

# Data-driven approach to satellite selection in multi-constellation GNSS receivers

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**Abstract**—In this work we developed an algorithm for multi-constellation GNSS receivers that would select satellites out of the tracked ones in order to optimize the location accuracy. As the receivers often have very limited computational resources, the complexity of the algorithm needed to be kept low.

The work began with an exploratory analysis of GNSS data. This analysis gave insight into the differences of the various satellite navigation systems as well as into the nature of the pseudorange residuals. These observations helped in shaping the algorithm that we proposed for the problem of satellite selection. The algorithm itself was developed using data science techniques to filter out bad pseudorange measurements and borrowed some earlier ideas to optimize the geometric dilution of precision of the solution set as well.

The approach we chose was shown to work very well when applied to real data measured from road tests in varying surroundings. Even with practically non-existent parameter tuning the algorithm was able to spot almost 90% of the bad pseudorange measurements, keeping the specificity, i.e., ability to hold on to the good measurements at over 90% level.

The ability to filter out bad pseudorange measurements translated to improved location accuracy as well. All in all, the results achieved in this work proved encouraging enough to begin implementing the algorithm in actual receiver software to study the performance of the data-driven approach in action.

## I. INTRODUCTION

Satellite systems are an integral part of positioning and navigation today. They have been in use since the 1960s when the United States military deployed the Transit system. Nowadays there are multiple global navigation satellite systems (GNSS), as well as a few regional ones. Most of the systems are made available also to civilian users. The global systems are designed so that when used alone they provide enough visible satellites for a positioning solution. Thus when using receivers that are able to track satellites from multiple systems there are often more visible satellites available than it is feasible to use. This leads to the question of selecting the optimal subset of visible satellites for the navigation solution.

The topic has properly risen only in the recent years as the BeiDou-2 and Galileo systems have not been operational for too long, and are still not in full operational capability. Most of the former work concerning the satellite selection has mainly focused on minimizing the geometric dilution of precision (GDOP) [1]–[9]. However satellite geometry works only to inflate the initial measurement errors. The actual

sources of this error include signal transmission delays caused by troposphere and ionosphere, satellite location inaccuracies and multipath caused by signal reflections on its way from the satellite to the receiver. As there are more things to consider than just the geometry when selecting the satellites, a more holistic approach is wanted.

So the main goal of this work is an algorithm to minimize the location error without a considerable computational complexity. This paper is organized as follows. In Section II we formulate the problem and Section II-A looks at the previous work done on the topic. This is followed by a description of our approach in Section II-B. We then present and discuss test results using real-world data in Section III. Finally, open questions and issues, and potential solutions are discussed in Section IV.

## II. SATELLITE SELECTION

The achieved location accuracy is a result of two factors: the satellite location dependent geometric dilution of precision and the pseudorange measurement inaccuracies. The latter includes transmission delays induced by ionosphere, troposphere and multipath reflections in addition to, e.g., satellite location inaccuracies. Some of these can be diminished by applying relevant corrections in the receiver, but some are harder to mitigate. For example there are models like Klobuchar and NeQuick for taking into account the ionosphere induced errors [10]–[12]. Hopfield and Saastamoinen models for the troposphere error [13], [14]. In addition there are services like the satellite based augmentation systems (SBAS), which provide correction data for these kind of errors. The multipath delays however are often more problematic and can have greater effects on the calculated location. These can be affected by eliminating the non-line-of-sight (NLOS) measurements from the solution.

The main problem of this work is thus the selection of an optimal subset. We are looking for a subset of satellites from the set of all visible satellites that minimizes the error in the receiver location  $e_{\hat{u}}$  with possibly an added limit to the cardinality of this subset.

Let  $S$  now define the set of all visible satellites. Similarly let  $S_i \subseteq S$  denote the subset selected for the location solution. Now we can formulate the problem as

$$\begin{aligned} & \underset{S_i \subseteq S}{\text{minimize}} && \|e_{\hat{u}}(S_i)\|_2 \\ & \text{subject to} && |S_i| \leq k \end{aligned} \quad (1)$$

where  $k$  is the maximum number of satellites allowed in the solution.

There are certain requirements for the selection algorithm. First, and foremost of these requirements is that the selection needs to be executable in the receiver hardware in real time. This rules out too complex solutions as well as brute force methods. Second, the selection criteria should preferably be a white box model, i.e., the selection criteria can be understood when inspecting the model.

#### A. Minimizing the GDOP

Previous work concerning satellite selection in GNSS has been mainly about minimizing the effect of the satellite geometry. This has been approached in numerous ways. One can naturally go through all the possible subsets and select the optimal one but this produces huge computational load. Greedy algorithms performing backward elimination [4] or forward selection based on GDOP [7] are obvious reliefs, but they still require multiple matrix inversions. To mitigate the need for matrix inversions alternative metrics have been proposed as well.

In [1] a quasi-optimal algorithm for the selection problem is proposed. The paper introduces a cost function for the satellites

$$J_j = \sum_{i=1}^N \cos(2\theta_{i,j}), \quad (2)$$

where  $\theta_{i,j}$  is the angle between the line of sight vectors to satellites  $i$  and  $j$  and  $N$  is the number of satellites. This stems from the idea that satellites with collinear line of sight vectors are redundant. Using this cost function the satellites are selected in a backward elimination manner, always removing the vehicle with the highest cost. The aforementioned algorithm often misses the optimal satellite geometries but the computational load required makes it suitable for real time applications. Indeed, since the cosine term can be expressed as

$$\cos(2\theta_{i,j}) = 2 \cos^2(\theta_{i,j}) - 1 = 2(v_i \cdot v_j)^2 - 1, \quad (3)$$

where  $v_i \in \mathbb{R}^3$  and  $v_j \in \mathbb{R}^3$  are the line of sight unit vectors to satellites  $i$  and  $j$ , the computation requires very low resources. The aforementioned paper has also inspired multiple other metrics using cosine for the ranking of the satellites [6], [8]. They incorporate additional ideas like requiring the highest elevation satellites to be included in the selection.

Other strategies have also been developed in search for the best geometries. In [5] an algorithm is proposed in which those satellites are selected that span a convex hull around the visible satellite vehicles. Such algorithms have also been proposed that pick those subsets which most resemble some predefined geometries [2], [3], [9].

However a recent paper [15] exposed the shortcomings of algorithms that rely on minimizing GDOP alone. Even though the GDOP would be optimal for a given size of satellite subsets, if there are satellites with very poor pseudorange measurements included, the location error can be quite significant. This is also depicted in the results of this work.

Indeed, when minimizing the location error, both the GDOP and measurement errors need to be considered. Thus we propose a more holistic approach for the satellite selection issue. This approach is presented in the next section.

#### B. Data-driven approach

The approach that was constructed for this work scores the satellites in two parts, the other tries to optimize the GDOP and the other aims at eliminating too large measurement errors. As calculating the GDOP can only happen after a subset of the visible satellites has been selected it is not practical to be computing it in the selection process. That is why we model the effect the satellites have on GDOP by something we call the redundancy score from now on. The redundancy scores depend on which satellites have already been picked in to the selection subset and are only calculated to those satellites that have not been picked yet. The other part, the large measurement errors, we try to predict with the logistic regression model.

Since the measurement errors are not in anyway dependent on the satellites already picked for selection, we initialize the algorithm by calculating the probabilities that a pseudorange measurement is over a certain threshold. For understanding how logistic regression achieves this one can see for example [16], [17]. In this case, the probability  $p$  a satellite  $j \in S$  has a bad pseudorange measurement is

$$p_j = \frac{1}{e^{w^T x_j} + 1}, \quad (4)$$

where  $x_j$  is information about the satellite and its signal, and  $w$  a weighting coefficient vector for these features. In this case  $x_j$  includes carrier-to-noise-density ratio, elevation and satellite system information.

Next the three best scoring satellites are added in to the selection subset  $S_i$ . So the first three satellites are selected purely based on their probability to have a good measurement. In the following selections also the redundancy score is taken into account. In the redundancy score part we borrow the idea of using line of sight vector cosines from the quasi-optimal algorithm [1].

The quasi-optimal algorithm has the nice idea of modeling the effect the satellite has on the GDOP by the cosines of the line of sight vectors. As the cosine can be easily computed via the dot product it is suitable for the requirement of low computational complexity present in this work as well.

Unlike  $\cos(2\theta_{i,j})$  in the quasi-optimal algorithm a simple  $\cos(\theta_{i,j})$  is used here. Fig. 1 visualizes the differences of these redundancy cost functions. As can be seen in this figure, the  $\cos(2\theta)$  cost function penalizes satellites if the angle between

them gets over  $90^\circ$ . This is not desirable in our scenarios and so the plain dot product is good enough in this case.

So now the satellites  $j \in S \setminus S_i$  get a redundancy score,  $r_j$ , which we define with regards to the satellites already in the selection,  $i \in S_i$ . That is, the redundancy is now

$$\begin{aligned} r_j &= \frac{1}{|S_i|} \sum_{i=1}^{|S_i|} \cos(\theta_{i,j}) + 1 \\ &= \frac{1}{|S_i|} \sum_{i=1}^{|S_i|} v_i \cdot v_j + 1. \end{aligned} \quad (5)$$

As the redundancy score was defined as the mean, it now suitably lies in the interval  $[0, 2]$ . The end score,  $s_j$ , is now simply the sum

$$s_j = p_j + r_j/2. \quad (6)$$

Then the satellite with the lowest end score  $s_j$  is added to the selection set.

For a new satellite to be selected the redundancy scores for the remaining satellites need to be recalculated as they are dependent on  $S_i$ . This is repeated until the lowest end score gets too high or the selection subset reaches a predetermined size. A simplified summarization of the selection algorithm could be given as

- 1) Calculate  $p_j$  (4)
- 2) Add those three satellites with the lowest  $p_j$  to  $S_i$
- 3) Calculate  $r_j$  (5), and then  $s_j$  (6), for the remaining satellites
- 4) Add the satellite with the lowest  $s_j$  to  $S_i$
- 5) Repeat steps 3. and 4. until  $\min(s_j) > s_{th}$  or  $|S_i| = k$

where  $s_{th}$  is a score threshold after which it is considered that adding a satellite into the selection yields no profit and  $k$  is a predetermined maximum size for the selection set.

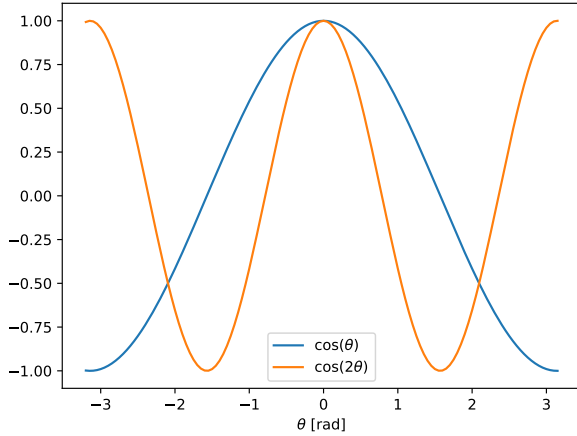


Fig. 1. Cost function for the satellite redundancy used in the quasi-optimal algorithm ( $\cos(2\theta)$ ) in comparison with the cost function ( $\cos(\theta)$ ) used in our approach.

### III. RESULTS

We compared the results our algorithm produced to those achieved with the quasi-optimal algorithm by Park et al [1]. The quasi-optimal algorithm was selected as a reference point due to it being very well established in the field. The GNSS data for these tests was recorded when driving in and around Zürich. A total of three runs was made with lengths varying between 58 minutes and 120 minutes. A truth recording was made using an Applanix POS LV product, post-processed with POSpac software and is accurate down to decimeter level [18]. 3-fold cross-validation was used first to get the prediction probabilities for logistic regression. Table I shows the results for predicting over 10m pseudorange errors and Fig. 2 gives the regression coefficients for the predictor.

	Test accuracy	Test recall	Test specificity
Fold 1	0.900	0.899	0.918
Fold 2	0.946	0.947	0.885
Fold 3	0.899	0.899	0.903
mean $\pm$ std	$0.915 \pm 0.022$	$0.915 \pm 0.023$	$0.902 \pm 0.013$

TABLE I  
THE RESULTS FROM PREDICTING OVER 10M PSEUDORANGE RESIDUALS IN THE ROAD TEST DATA.

The prediction results seem rather promising, with the accuracy, recall and specificity numbers all averaging over 90%. However the size of the dataset used in the work could be criticized. The effect of the small dataset can be seen in the feature coefficients with the error whiskers being quite significant. The most notable implication here would be that the measurements from the GLONASS system are more susceptible to bad pseudoranges. This is more thoroughly shown in [19], where the pseudorange errors from different systems are compared.

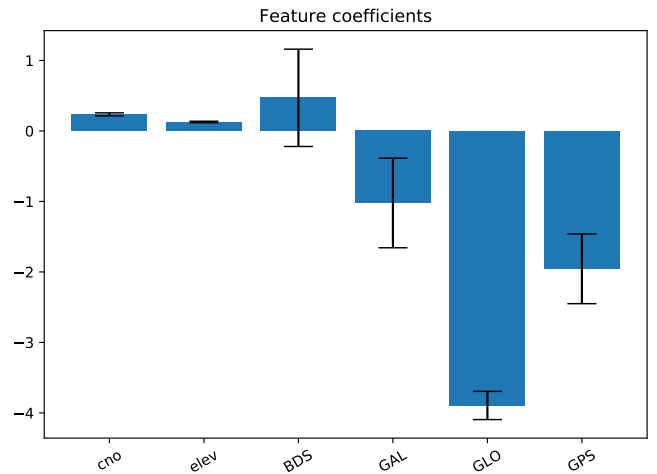


Fig. 2. The logistic regression coefficients used for predicting if the residual of a pseudorange measurement is going to be over 10m. The bars are the means from the cross-validation folds and the whiskers present one-sigma errors.

For the actual comparison of the selection algorithms one of the recorded runs was chosen and then the regression model was trained with the two remaining runs. Table II and Fig. 3 show the location errors acquired with different selection strategies.

Error [m]	mean	std	min	25%	50%	75%	max
AIV	3.38	3.37	0.08	1.29	2.18	4.36	28.52
DDA 10	2.01	0.90	0.11	1.40	1.86	2.44	7.40
DDA 20	1.37	0.87	0.07	0.83	1.18	1.66	7.69
DDA 30	1.37	0.87	0.05	0.82	1.18	1.67	7.78
QO 10	8.51	7.83	0.10	3.58	5.99	11.21	90.18
QO 20	3.72	3.55	0.14	1.48	2.53	4.78	30.26
QO 30	3.38	3.37	0.08	1.29	2.18	4.36	28.52

TABLE II

A SUMMARY OF THE LOCATION ERRORS OBTAINED FROM THE TEST DATA USING DIFFERENT METHODS FOR THE SATELLITE SELECTION. THE ERRORS ARE MEASURED AS DISTANCE FROM THE TRUTH AND GIVEN IN METERS. AIV REFERS TO THE ALL IN VIEW CASE, DDA TO DATA-DRIVEN APPROACH AND QO TO THE QUASI-OPTIMAL METHOD. THE NUMBERS BEHIND THE LATTER TWO REFER TO THE NUMBER OF SATELLITES SELECTED.

The All in view uses all of the satellites in view for the location solution. DDA is an abbreviation for our data-driven approach, whereas QO is for Park's [1] quasi-optimal method. The abbreviations are accompanied by the number of satellites selected in each case, for data-driven approach being only a maximum number as less can be used as well.

The errors are binned into 9 bins with a width of one meter. The 1-2m error range has the most counts for almost all of the selections, only the quasi-optimal with 10 satellites differs in this sense. This representation shows how the data driven approach with a maximum of 30 satellites selected has slightly higher counts in the first two bins than when allowing a maximum of 20 satellites. However these differences are very minor and as a whole the 20 and 30 satellite cases for the data-driven method can be considered equally accurate. Also visible here is how neither of them induce errors of over

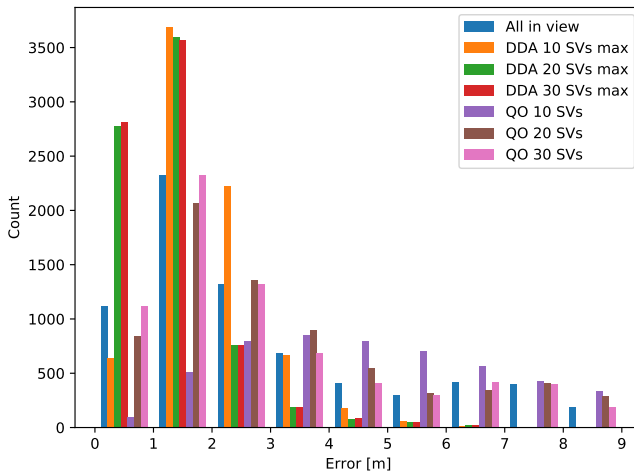


Fig. 3. Histograms of the location errors acquired in the test data with the all in view, our data driven and the quasi-optimal satellite selection methods.

8m.

These results indicated how the logistic regression works surprisingly well in predicting the bad pseudoranges. The good and bad residuals seem to be rather well linearly separable with those prediction variables that were selected for the model. This primes the data-driven method to succeed in weeding out the bad pseudorange measurements and thus provides for a more accurate positioning solution. In addition the redundancy score based optimization for the geometric dilution of precision appeared to work adequately well.

The next section looks at the computational complexity of the data-driven method and again compares it to the quasi-optimal algorithm. After all, one of the requirements for the satellite selection was that it needs to work in real time in the receiver hardware.

### A. Computational complexity

Let us consider the quasi-optimal algorithm first. In the trivial case that the number of visible satellites happens to be the same as (or less than) the number of satellites we wish to select then there is no computation needed, we just simply select all of the satellites in view. Then consider that we need to drop one satellite, i.e., the number of visible satellites,  $|S|$ , is one greater than the number of satellites we wish to select for the solution. Now, since the quasi-optimal algorithm works via backward elimination, we need to calculate the cosines for all of the angles between the visible satellites, see (3). As one angle requires two satellite line of sight vectors there are  $\binom{|S|}{2}$  angles. Thus when we consider the number of satellites to be selected constant, as it is, the complexity of the quasi-optimal algorithm w.r.t. the number of visible satellites is

$$\mathcal{O}\left(\binom{|S|}{2}\right) = \mathcal{O}\left(\frac{|S|(|S|-1)}{2!}\right) = \mathcal{O}(|S|^2). \quad (7)$$

Considering the computational complexity of the data-driven method is not quite as straightforward. In the initial stage, we need to compute the signal scores for all of the satellites, hence that part is  $\mathcal{O}(|S|)$ . As was stated in Section II-B the first three satellites are added to the selection according to these signal scores. Then, for more satellites to be added we start calculating the redundancy scores. So for the fourth satellite the number of angles we need to consider is  $3(|S|-3)$ . In general, for  $|S_i|$  satellites to be selected out of  $|S|$  we need to consider

$$\sum_{i=3}^{|S_i|-1} i(|S|-i) \quad (8)$$

angles. For this the complexity can be given as  $\mathcal{O}(|S_i||S|)$ , and since  $|S_i|$  is a constant it can be simplified to  $\mathcal{O}(|S|)$ . Thus the complexity for the data-driven method is

$$\mathcal{O}(|S| + |S|) = \mathcal{O}(|S|). \quad (9)$$

Worth remembering is that the signal scores need to be calculated only once in an epoch.

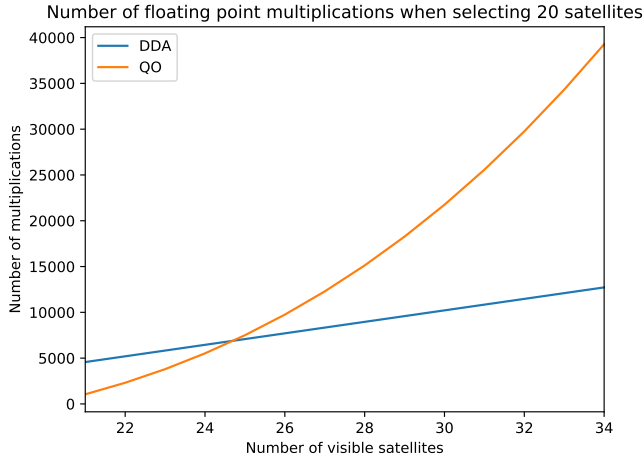


Fig. 4. Number of floating point operations needed as a function of the number of visible satellites when 20 satellites are to be selected.

To verify, we calculated the numbers of floating point multiplications needed when selecting 20 satellites out of a varying number of visible satellites. Fig. 4 illustrates these numbers which support the complexities attained above. When there is only one satellite to dismiss the backward elimination of the quasi-optimal algorithm does this with far fewer calculations than the data-driven approach, which works in a forward selection manner. However, the number of multiplications needed in the quasi-optimal algorithm grows rapidly as the number of visible satellites increases, whereas in the data-driven approach the growth is rather linear. If we were to select only 10 satellites the difference would grow even more substantial in favor of the data-driven approach.

#### IV. DISCUSSION AND CONCLUSIONS

Previous work concerning the selection has been predominantly about optimizing the geometrical dilution of precision in the solution set. However that is only a scaling factor in the ensuing location accuracy, the errors stem from the pseudorange inaccuracies. Nonetheless there are very few papers published where the satellite selection has happened from the measurement error minimization point of view. This directed most of our efforts in to the direction of the pseudorange residuals.

The first approach was naturally to apply linear regression to the data. This however produced rather poor results in the initial tests. Discretizing the dependent variable proved a simple solution and logistic regression gave surprisingly good results already from the first tests onwards.

Table I presented the results from the logistic regression model when predicting over 10m pseudorange residuals. It can be seen that the model works well in the road test scenarios and that the results from the three cross-validation folds are in unison. The recall of nearly 90% is especially pleasing as it means that the model was able to spot 9 out of 10 bad pseudorange measurements. However when looking at the

coefficients in Fig. 2 we see that there is considerable variation in them, especially for the BeiDou system. To get more rigid estimates for the coefficients more data would be needed for training. Also if more data of sufficient quality would be available there would be a possibility to even calculate weights for each individual satellite instead of using the systems as we do here. All in all, the efficiency of applying logistic regression for the problem at hand proved sufficient to say the least.

The location accuracies, given in Table II, obtained with the data-driven approach were in a completely different class when compared to the satellite selections of the quasi-optimal algorithm. This was mainly due to the ability of the DDA to eliminate the measurements with significant error in them. However it needs to be noted that when using 10 or 20 satellites the data-driven approach achieved lower GDOP as well [19]. Worth remembering is that all the results are without the receiver autonomous integrity monitoring (RAIM) applied. The fault detection and exclusion part of RAIM performs certain basic checks to the satellite signals and can weed out some of the bad pseudorange measurements [20]. The addition of RAIM would most likely improve the acquired results to some extent.

Another interesting aspect in the location accuracy results was that there was virtually no difference in using 20 or 30 satellites in the data-driven approach. Even in few epochs the accuracy was slightly poorer when allowing the DDA to use 30 satellites in the solution. This would indicate to having a slightly too high overall score (6) threshold after which no more satellites are to be added to the solution.

As one of the main reasons for wanting to limit the amount of measurements in the solution was to cut down on the cost of the computations the satellite selection algorithm needs to be computationally light as well. This was studied in Section III-A, where it was shown that the complexity of the data-driven approach grows linearly with the number of visible satellites. For the quasi-optimal method the growth is quadratic, causing it to take up more computation time with the numbers of visible satellites there are available nowadays.

#### A. Conclusions and future work

The main goal of this work was to develop an algorithm for multi-constellation GNSS receivers that would select satellites out of the tracked ones to be used in the location solution. As the receiver has very limited computational resources, the complexity of the algorithm needed to be kept low. The work showed that optimizing the GDOP only lacks the important aspect of considering the pseudorange measurement errors. These have been shown to have an even greater influence on the accuracy of the position solution than optimizing the GDOP does [15].

The work began by exploratory analysis of GNSS data from near optimal situations. This analysis gave already some insight into the differences of the various satellite navigation systems as well as into the nature of the pseudorange residuals. These observations helped in shaping the algorithm that we proposed for the problem of satellite selection. The algorithm

itself was developed using data science techniques to filter out bad pseudorange measurements and borrowed some earlier ideas to optimize the geometric dilution of precision of the solution set as well.

The approach we chose was shown to work very well when applied to real data measured from road tests in varying surroundings. Even with practically non-existent parameter tuning the algorithm was able to spot almost 90% of the bad pseudorange measurements, keeping the specificity, i.e., ability to hold on to the good measurements at over 90% level. As there was practically no parameter tuning done here, optimizing the weights and thresholds of the selection model would most likely improve the results even further. Another development aspect would be to have weights for each of the individual satellites instead of the global navigation satellite systems. This would however require far more data of sufficient quality.

The ability to filter out bad pseudorange measurements translated to improved location accuracy as well. The data-driven approach outdid the quasi-optimal method clearly and in addition was shown to have lower computational complexity already with the present number of navigation satellites.

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