Pairwise Error Probability of Non-Orthogonal Multiple Access with I/Q Imbalance

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***Abstract*—In this paper, we investigate the performance of a NOMA system under the effect of in-phase/quadrature-phase imbalance (IQI) over Rayleigh fading channels. Specifically, a closed-form pairwise error probability (PEP) expression for the underlying NOMA scenario is derived, which is subsequently utilized to analyze the achievable diversity order of NOMA users under IQI. Monte Carlo simulation results are provided to corroborate the analysis and to illustrate the detrimental effect of IQI on the performance of NOMA users.**

***Index terms*—** Diversity order, IQI, NOMA, pairwise error probability.

I. Introduction

Non-orthogonal multiple access (NOMA) has been identified as a key enabling technology for the next generation of wireless networks, which is envisaged to enhance the spectral efficiency and support massive connectivity. NOMA is realized by allowing a controlled interference among users in the power-domain or code-domain using the same time and frequency resources. At the users' terminals, the corresponding users' signals are decoded using multi-user detection, which, in the context of power domain NOMA, is performed using successive interference cancellation (SIC). Recently, the authors in [1], [2] demonstrated that NOMA is a promising solution that is able to provide enhanced spectral efficiency, relaxed channel feedback, and increased cell-edge throughput.

On the other hand, the increasing demand for wireless applications have yielded stringent design targets on radio frequency (RF) transceivers. These targets include low power dissipation, low cost, and small form factor. Accordingly, integrable direct-conversion transceivers, which operate without the need for external intermediate frequency filters or image rejection filters, were introduced as an effective low-cost RF front-end solution. However, these monolithic architectures are subject to RF impairments in practical systems, resulting from components mismatch and manufacturing non-idealities. These impairments comprise the in-phase (I) and the quadrature-phase (Q) imbalance, which stems from the amplitude and phase mismatch between the I and Q branches of a transceiver.

In practical scenarios, phase shift errors as well as the mismatch between the amplitudes of the I and Q branches are practically inevitable, leading to imperfect image rejection, which corrupts the down-converted signal constellation, and thereby, increases the associated error rate [3], [4]. Recently, significant research efforts have been dedicated to model, analyze and propose compensation schemes for I/Q imbalance (IQI), see, e.g., [5]–[7] and the references therein. Nonetheless, RF impairments still present several challenges in the context of NOMA systems, which need to be addressed before the effective deployment of such systems in future networks. Apart from our recent results on the outage analysis, which was reported in [8], the detrimental effects of IQI on NOMA systems have not been well-studied in the open literature.

To the best of our knowledge, none of the reported studies considered the pairwise error probability (PEP) analysis for NOMA systems under RF impairments. PEP is considered as the basic building block for the derivation of union bounds on the error rate performance of a digital communication system. Therefore, the aim of this work is to analyze the PEP performance, which can reveal many useful insights into the quality of service (QoS) of all the users. More specifically, this work investigates the effects of IQI on the performance of a downlink power-domain NOMA system consisting of a base station (BS) and a number of M users. Consequently, efficient compensation mechanisms for implementing reliable NOMA-based systems may be developed.

The rest of this paper is organized as follows: Section II presents the system model of the considered NOMA scenario under IQI. The analytical framework for the PEP expression derivation is provided in Section III. Numerical results and discussions are given in Section IV. Finally, concluding remarks are provided in Section V.

II. System Model

We consider a single-cell multi-carrier NOMA downlink system consisting of a BS and M users, each with single antenna transceivers. Without loss of generality, it is assumed that the weakest channel gain is associated with the first user while the strongest is associated with the Mth user such that, $|h_1|^2 \leq |h_2|^2 \leq ... \leq |h_M|^2$, where h_l denotes the fading coefficient between the lth user and the BS.

Assuming that the IQI is perfectly compensated at the BS, the baseband equivalent transmitted signal at the kth subcarrier can be expressed as [9]

$$x(k) = \sum_{i=1}^{M} \sqrt{\alpha_i P} s_i(k) \tag{1}$$

where P and $s_i(k)$ are the transmit power of the BS and information symbol of the ith sorted user, respectively. Moreover, α_i represents the power allocation coefficient of the ith ordered user, where $\sum_{i=1}^{M} a_i = 1$ and $a_1 > a_2 > ... > a_M$.

After some necessary processing stages at the receiver RF front-end, which involve filtering, amplification, analog I/Q demodulation (down-conversion) to baseband, sampling and assuming IQI at the receivers' sides only, the baseband equivalent received signal at the lth sorted user is given by [8]

$$\begin{aligned} r_{l_{\text{IQI}}}(k) =& \mu_{r_l} h_l(k) \sum_{i=1}^{M} \sqrt{\alpha_i P} s_i(k) \\ &+ \nu_{r_l} h_l^*(-k) \sum_{i=1}^{M} \sqrt{\alpha_i P} s_i^*(-k) \\ &+ \mu_{r_l} n_l(k) + \nu_{r_l} n_l^*(-k). \end{aligned} \tag{2}$$

where $(\cdot)^*$ denotes complex conjugation and the subscript l is the lth sorted user. Furthermore, h and n denote the channel coefficient and the circularly symmetric complex additive white Gaussian noise (AWGN) signal, respectively. Moreover, the IQI parameters μ_r and ν_r are given by

$$\mu_r = \frac{1}{2}\left(1 + \epsilon_r \exp(-j\phi_r)\right) \tag{3}$$

and

$$\nu_r = \frac{1}{2}\left(1 - \epsilon_r \exp(j\phi_r)\right) \tag{4}$$

where $j = \sqrt{-1}$, whereas ϵ_r and ϕ_r denote the receiver amplitude and phase mismatch levels, respectively. It is worth mentioning that for ideal RF front-ends, $\phi_r = 0°$ and $\epsilon_r = 1$, and accordingly, $\mu_r = 1$ and $\nu_r = 0$. Also, the image rejection ratio (IRR) is given by

$$\text{IRR}_r = \frac{|\mu_r|^2}{|\nu_r|^2} \tag{5}$$

where $|\cdot|$ denotes the absolute value operation. The lth user performs SIC to cancel the interference associated with all users i, where $i < l$. Meanwhile, the signals intended for all other users $i > l$ are treated as noise. Therefore, assuming perfect channel state information (CSI) at the receiver and perfect cancellation, one obtains (6), at the top of the next page.

III. Pairwise Error Probability Analysis

PEP is the basic building block in the derivation of union bounds on the error rate performance of digital communication systems. It gives valuable insights into the system performance, particularly, when exact bit error rate (BER) expressions are intractable. Let $s_l(k)$ and $\bar{s}_l(k)$ denote the transmitted and the erroneously decoded symbols of the lth user, respectively. The PEP represents the probability of choosing $\bar{s}_l(k)$ when indeed $s_l(k)$ was transmitted. Consequently, under the assumption of perfect CSI, the conditional PEP is given by [10]

$$\begin{aligned} \Pr\left(s_l(k), \bar{s}_l(k)\right) = \Pr\Bigg(& \left| r_{l_{\text{IQI}}}(k) - \sqrt{\alpha_l P} h_l(k) \bar{s}_l(k) \right|^2 \\ & \leq \left| r_{l_{\text{IQI}}}(k) - \sqrt{\alpha_l P} h_l(k) s_l(k) \right|^2 \Bigg) \end{aligned} \tag{7}$$

where $s_l(k) \neq \bar{s}_l(k)$. Substituting (6) into (7) and after some mathematical simplifications, the conditional PEP can be evaluated as in (8), shown on the top of the next page, where $\lambda_l = \Lambda_l - (\mu_{r_l} - 1)s_l(k)$, $\Lambda_l = \mu_{r_l} s_l(k) - s_l(-k)$,

$$\beta = \sqrt{\alpha_l P} \nu_{r_l}^* \sum_{i=1}^{M} \sqrt{\alpha_i P} s_i(-k) \left[\Lambda_l - s_l(k)\left(\mu_{r_l} - 1\right)\right] \tag{9}$$

and ζ is given at the top of the next page. It should be pointed out here that the decision variable N in (8) follows the normal distribution with zero mean and variance

$$\sigma_N^2 = 2\alpha_l P |\lambda_l|^2 |h_l(k)|^2 \sigma_n^2 \left(|\mu_{r_l}|^2 + |\nu_{r_l}|^2\right), \tag{11}$$

where σ_n^2 is the variance of the AWGN. Therefore, based on the cumulative distribution function (CDF) of a Gaussian random variable, the conditional PEP is given by

$$\begin{aligned} \Pr\Big(s_l(k), \bar{s}_l(k) \mid (h_l(k), h_l(-k)) \Big) = \\ \text{Q}\left(\frac{\zeta |h_l(k)|^2 + 2\text{Re}\left\{\beta h_l(k) h_l(-k)\right\}}{\sqrt{2\alpha_l P \left(|\mu_{r_l}|^2 + |\nu_{r_l}|^2\right)} |\lambda_l| |h_l(k)| \sigma_n} \right) \end{aligned} \tag{12}$$

where for practical ϵ_r values, i.e., $\epsilon_r \leq 1.3$, the conditional PEP can be accurately approximated as [11]

$$\begin{aligned} \Pr\Big(s_l(k), \bar{s}_l(k) \mid (|h_l(k)|) \Big) = \\ \text{Q}\left(\frac{\zeta |h_l(k)|}{\sqrt{2\alpha_l P \left(|\mu_{r_l}|^2 + |\nu_{r_l}|^2\right)} |\lambda_l| \sigma_n} \right). \end{aligned} \tag{13}$$

In the sequel, the index k will be dropped to simplify the ensuing presentation. To obtain the unconditional PEP, the expression in (13) is averaged over the probability density function (PDF) of $Z_l = |h_l|$. Recalling that users are ordered based on their channel gains, order statistics should be em-

$$r_{l_{\text{IQI}}}(k) = (\mu_{r_l} - 1) h_l(k) \sum_{i=1}^{l-1} \sqrt{\alpha_i P} s_i(k) + \mu_{r_l} h_l(k) \sum_{i=l}^{M} \sqrt{\alpha_i P} s_i(k) + \nu_{r_l} h_l^*(-k) \sum_{i=1}^{M} \sqrt{\alpha_i P} s_i^*(-k) + \mu_{r_l} n_l(k) + \nu_{r_l} n_l^*(-k). \tag{6}$$

$$\Pr\left(s_l(k), \bar{s}_l(k) \mid (h_l(k), h_l(-k))\right) = \Pr\Bigg(\underbrace{2\text{Re}\left\{\sqrt{a_l P} h_h \mu_{r_l}^* \lambda_l n_l(k)^* + \sqrt{a_l P} h_h \nu_{r_l}^* \lambda_l n_l(-k)^*\right\}}_{N} \le -\Big[\zeta \left|h_l(k)\right|^2 + 2\text{Re}\left\{\beta h_l(k) h_l(-k)\right\}\Big]\Bigg) \tag{8}$$

$$\zeta = 2\text{Re}\left\{\sqrt{\alpha_l P} \lambda_l^* \left[\mu_{r_l} \sum_{i=l+1}^{M} \sqrt{\alpha_i P} s_i(k) + (\mu_{r_l} - 1) \sum_{i=1}^{l-1} s_i(k)\right] + \alpha_l P \left(\left|\Lambda_l\right|^2 - \left|\mu_{r_l} - 1\right|^2 \left|s_l(k)\right|^2\right)\right\} \tag{10}$$

ployed to evaluate the PDF of Z_l. Hence, the ordered PDF of the lth user can be evaluated as the following [12]

$$f_l(Z_l) = A_l f(Z_l) \left[F(Z_l)\right]^{l-1} \left[1 - F(Z_l)\right]^{M-l} \tag{14}$$

where

$$A_l = \frac{M!}{(l-1)!(M-l)!} \tag{15}$$

Given that the channel gains follow the Rayleigh distribution with the following PDF and CDF

$$f(Z) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right) \tag{16}$$

and

$$F(Z) = 1 - \exp\left(-\frac{x^2}{2\sigma^2}\right) \tag{17}$$

respectively, the ordered PDF of Z_l can be written as

$$f_l(Z_l) = \frac{A_l}{\sigma^2} Z_l \sum_{u=0}^{l-1} \binom{l-1}{u} (-1)^u \exp\left(-\frac{(M-l+1+u)Z_l^2}{2\sigma^2}\right). \tag{18}$$

Hence, using (14), the unconditional PEP is evaluated as in (19), which is given at the top of the next page. Using the representation of the Gaussian-Q function $\mathrm{Q}(\mathrm{x}) = 0.5\,\mathrm{erfc}\left(\frac{\mathrm{x}}{\sqrt{2}}\right)$ and [13, Eq. 2.8.5.9], the unconditional PEP can be expressed as (20), given at the top of the next page.

IV. Numerical and Simulation Results

In this section, we present analytical and Monte Carlo simulation results for the considered NOMA system. Particularly, we investigate the effect of IQI on the performance of multi-carrier NOMA downlink system with M users. Without loss of generality, we consider a scenario with three NOMA users with power coefficients $\alpha_1 = 0.7$, $\alpha_2 = 0.2$ and $\alpha_3 = 0.1$. The total transmission power is normalized to unity, $P = 1$, which implies that the transmitted signal is normalized by $\left(\left|\mu_r\right|^2 + \left|\nu_r\right|^2\right)$. Furthermore, in our results, we refer to $\bar{\gamma} = P/\sigma_n^2$ as the transmit average system signal to noise (SNR) ratio. The transmitted symbols are selected randomly from binary phase shift keying (BPSK) constellation. Considering a practical scenario, the receiver amplitude and phase mismatch levels are selected as $\epsilon_r = 1.2$ and $\phi_r = 2°$, respectively.

Fig. 1 compares the analytical and simulated PEP of the impaired users, where $\epsilon_r = 1.2$ and $\phi_r = 2°$, versus the SNR with the ideal scenario, i.e., $\epsilon_r = 1$ and $\phi_r = 0°$. It can be observed from Fig. 1 that there is a perfect match between the analytical and Monte-Carlo simulation results, confirming that, for practical ϵ_r values, the adopted approximation in (13) is accurate. Moreover, it is shown that for the selected IQI parameters and the considered SNR range, the diversity order achieved by NOMA users is preserved under IQI scenario, where it is noticed that the slope of the PEP of all IQI impaired users matches the ideal scenario.

Fig. 2 depicts the achievable diversity order of NOMA users under practical IQI values. Considering practical SNR values and IQI levels, i.e., $\epsilon_r < 1.3$, it is clear that the achievable diversity order converges to l for the ideal and non-ideal scenarios, which implies that, for practical ϵ_r and ϕ_r values, the IQI has no effect on the diversity order of NOMA users. It is worth noting that the adopted approximation in (13) is valid for the entire practical IQI range. Additionally, we stress that, at high SNR values, the exact PEP experiences an error floor when $\epsilon_r > 1.3$, yielding a diversity order equals to zero for all NOMA users.

V. Conclusions

In this paper we investigated the effect of IQI on the error performance of NOMA users under imperfect SIC conditions. In particular, we derived an accurate PEP expression, which

$$\Pr(s_l,\bar{s}_l)=\frac{A_l}{\sigma^2}\sum_{u=0}^{l-1}\binom{l-1}{u}(-1)^u\int_0^\infty Z_l\exp\left(-\frac{(M-l+1+u)Z_l^2}{2\sigma^2}\right)\mathrm{Q}\left(\frac{\zeta Z_l}{\sqrt{2\alpha_l P\left(|\mu_{r_l}|^2+|\nu_{r_l}|^2\right)}|\lambda_l|\sigma_n}\right) \tag{19}$$

$$\Pr(s_l,\bar{s}_l)=\frac{A_l}{2}\sum_{u=0}^{l-1}\binom{l-1}{u}\frac{(-1)^u}{(M-l+1+u)}\left(1-\frac{\zeta\sigma}{\sqrt{\zeta^2\sigma^2+2\alpha_l P(M-l+1+u)\left(|\mu_{r_l}|^2+|\nu_{r_l}|^2\right)|\lambda_l|^2\sigma_n^2}}\right) \tag{20}$$

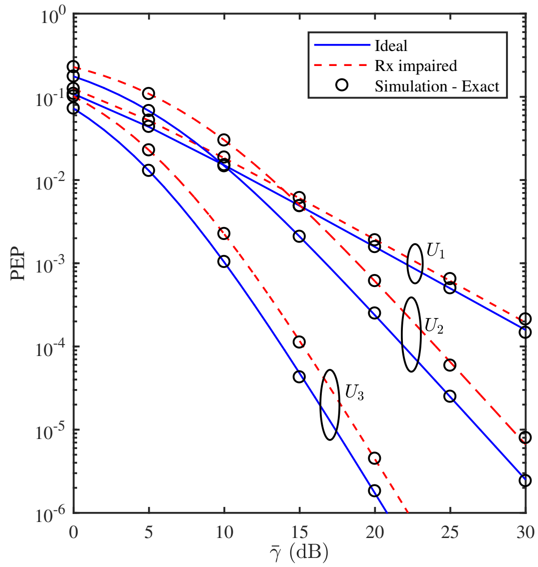

Fig. 1: Analytical and simulated PEP for different users, $\epsilon_r=1.2$ and $\phi_r=2°$.

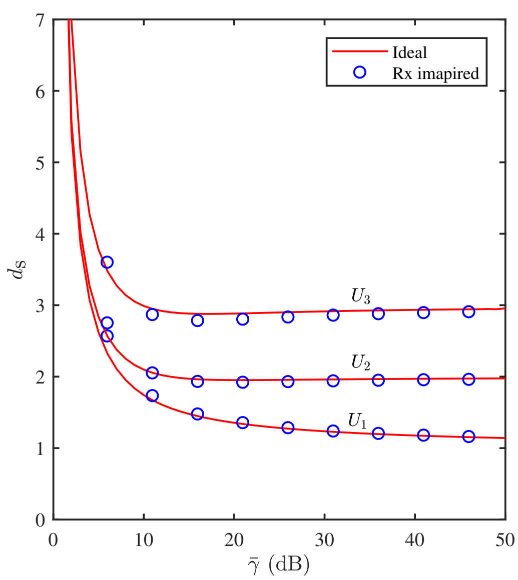

Fig. 2: Achievable diversity order of different users under I/Q imbalance, $\epsilon_r=1.2$ and $\phi_r=2°$.

was then used to analyze the diversity order of NOMA in practical impairment scenarios. The derived expression was verified with the corresponding simulation results. Our analysis demonstrated that the performance under IQI is determined by different factors, such as the user order and the level of impairment. Moreover, it was shown that, for practical IQI levels, the diversity order is preserved under IQI, where, as in the case of ideal RF front-ends, it was observed that the diversity order of the lth user converges to l.

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