization, *Computers & Structures*, 79, 2527–2539. DOI:10.1016/S0045-7949(01)00137-7

Azad Javanmiri, Jari Mäkinen
Tampere University
P.O.Box 600, FI-33101 Tampere, Finland
azad.javanmiri@tuni.fi, jari.makinen@tuni.fi