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Robust Output Regulation of a Flexible Satellite ⋆

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Abstract: We consider a PDE-ODE model of a satellite and robust output regulation of the corresponding model. The satellite is composed of two flexible solar panels and a rigid center body. Exponential stability of the model is proved using passivity and resolvent estimates in the port-Hamiltonian framework. In addition, we construct a simple low-gain controller for robust output regulation of the satellite model.

Keywords: Port-Hamiltonian system, stability analysis, output regulation, distributed parameter systems.

1. INTRODUCTION

Flexible structures are widely used in the modern technology because of their advantages such as light weight, cost effectiveness and low energy consumption. Flexibility of these structures leads to problems of structural vibration and shape deformation, hence control problems of flexible systems have become a very interesting topic in research. Moreover, flexible structures are distributed parameter systems and they are often modeled as partial differential equations. Applications of flexible structures can be found, e.g., in robotics, satellites and wind turbines.

For the past few decades, satellite models have attracted many researchers in science and engineering as they are increasingly used, for instance, in communication systems, remote sensors, navigation and earth sciences. There are a number of satellites that are modeled as two flexible solar panels connected to a center rigid body. However, the flexibility of the panels affects the model dynamics such as shape deformation, which leads to challenges in controlling these type of systems. Control problems for satellite models can be found, for example, in Bontsema (1989), Aoues, Cardoso-Rebeiro, Matignon and Alazard (2018), Souza (2015) and Wei and Shuzhi Sam (2015). Robust output regulation of a coupled PDE-ODE system is considered, e.g., in Zhao and Weiss (2018). However, output regulation of satellite models has not been considered in the literature to our knowledge.

The goal of robust output regulation is designing a controller in such a way that the output of the controlled system converges to a given reference signal asymptotically despite perturbations, disturbances and uncertainties in the system.The main key in the construction of a robust regulating controller is the internal model principle which provides complete knowledge of the controllers and the ability to solve the robust output regulation problem. The investigation of robust output regulation theory was started in the 1970's for finite dimensional systems by Davison (1976), Francis and Wonham (1976), and Francis and Wonham (1975), and since then it has been developed for infinite dimensions by many authors, see for example, Paunonen and Pohjolainen (2014) and the references therein.

Many physical systems can be modeled as port-Hamiltonian systems(PHSs) (see Jacob, Zwart (2011)). The class of port-Hamiltonian systems includes a wide range of models including flexible structures, traveling waves in acoustics, heat exchangers, suspension systems and bio reactors. Moreover, several interconnected PHSs via standard feedback interconnection is again a PHS. Stability analysis of port-Hamiltonian systems is considered in Augner (2019), Augner and Jacob (2013) and Augner (2018). Robust output regulation problem of boundary controlled port-Hamiltonian systems can be found, e.g., in Humaloja and Paunonen (2018).

In this paper, we consider a satellite system that is composed of two symmetric flexible solar panels and a center rigid body. The panels are modeled as Euler-Bernoulli beams. In addition, it is assumed that the beams have distributed viscous damping. Both panels are modeled in the port-Hamiltonian framework and the passivity of the system is proved by computing the energy balance equation.

As the main contribution of the paper, a power-preserving interconnection is shown between the satellite panels and the center rigid body. This interconnection results in an impedance passive port-Hamiltonian system. We stabilize the rigid body by negative output feedback and we utilize the passivity property in proving that the satellite system generates an exponentially stable semigroup. Due to the exponential stability of the model, using the theories from

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Pohjolainen (1985) and Paunonen (2016), we construct a simple low-gain controller that solves the robust output regulation problem.

The paper is organized as follows. In section 2, we formulate our satellite model as an abstract PDE-ODE system and establish a power-preserving interconnection between the satellite panels and the center rigid body in the port-Hamiltonian framework. In section 3, we prove the exponential stability of the satellite system. In section 4, we consider robust output regulation of the satellite model and we construct a low-gain controller that achieves robust output regulation of the satellite model. In section 5, we conclude our work and present topics for future research.

1.1 Notation

For normed linear spaces X and Y, $\mathcal{L}(X,Y)$ denotes the set of all bounded linear operators from X into Y. For a linear operator A, $D(A), \mathcal{R}(A)$ and $\mathcal{N}(A)$ denote the domain, range and the kernel of A, respectively. The resolvent and the spectrum of A are denoted by $\rho(A)$ and $\sigma(A)$, respectively. The resolvent operator is denoted by $R(\lambda, A) = (\lambda - A)^{-1}$ for $\lambda \in \rho(A)$. We denote by X_{-1} the completion of X with respect to the norm $\|x\|_{-1} = \|((\beta I - A)^{-1}x)\|,\ x \in X, \beta \in \rho(A)$ and by $A_{-1} \in \mathcal{L}(X, X_{-1})$ the extension of A to X_{-1}. For $x(t,\xi) \in X$, $\dot{x}$ and x' denote time and spatial derivatives of x, respectively.

2. THE SATELLITE MODEL

We consider a dynamic model of a satellite composed of a center rigid body and two symmetric flexible solar panels. The panels are modeled as Euler-Bernoulli beams. Let us assume that both beams are of length 1 with cross sectional area a, mass density ρ, Young's modulus of elasticity E, second moment of area of the cross section I and the viscous damping coefficient γ.

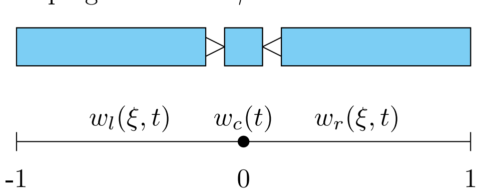

Fig. 1. Satellite with flexible solar panels

Let m and I_m denote the mass and the mass moment of inertia of the center rigid body. If $w_l(\xi,t)$ and $w_r(\xi,t)$ are the transverse displacements of the left and the right beam, respectively, and $w_c(t)$ and $\theta_c(t)$ are the linear and angular displacements of the rigid body respectively, then the governing equations of motion of the satellite are given by (similar models can be found in Bontsema (1989), Wei and Shuzhi Sam (2015)),

$$\ddot{w}_l(\xi,t) + \frac{EI}{\rho a} w_l''''(\xi,t) + \frac{\gamma}{\rho a}\dot{w}_l(\xi,t) = 0,\ -1 < \xi < 0, t > 0,$$

$$\ddot{w}_r(\xi,t) + \frac{EI}{\rho a} w_r''''(\xi,t) + \frac{\gamma}{\rho a}\dot{w}_r(\xi,t) = 0,\ 0 < \xi < 1, t > 0,$$

with the boundary conditions,

$$m\ddot{w}_c(t) = EIw_l'''(0,t) - EIw_r'''(0,t) + u_1(t),$$
$$I_m\ddot{\theta}_c(t) = -EIw_l''(0,t) + EIw_r''(0,t) + u_2(t),$$
$$w_l''(-1,t) = 0, \quad w_r''(1,t) = 0,$$
$$w_l'''(-1,t) = 0, \quad w_r'''(1,t) = 0,$$
$$\dot{w}_l(0,t) = \dot{w}_r(0,t) = \dot{w}_c(t),$$
$$\dot{w}_l'(0,t) = \dot{w}_r'(0,t) = \dot{\theta}_c(t),$$
$$y_1(t) = \dot{w}_c(t),\ y_2(t) = \dot{\theta}_c(t).$$

where $u_1(t)$ and $u_2(t)$ are external control inputs and $y_1(t)$ and $y_2(t)$ are outputs of the satellite model. Here $\dot{w}_c(t) = \dot{w}_l(\xi,t)|_{\xi=0} = \dot{w}_r(\xi,t)|_{\xi=0}$ and $\dot{\theta}_c(t) = \dot{w}_l'(\xi,t)|_{\xi=0} = \dot{w}_r'(\xi,t)|_{\xi=0}$ are the linear and the angular velocities of the rigid body respectively. We formulate this system as an abstract system of a PDE and an ODE in the port-Hamiltonian framework similarly as in Augner (2019).

2.1 Abstract Formulation of the Beams

The standard boundary control and boundary observation problem for port-Hamiltonian systems of order $N = 2$ on the spatial interval $[a,b]$ takes the form,

$$\dot{x}(t,\xi) = P_2(\mathcal{H}x)''(t,\xi) + P_1(\mathcal{H}x)'(t,\xi) + P_0(\mathcal{H}x)(t,\xi),$$
$$u(t) = \mathcal{B}x(t,\xi),$$
$$y(t) = \mathcal{C}x(t,\xi),$$

where, $P_0, P_1, P_2 \in \mathbb{R}^{n\times n}$, and $\mathcal{H} : [a,b] \to \mathbb{R}^{n\times n}$ is the Hamiltonian density matrix function.

Now, we formulate the beam systems in the satellite model as boundary controlled port-Hamiltonian systems of order $N = 2$.

The left beam in the satellite system can be modeled as a boundary controlled port-Hamiltonian system of order $N = 2$ on the energy space $X_l = L^2([-1,0];\mathbb{R}^2)$. The space X_l is a Hilbert space equipped with the energy norm $\|x_l(t)\|^2_{X_l} := \frac{1}{2}\langle x_l(t), \mathcal{H}_l x_l(t)\rangle_{L^2},\ x_l \in X_l$, where $\mathcal{H}_l$ given in (2) is the Hamiltonian density matrix function associated with the left beam.

The left beam that we detach from the satellite system has $u_{l1}(t) = \dot{w}_l(0,t),\ u_{l2}(t) = \dot{w}_l'(0,t)$ as boundary inputs and $y_{l1}(t) = -EIw_l'''(0,t),\ y_{l2}(t) = EIw_l''(0,t)$ as outputs. Then choosing the energy state variable $x_l(t) = \begin{bmatrix} \rho a\dot{w}_l(\xi,t) \\ w_l''(\xi,t)\end{bmatrix}$, we have

$$\frac{d}{dt}x_l(t) = \mathcal{A}_l x_l(t),\ u_l(t) = \mathcal{B}_l x_l(t),\ y_l(t) = \mathcal{C}_l x_l(t), \quad (1)$$

where,

$$\mathcal{A}_l = \begin{bmatrix} -\gamma(\rho a)^{-1} & -EI\partial_{\xi\xi} \\ (\rho a)^{-1}\partial_{\xi\xi} & 0 \end{bmatrix},$$

$$\mathcal{B}_l x_l(t) = \begin{bmatrix} \dot{w}_l(0,t) \\ \dot{w}_l'(0,t)\end{bmatrix} \text{ and,}$$

$$\mathcal{C}_l x_l(t) = \begin{bmatrix} -EIw_l'''(0,t) \\ EIw_l''(0,t)\end{bmatrix}.$$

Here

$$P_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0\end{bmatrix},\ P_1 = 0,\ P_0 = \begin{bmatrix} -\gamma & 0 \\ 0 & 0\end{bmatrix},$$

$$\mathcal{H}_l = \begin{bmatrix} (\rho a)^{-1} & 0 \\ 0 & EI\end{bmatrix} \quad (2)$$

and

$$D(\mathcal{A}_l) = \{x_l \in X_l \mid \mathcal{H}_l x_l \in H^2([-1,0];\mathbb{R}^2), \ x_{l2}(-1) = x_{l2}'(-1) = 0\}.$$

The Hamiltonian i.e., energy for the left beam is given by,

$$H_l = \frac{1}{2}\|x_l\|^2_{X_l} = \frac{1}{2}\int_{-1}^{0} (\rho a|\dot{w}_l(t,\xi)|^2 + EI|w_l''(t,\xi)|^2)d\xi.$$

Differentiating,

$$\begin{aligned}\dot{H}_l &= \int_{-1}^{0} (\rho a\dot{w}_l(t,\xi)\ddot{w}_l(t,\xi) + EIw_l''(t,\xi)\dot{w}_l''(t,\xi))d\xi,\\ &= \int_{-1}^{0} \frac{\partial}{\partial\xi}(EIw_l''(t,\xi)\dot{w}_l'(t,\xi) - \dot{w}_l(t,\xi)EIw_l'''(t,\xi))d\xi\\ &\quad - \gamma\int_{-1}^{0} \dot{w}_l(t,\xi)^2 d\xi,\\ &\le EIw_l''(t,0)\dot{w}_l'(t,0) - \dot{w}_l(t,0)EIw_l'''(t,0),\\ &= u_l(t)^T y_l(t).\end{aligned}$$

This implies that the energy satisfies

$$\frac{1}{2}\frac{d}{dt}\|x_l(t)\|^2_{X_l} \le u_l(t)^T y_l(t).$$

Hence, the left beam is an impedance passive system on the Hilbert space $X_l = L^2([-1,0];\mathbb{R}^2)$, and thus, the operator $A_l = \mathcal{A}_l|_{\mathcal{N}(\mathcal{B}_l)}$ generates a contraction semigroup $T_l(t)$ on X_l. That is, $\|T_l(t)\| \le 1$ on X_l.

In the same way, the right beam that we detach from the satellite system can be modeled as a boundary controlled port-Hamiltonian system on the Hilbert space $X_r = L^2([0,1];\mathbb{R}^2)$ with $u_{r1}(t) = \dot{w}_r(0,t)$, $u_{r2}(t) = \dot{w}_r'(0,t)$ as boundary inputs and $y_{r1}(t) = EIw_r'''(0,t)$, $y_{r2}(t) = -EIw_r''(0,t)$ as outputs. Choosing the energy state variable $x_r(t) = \begin{bmatrix}\rho a\dot{w}_r(\xi,t)\\ w_r''(\xi,t)\end{bmatrix}$, we have

$$\frac{d}{dt}x_r(t) = \mathcal{A}_r x_r(t),\ u_r(t) = \mathcal{B}_r x_r(t),\ y_r(t) = \mathcal{C}_r x_r(t), \tag{3}$$

where,

$$\mathcal{A}_r = \begin{bmatrix}-\gamma(\rho a)^{-1} & -EI\partial_{\xi\xi}\\ (\rho a)^{-1}\partial_{\xi\xi} & 0\end{bmatrix},$$

$$\mathcal{B}_r x_r(t) = \begin{bmatrix}\dot{w}_r(0,t)\\ \dot{w}_r'(0,t)\end{bmatrix} \text{ and,}$$

$$\mathcal{C}_r x_r(t) = \begin{bmatrix}EIw_r'''(0,t)\\ -EIw_r''(0,t)\end{bmatrix}.$$

Here P_0, P_1, P_2 and $\mathcal{H}_r$ are defined the same as of the left beam and

$$D(\mathcal{A}_r) = \{x_r \in X_r \mid \mathcal{H}_r x_r \in H^2([0,1];\mathbb{R}^2),\ x_{r2}(1) = x_{r2}'(1) = 0\}.$$

Furthermore, it can be shown analogously to the case of the left beam that the energy satisfies

$$\frac{1}{2}\frac{d}{dt}\|x_r(t)\|^2_{X_r} \le u_r(t)^T y_r(t),$$

which shows that the right beam is also an impedance passive system on the Hilbert space $X_r = L^2([0,1];\mathbb{R}^2)$,thus, the operator $A_r = \mathcal{A}_r|_{\mathcal{N}(\mathcal{B}_r)}$ generates a contraction semigroup $T_r(t)$ on X_r.

2.2 Combined Beam System

The two beam systems (1) and (3) can be combined into a single open loop system as follows:

$$\frac{d}{dt}x(t) = \mathcal{A}x(t),\quad \hat{\mathcal{B}}x(t) = \hat{u}(t),\quad \hat{\mathcal{C}}x(t) = \hat{y}(t),$$

where

$$x(t) = \begin{bmatrix}x_l(t)\\ x_r(t)\end{bmatrix},\quad \hat{u}(t) = \begin{bmatrix}u_l(t)\\ u_r(t)\end{bmatrix},\quad \hat{y}(t) = \begin{bmatrix}y_l(t)\\ y_r(t)\end{bmatrix},$$

$$\mathcal{A} = \begin{bmatrix}\mathcal{A}_l & 0\\ 0 & \mathcal{A}_r\end{bmatrix},\quad \hat{\mathcal{B}} = \begin{bmatrix}\mathcal{B}_l & 0\\ 0 & \mathcal{B}_r\end{bmatrix},\quad \hat{\mathcal{C}} = \begin{bmatrix}\mathcal{C}_l & 0\\ 0 & \mathcal{C}_r\end{bmatrix},$$

and $D(\mathcal{A}) = D(\mathcal{A}_l) \times D(\mathcal{A}_r)$.

Using the boundary conditions $u_{l1}(t) = \dot{w}_l(0,t) = \dot{w}_r(0,t) = u_{r1}(t)$ and $u_{l2}(t) = \dot{w}_l'(0,t) = \dot{w}_r'(0,t) = u_{r2}(t)$, the energy of the combined system is given by,

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\|x(t)\|^2 &= \frac{1}{2}\frac{d}{dt}\|x_l(t)\|^2_{X_l} + \frac{1}{2}\frac{d}{dt}\|x_r(t)\|^2_{X_r}\\ &\le u_l(t)^T y_l(t) + u_r(t)^T y_r(t),\\ &= u_l(t)^T(y_l(t) + y_r(t)).\end{aligned} \tag{4}$$

Let us define a new output function

$$\begin{aligned}y(t) = y_l(t) + y_r(t) &= \mathcal{C}_l x_l(t) + \mathcal{C}_r x_r(t)\\ &= (\mathcal{C}_l\ \mathcal{C}_r)\begin{pmatrix}x_l(t)\\ x_r(t)\end{pmatrix}\end{aligned}$$

and an input function

$$u(t) = \begin{pmatrix}\frac{1}{2}\mathcal{B}_l & \frac{1}{2}\mathcal{B}_r\end{pmatrix}\begin{pmatrix}x_l(t)\\ x_r(t)\end{pmatrix}.$$

With this input $u(t)$ and output $y(t)$, it follows from (4) that the system

$$\frac{d}{dt}x(t) = \mathcal{A}x(t),\quad \mathcal{B}x(t) = u(t),\quad \mathcal{C}x(t) = y(t) \tag{5}$$

is an impedance passive port-Hamiltonian system on $X = X_l \times X_r$ and $A = \mathcal{A}|_{\mathcal{N}(\mathcal{B})}$ generates a contraction semigroup $T(t)$ on X.

2.3 Abstract Formulation of the Rigid Body

The center rigid body that we detach from the satellite system has $u_{c1}(t) = EIw_l'''(0,t) - EIw_r'''(0,t)$ and $u_{c2}(t) = -EIw_l''(0,t) + EIw_r''(0,t)$ as inputs and $y_{c1}(t) = \dot{w}_c(t)$ and $y_{c2}(t) = \dot{\theta}_c(t)$ as outputs. Then, with the state variable $x_c(t) = \begin{bmatrix}m\dot{w}_c(t)\\ I_m\dot{\theta}_c(t)\end{bmatrix}$, the rigid body on the Hilbert space $X_c = \mathbb{R}^2$ can be written as,

$$\frac{d}{dt}\begin{bmatrix}m\dot{w}_c(t)\\ I_m\dot{\theta}_c(t)\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}m\dot{w}_c(t)\\ I_m\dot{\theta}_c(t)\end{bmatrix} + \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}u_{c1}(t)\\ u_{c2}(t)\end{bmatrix},$$

$$y_c(t) = \begin{bmatrix}\dot{w}_c(t)\\ \dot{\theta}_c(t)\end{bmatrix}.$$

Equivalently,

$$\begin{aligned}\frac{d}{dt}x_c(t) &= A_c x_c(t) + B_c u_c(t),\\ y_c(t) &= C_c x_c(t),\end{aligned} \tag{6}$$

where,

$$A_c = 0,\ B_c = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ C_c = \begin{bmatrix}\frac{1}{m} & 0\\ 0 & \frac{1}{I_m}\end{bmatrix}, \text{ and}$$

$$u_c(t) = \begin{bmatrix}u_{c1}(t)\\ u_{c2}(t)\end{bmatrix}.$$

The Hamiltonian of the center rigid body is given by,

$$H_c = \frac{1}{2}m\dot{w}_c(t)^2 + \frac{1}{2}I_m\dot{\theta}_c(t)^2 = \frac{1}{2}\|x_c\|^2_{X_c}$$

Differentiating,

$$\dot{H}_c = \dot{w}_c(t)u_{c1}(t) + \dot{\theta}_c(t)u_{c2}(t) = u_c(t)^T y_c(t).$$

Equivalently,

$$\frac{1}{2}\frac{d}{dt}\|x_c(t)\|^2_{X_c} = u_c(t)^T y_c(t).$$

Hence, the rigid body is an impedance passive system on X_c.

2.4 The Satellite System as a Coupled PDE-ODE System

From the previous sections, we are able to write our satellite system as an abstract PDE-ODE system with the power-preserving interconnection $u(t) = y_c(t),\ u_c(t) = -y(t)$ as follows:

$$\begin{aligned}\frac{d}{dt}x(t) &= \mathcal{A}x(t),\\ \frac{d}{dt}x_c(t) &= B_c u_c(t) + B_c u_{sat}(t),\\ \mathcal{B}x(t) &= C_c x_c(t),\\ u_c(t) &= -\mathcal{C}x(t),\end{aligned} \tag{7}$$

or equivalently,

$$\frac{d}{dt}\begin{bmatrix} x(t)\\ x_c(t)\end{bmatrix} = \begin{bmatrix}\mathcal{A} & 0\\ -B_c\mathcal{C} & 0\end{bmatrix}\begin{bmatrix} x(t)\\ x_c(t)\end{bmatrix} + \begin{bmatrix}0\\ B_c\end{bmatrix}u_{sat}(t),$$

$$[\mathcal{B}\ -C_c]\begin{bmatrix} x(t)\\ x_c(t)\end{bmatrix} = 0.$$

where $u_{sat}(t) = \begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix}$.

The operator $\tilde{A}_{sat} := \begin{bmatrix}\mathcal{A} & 0\\ -B_c\mathcal{C} & 0\end{bmatrix}$ with $D(\tilde{A}_{sat}) = \{(x, x_c) \in D(\mathcal{A}) \times X_c\ :\ \mathcal{B}x = C_c x_c\}$ is dissipative, since

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\left\|\begin{bmatrix} x(t)\\ x_c(t)\end{bmatrix}\right\|^2 &= \frac{1}{2}\frac{d}{dt}\|x(t)\|^2_X + \frac{1}{2}\frac{d}{dt}\|x_c(t)\|^2_{X_c}\\ &\le u(t)^T y(t) + u_c(t)^T y_c(t),\\ &= y_c(t)^T y(t) - y(t)^T y_c(t),\\ &= 0.\end{aligned}$$

and thus, according to Augner (2019)(see, example 3.4), $\tilde{A}_{sat}$ generates C_0-semigroup of contractions on the Hilbert space $X_{sat} = X \times X_c$.

3. STABILITY OF THE SATELLITE MODEL

An important step in constructing a robust regulating controller is to analyze the stability of the system. In this section, we analyze the stability of the satellite system (7).

3.1 Stabilization of the Finite Dimensional System

Since the eigenvalues of the rigid body are zeros, it is not asymptotically stable. We stabilize the rigid body by negative output feedback, hence the new input is given by $\tilde{u}_c(t) = u_c(t) - y_c(t)$. Now, from (6), we have,

$$\begin{aligned}\frac{d}{dt}x_c(t) &= B_c\tilde{u}_c(t),\\ &= B_c u_c(t) - B_c y_c(t),\\ &= B_c u_c(t) - B_c C_c x_c(t),\\ &= -B_c C_c x_c(t) + B_c u_c(t),\\ &= \tilde{A}_c x_c(t) + B_c u_c(t),\end{aligned}$$

where $\tilde{A}_c = -B_c C_c$. The stabilized rigid body is an impedance passive system. Hence the whole satellite system (7) can be written as,

$$\begin{aligned}&\frac{d}{dt}\begin{bmatrix} x(t)\\ x_c(t)\end{bmatrix} = \begin{bmatrix}\mathcal{A} & 0\\ -B_c\mathcal{C} & -B_cC_c\end{bmatrix}\begin{bmatrix} x(t)\\ x_c(t)\end{bmatrix} + \begin{bmatrix}0\\ B_c\end{bmatrix}u_{sat}(t),\\ &[\mathcal{B}\ -C_c]\begin{bmatrix} x(t)\\ x_c(t)\end{bmatrix} = 0.\end{aligned} \tag{8}$$

where $A_{sat} := \begin{bmatrix}\mathcal{A} & 0\\ -B_c\mathcal{C} & -B_cC_c\end{bmatrix}$ with $D(A_{sat}) = \{(x, x_c) \in D(\mathcal{A}) \times X_c\ :\ \mathcal{B}x = C_c x_c\}$ generates a contraction semigroup $T_{sat}(t)$ on X_{sat}.

3.2 Stability of the Beam System

Lemma 1. The left beam system is exponentially stable.

Proof. Let $x_l(t) \in D(\mathcal{A}_l)$ be the classical solution of the left beam. If $\mathcal{A}_0 = \begin{bmatrix}0 & -EI\partial_{\xi\xi}\\ (\rho a)^{-1}\partial_{\xi\xi} & 0\end{bmatrix}$ and $C_0 = \left[(\gamma(\rho a)^{-1})^{\frac{1}{2}}\ 0\right]$, then $\mathcal{A}_l = \mathcal{A}_0 - C_0C_0^*$. Here $A_0 = \mathcal{A}_0|_{\mathcal{N}(\mathcal{B}_l)}$ generates a unitary group on X_l. It can be shown that (A_0, C_0) is exactly observable(see Ch.6, Tucsnak and Weiss (2009) for more details on exact observability).

Using the skew-adjoint property of the operator $\mathcal{A}_0$, we have

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\|x_l(t)\|^2 &= \left\langle \frac{d}{dt}x_l(t), x_l(t)\right\rangle,\\ &= \langle \mathcal{A}_l x_l(t), x_l(t)\rangle,\\ &= \left\langle \begin{bmatrix}0 & -EI\partial_{\xi\xi}\\ (\rho a)^{-1}\partial_{\xi\xi} & 0\end{bmatrix}x_l(t), x_l(t)\right\rangle\\ &\quad - \gamma(\rho a)^{-1}\left\langle\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}x_l(t), x_l(t)\right\rangle,\\ &= -\gamma\int_{-1}^{0}\dot{w}_l^2(t,\xi)d\xi.\end{aligned}$$

Now,

$$\begin{aligned}&\|x_l(T)\|^2 - \|x_l(0)\|^2\\ &= \int_0^T \frac{d}{dt}\|x_l(t)\|^2 dt,\\ &= -2\gamma\int_0^T\int_{-1}^0 \dot{w}_l^2(t,\xi)d\xi dt,\\ &\le -2\gamma C_1\int_{-1}^0 (\rho a\dot{w}_l^2(0,\xi) + (w_l''(0,\xi))^2)d\xi,\end{aligned}$$

for some $0 < C_1 < 1$ where we used the exact observability of the pair (A_0, C_0). This yields,

$$\begin{aligned}\|x_l(T)\|^2 - \|x_l(0)\|^2 &\le -C_2\|x_l(0)\|^2, \quad 0 < C_2 < 1,\\ \|x_l(T)\|^2 &\le (1 - C_2)\|x_l(0)\|^2,\\ \|x_l(T)\| &\le C\|x_l(0)\|, \quad 0 < C < 1,\\ \Leftrightarrow \quad \|T_l(T)x_l(0)\| &\le C\|x_l(0)\|.\end{aligned}$$

That is, $\|T_l(T)\| < 1$ for some $T > 0$. We obtain, [Engel and Nagel (2000), Prop.V.1.7]

$$\|T_l(t)\| \le Me^{-\omega t}, \quad M \ge 1, \quad \omega > 0,$$

by which the left beam system (1) is exponentially stable.

Corollary 2. The beam system (5) is exponentially stable.

Proof. By symmetry, it follows from lemma 1 that the right beam system (3) is exponentially stable. Hence the semigroup $T(t)$ generated by $A = \mathcal{A}|_{\mathcal{N}(\mathcal{B})}$ is exponentially stable.

3.3 Stability of the satellite system

In this section, we sketch a proof for exponential stability of the satellite model. A detailed proof will be presented in a later paper.

Theorem 3. The satellite system (8) is exponentially stable.

Proof. By Gearheart-Greiner-Prüss theorem, the semigroup $T_{sat}(t)$ generated by A_{sat} is exponentially stable on a Hilbert space if and only if the spectrum of A_{sat} lies in the complex left half-plane and $\sup_{\omega\in\mathbb{R}}\|(i\omega - A_{sat})^{-1}\| < \infty$ (see, Engel and Nagel (2000), Thm.V.1.11). Since A_{sat} generates a contraction semigroup, the spectrum $\sigma(A_{sat})$ lies in the closed complex left-half plane. It remains to prove that the resolvent $R(i\omega, A_{sat})$ of the system exists and is uniformly bounded on the imaginary axis.

According to Tucsnak and Weiss (2009)(Prop.10.1.2), there exists a unique $B \in \mathcal{L}(U, X_{-1})$ such that the equations (8) of the satellite system can be written as,

$$\frac{d}{dt}\begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix} = \begin{bmatrix} A_{-1} & BC_c \\ -B_c\mathcal{C} & -B_cC_c \end{bmatrix}\begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_c \end{bmatrix} u_{sat}(t),$$

and the resolvent of the satellite system is given by,

$$R(i\omega, A_{sat}) = \begin{bmatrix} (i\omega - A_{-1}) & -BC_c \\ B_c\mathcal{C} & (i\omega + B_cC_c) \end{bmatrix}^{-1}.$$

Let $P(i\omega)$ and $P_c(i\omega)$ be the transfer functions of $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ and $(\tilde{A}_c, B_c, C_c)$ respectively. Then the passivity of the systems implies that $\mathrm{Re}P(i\omega) \geq 0$ and $\mathrm{Re}P_c(i\omega) \geq 0$ for all $i\omega \in \rho(A)$ and $i\omega \in \rho(\tilde{A}_c)$. Also, it can be shown that $(I + P(i\omega)P_c(i\omega))$ and $(I + P_c(i\omega)P(i\omega))$ are boundedly invertible for all $\omega \in \mathbb{R}$. For more details on passive systems, see Paunonen (2017)(Appendix).

Using the Schur complement $S(i\omega) = [(i\omega + B_cC_c) + B_c\mathcal{C}(i\omega - A_{-1})^{-1}BC_c]^{-1}$, we obtain,

$$R(i\omega, A_{sat}) = \begin{bmatrix} R_{11}(i\omega, A_{sat}) & R_{12}(i\omega, A_{sat}) \\ R_{21}(i\omega, A_{sat}) & R_{22}(i\omega, A_{sat}) \end{bmatrix},$$

where,

$$\begin{aligned} R_{11}(i\omega, A_{sat}) &= R(i\omega, A) \\ &\quad - R(i\omega, A_{-1})BC_cS(i\omega)B_c\mathcal{C}R(i\omega, A), \\ R_{12}(i\omega, A_{sat}) &= R(i\omega, A_{-1})BC_cS(i\omega), \\ R_{21}(i\omega, A_{sat}) &= -S(i\omega)B_c\mathcal{C}R(i\omega, A), \\ R_{22}(i\omega, A_{sat}) &= S(i\omega). \end{aligned}$$

Using Kato perturbation formula, we have

$$\begin{aligned} &S(i\omega) \\ &= [(i\omega + B_cC_c) + B_c\mathcal{C}(i\omega - A_{-1})^{-1}BC_c]^{-1}, \\ &= R(i\omega, \tilde{A}_c) \\ &\quad - R(i\omega, \tilde{A}_c)B_cP(i\omega)(I + P(i\omega)P_c(i\omega))^{-1}C_cR(i\omega, \tilde{A}_c). \end{aligned}$$

From the stability of the beam system we have that $\|R(i\omega, A)\|$ is uniformly bounded and from the stability of the rigid body we have that $\|R(i\omega, \tilde{A}_c)\|$, $\|C_cR(i\omega, \tilde{A}_c)\|$, $\|R(i\omega, \tilde{A}_c)B_c\|$ and $\|P_c(i\omega)\|$ are all uniformly bounded and tend to zero as $|\omega| \to \infty$. Furthermore, $\|P_c(i\omega)\|$ tends to zero sufficiently fast such that $P(i\omega)P_c(i\omega)$ and $(I + P(i\omega)P_c(i\omega))^{-1}$ are uniformly bounded. This implies that the Schur complement $S(i\omega)$ is uniformly bounded. Moreover, $\|S(i\omega)\|$ tends to zero sufficiently fast as $|\omega| \to \infty$ such that $R_{11}(i\omega, A_{sat})$, $R_{12}(i\omega, A_{sat})$, and $R_{21}(i\omega, A_{sat})$ are also uniformly bounded. Hence, the resolvent $R(i\omega, A_{sat})$ is uniformly bounded and therefore A_{sat} generates an exponentially stable semigroup.

4. ROBUST OUTPUT REGULATION OF THE SATELLITE MODEL

In this section, we present the satellite system and the controller that solves the robust output regulation problem for the system. Our goal is to design a controller in such a way that the linear and angular velocities of the center rigid body converge to given reference signals of the form (11).

From the previous sections, the satellite system with control and observations on the rigid body is given by,

$$\begin{aligned} &\frac{d}{dt}\begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix} = \begin{bmatrix} \mathcal{A} & 0 \\ -B_c\mathcal{C} & 0 \end{bmatrix}\begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_c \end{bmatrix} u_{sat}(t), \\ &[\mathcal{B} \ -C_c]\begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix} = 0, \qquad (9) \\ &y_{sat}(t) = [0 \ C_c]\begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix}. \end{aligned}$$

We construct a dynamic error feedback controller of the form

$$\begin{aligned} \dot{z}(t) &= \mathcal{G}_1 z(t) + \mathcal{G}_2 e(t), \quad z(0) = z_0, \\ u_{sat}(t) &= Kz(t) - y_c(t), \end{aligned} \qquad (10)$$

on a Banach space Z, where $e(t) = y_{sat}(t) - y_{ref}(t)$, is the regulation error, $y_{ref}(t)$, a given reference signal, $\mathcal{G}_1 \in \mathcal{L}(Z)$, $\mathcal{G}_2 \in \mathcal{L}(Y_c, Z)$ and $K \in \mathcal{L}(Z, U_c)$, such that robust output regulation of the satellite system is achieved with a suitable choice of the parameters $(\mathcal{G}_1, \mathcal{G}_2, K)$. Here U_c and Y_c are the input and the output spaces of the satellite system. The term $-y_c(t)$ appears in the controller (10) because it is used to stabilize the rigid body of the satellite system, see section 3.1. The reference signals to be tracked are of the form,

$$y_{ref}(t) = a_0 + \sum_{k=1}^{q} [a_k \cos(\omega_k t) + b_k \sin(\omega_k t)], \qquad (11)$$

with $0 = \omega_0 < \omega_1 < \cdots < \omega_q$ as the known frequencies and $\{a_k\}_{k=0}^q$, $\{b_k\}_{k=1}^q \subset Y_c$ as the unknown coefficients.

The Robust Output Regulation Problem. Choose the controller $(\mathcal{G}_1, \mathcal{G}_2, K)$ in such a way that

(a) The closed loop semigroup $T_{cl}(t)$ comprised of the satellite system (9) and the controller (10) is exponentially stable.
(b) For all initial states $x(0) \in D(\mathcal{A})$ and $x_c(0) \in X_c$ satisfying $\mathcal{B}x(0) = C_cx_c(0)$, the regulation error $e(t)$ satisfies $e^{\alpha t}\|y_{sat}(t) - y_{ref}(t)\| \to 0$ as $t \to \infty$, for some $\alpha > 0$.
(c) If the system $(\mathcal{A}, \mathcal{B}, \mathcal{C}, A_c, B_c, C_c)$ is perturbed in such a way that the perturbed closed loop system is still exponentially stable, the perturbed $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ is a boundary controlled impedance passive port-Hamiltonian system and the perturbed (A_c, B_c, C_c)

is an impedance passive system, then (b) continues to hold for some $\tilde{\alpha} > 0$.

4.1 Controller for the Satellite Model

Since the system is exponentially stable, using the theories in Rebarber and Weiss (2003), Paunonen (2016) and Pohjolainen (1985), a simple low-gain controller can be constructed for obtaining robust output regulation of the model with the following choices of parameters. Defining $Z = Y_c^{2q+1}$, and $\omega_{-k} = -\omega_k, k = 1, 2, \cdots, q$,

$$\mathcal{G}_1 = \operatorname{diag}(i\omega_{-q}I_{Y_c}, \cdots i\omega_0 I_{Y_c}, \cdots, i\omega_q I_{Y_c}),$$
$$K = \epsilon(K_0^{-q}, \cdots, K_0^0, \cdots, K_0^q), \text{ where, } K_0^k = P_{sat}(i\omega_k)^\dagger,$$
$$\mathcal{G}_2 = (-(P_{sat}(i\omega_k)K_0^k)^*)_{k=-q}^q.$$

Here $P_{sat}(i\omega_k) = C_c S(i\omega_k) B_c, S(i\omega_k)$ is the Schur complement, is the transfer function of the satellite system (9) which can be obtained by frequency response measurement from the system, $P_{sat}(i\omega_k)^\dagger$ is the Moore-Penrose pseudoinverse of $P_{sat}(i\omega_k)$ and the tuning parameter $\epsilon > 0$ is to be chosen sufficiently small such that the closed loop system is exponentially stable.

5. CONCLUSION

We considered a PDE-ODE model of a flexible satellite. The model was formulated as an abstract system in the port-Hamiltonian framework and it was shown that there is a power-preserving interconnection between the satellite panels and the center rigid body of the model. The exponential stability of the model was proved using passivity and the resolvent estimate where we used Schur complement and Kato perturbation formula. Exponential stability of the satellite model enabled us to construct a simple low-gain controller for robust output regulation of the model.

Future works are possible for the same model. Numerical simulations testing the effectiveness of the controller and technical details in the proofs of exponential stability will be presented in a later paper. Since the model is an exponentially stable impedance passive system, a passive controller can be constructed for this model. In this paper, the beams are assumed to have damping, one could also consider an undamped model.

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