Analyzing D2D Mobility: Framework for Steady Communications and Outage Periods Prediction

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***Abstract*—The use of direct device-to-device (D2D) communications is expected to drastically increase the spatial frequency reuse significantly improving the capacity of forthcoming fifth generation (5G) cellular systems. The number of D2D pairs operating within the same channel that can be supported in a certain area of interest is limited by the interference imposed by them on each other. In an inherently mobile environment where the users are constantly mobile the decision on enabling a certain number of direct connections should not only be based on the static interference picture but shall consider the changes in the signal-to-interference ratio caused by the user mobility. These changes manifest themselves into steady communications and outages time periods interchanging each other. In this paper, the statistical characteristics of the user mobility periods as a function of the D2D communications range, speed of users, number of communicating pairs and the area of interest is characterized utilizing the generic approach based on the Fokker-Plank equation. We show that although the time-evolution equation for the cumulative distribution function (CDF) of SIR can be written in explicit form for non-stationary movements of nodes, the solution cannot be obtained in closed-form. Using the mixed simulation-analytic approach we propose a generic methodology for performance assessment of time-dependent characteristics such as steady communications and outage time periods.**

***Index Terms*—5G; analytical model; D2D; Fokker-Planck equation; mobility.**

I. Introduction and Background

Direct device-to-device (D2D) communications is expected to be an integral part of prospective fifth generation (5G) systems greatly enhancing their spatial frequency reuse, thus, drastically improving the overall system capacity [1], [2], [3]. Conceptually, D2D enables establishing direct connections between users being in geographical proximity avoiding the use of long-distance infrastructure connections via base stations (BSs) [4]. This allows to utilize smaller emitted power levels at transmitting stations, thus, creating less interference for other users using the same communications channel. We distinguish between in-band D2D that use the licensed LTE spectrum and out-of-band D2D using the Wi-Fi technology [5]. We are interested in the former that is expected to be deployed in 5G systems [6], [7], [8].

The performance gains of deploying D2D technology primarily depends on the availability of users nearby that are interested in communications [9]. Such conditions are expected to be met in crowded use-cases, e.g. stadiums, squares. In these scenarios, the efficiency of D2D technology depends on careful decision making about the number of D2D communications pairs operating over the same frequency [10]. Such decisions are expected to be taken at the BSs and need to consider the interference created by other D2D communicating users in proximity [11], [12]. The interference affects the signal-to-interference ratio (SIR) that further determines the achievable bitrate of a D2D connection [13]. If SIR falls below some predetermined level – communicating stations may find themselves in outage. The latter could happen due to both high number of communicating D2D pairs and/or the user movement over the landscape. To carefully plan D2D communications in a certain area of interest, the analytical models are required to be developed.

The performance of D2D communications is conventionally studied using the stochastic geometry approach [14], [15]. According to this methodology, the communicating pairs are modeled using by realization of some spatial stochastic process, usually, Poisson point process (PPP) [16]. The SIR is then modeled using the random variables (RV) transformation technique [17] as the ratio between RVs representing the useful signal and aggregated interference. The performance metrics are therefore ergodic ones, i.e., the probability of

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outage is a time-averaged over all realizations of the spatial process of stations, see e.g., [18], [19], [20], [21]. For many applications, such as video streaming or file downloading, this metric is not representative as environmental conditions including the relative positions of D2D communications pairs may vary dynamically, and the time spent in outage might be tolerable. Therefore, the decision about accepting a new D2D communications pair requires not only be based on the ergodic outage probability, but to consider specifics of applications characterizing distributions of steady-state communications periods and outages as well.

In this paper, we develop an approach to characterize distributions of steady-state communications periods and outages as a function of various parameters including the speed of users, the square of the area of interest, and the D2D communications radius using the uniform theory of kinetic equations. Particularly, the Fokker-Planck equation is used for generation of ensemble of trajectories representing the stochastic motion of D2D communications pairs in the area of interest. The quantile of SIR distribution function is derived as a function depending on values of drift and diffusion in kinetic equation. The time-evolution equation for this quantile is then obtained. The steady communication quantile of SIR distribution is numerically analyzed as a function of the diffusion coefficient of Fokker-Planck equation. We formulate our model in three-dimensions (3D) as the reduction to two-dimensional case is straightforward and widely met in the literature.

The rest of the paper is organized as follows. In Section II we formulate the system model, provide the background information on kinetic equations and introduce metrics of interest. Then, in Section III we qualitatively analyze the system and also provide the time-evolution equation for SIR. The generic case and numerical results are then addressed in Section IV. The paper is concluded in the last section.

II. Deployment and System Metrics

The system model we consider consists of N D2D communication pairs located in three dimensional "sphere", $V \subset R^3$. From now on, we solely concentrate on transmitters and assume that they operate using the same frequency. An additional node, the so-called tagged receiver, is also assumed to be uniformly distributed in $V \subset R^3$. All the transmitters are assumed to create interference to the tagged receiver. In our study, we are interested in CDF of steady communications and outages at the tagged receiver. The level of communication quality is characterized by the SIR (S) with the critical value set as $s_{cr} = 0.01$. The connection between devices is, thus, assumed to be reliable if $S > s_{cr}$.

The SIR is defined as follows. Let x_i^{α} is a coordinate of i^{th} point inside the region $V \subset R^3$, upper index numerates coordinates in R^3 basis, connected with one of the points. The total number of points in the region V is assumed to be a constant equal to $N{+}2$. Those points have a mobility pattern in accordance with some stochastic process. The distance $r_{ij} = |r_i - r_j|$ between the points is calculated in Cartesian system of coordinates, that is,

$$r_{ij} = \sum_{\alpha=1}^{3} \left(x_i - x_j\right)^2. \tag{1}$$

Assuming the following propagation function

$$\phi_{ij} \equiv (r_{ij}) = \frac{1}{r_{ij}^2}, \tag{2}$$

we choose some arbitrary pair of points, e.g., r_1 and r_2, as the reference ones.

Now, we introduce SIR as follows

$$S(r_1, r_2) = \frac{\phi_{12}}{\sum\limits_{j=3}^{N+2} \phi_{1j}}. \tag{3}$$

The denominator in (3) can be considered as the average value of function ϕ in the region V, when the number of points N is sufficiently large. Let further $f(r)$ be the probability density function (pdf) of the points. It is convenient to relate all the points to the reference one, r_2, denoting $r = r_{12} \equiv r_1 - r_2$.

Now, SIR in (3) can be written as

$$S \equiv S(r) = \frac{\phi(r)}{NU(r)}, \tag{4}$$

where

$$U(r) = \int_V \phi\big(|r - r'|\big) f(r') dr'. \tag{5}$$

Note, the continuous representation of (5) is considered from now on. This approximation of point-to-point interaction by its continuous analog provides a simple way to study the SIR process using the general framework of kinetic movement equation. Using this approach, we may relate pdf $g(S)$ to CDF as $G(s) = \int_0^s g(S) dS$. The value of $g(S)$ is equal to the part of devices having reliable communication at any given instant of time. However, observe that, generally, $g(S)$ cannot be fully constructed using the function $f(r)$ in the explicit form. However, due to the random behavior of each trajectories configuration and the mobility of both real tagged receiver and sender, the efficiency of connection in total community of moving D2D communications pairs could be interpreted in terms of efficiency of the connection between given pair along their trajectories.

III. General Approach

A. Qualitative Analysis

Each device trajectory is modeled as a realization of random walk with non-stationary, in general, pdf $f(r,t)$ using the algorithm described in detail in [22]. We further briefly summarize it. Observe that for the proposed deployment, any random position of the point (alternatively, an increment of each coordinate) at a given moment of time t can be generated using the corresponding CDF

$$F(x,t) = \int_0^x f(y,t) dy. \tag{6}$$

We consider a unit region V such that all the coordinates are in $[0;1]$. Further, assuming unit step time, we generate stationary series of numbers $\{y_k\}$ of length T that is uniformly distributed in $[0;1]$. The corresponding series $\{x_k\}$ with the CDF $F(x,t)$ is obtained based on the inverse function of a given CDF, i.e.,

$$y_k = F(x_k, k). \tag{7}$$

Generating a set of N uniformly distributed samples, the corresponding set of trajectories is obtained, i.e., at any moment of time $t_n = n$ we can generate positions, $r(n)$, of all the points with given pdf $f(r,n)$. The value of SIR for an arbitrary pair of points is then

$$\begin{aligned} S(r,t) &= \frac{\phi(r)}{NU(r,t)}, \\ U(r,t) &= \int_V \phi\big(|r-r'|\big) f(r',t)dr', \end{aligned} \tag{8}$$

and the average SIR over ensemble of N points is calculated as

$$q(t) = \frac{1}{N}\int_V \frac{\phi(r)}{U(r)} f(r)dr. \tag{9}$$

Observe that in the stationary case (9) becomes

$$q = \int_V S(r)f(r)dr = \frac{1}{N}\int_V \frac{\phi(r)}{U(r)} f(r)dr. \tag{10}$$

In the non-stationary case, the sample distribution function $G(s,t)$ of the corresponding time-series $S(r,t)$ depends on the ensemble of trajectories of moving devices, that is, on the pdf $f(r,t)$, via the complex non-linear law. The actual challenge in this case is a numerical modeling of the trajectories and collecting the corresponding statistics related to the SIR distribution density $g(S,t)$. One of the questions to be answered is: *may the distribution density $g(S,t)$ be stationary even when density $f(r,t)$ satisfies the solution of non-stationary kinetic equation?* Another question is related to communication reliability during finite period. In the discrete time moments with unit step $t_n = n$, we have

$$P_n = P\{S \le s_{cr}\} = \int_0^{s_{cr}} g(S,n)dS. \tag{11}$$

Implying that the communications between a pair of devices is reliable during the period from $t_n = n+1$ to $t_k = k = n+m, m \ge 1$ with probability

$$Q_{n,m} = P_{n+m+1}\prod_{i=1}^{m}(1-P_{n+i}). \tag{12}$$

In a stationary case, the probability in (12) is

$$Q_m = P(1-P)^m. \tag{13}$$

Thus, the task of interest is analysis of (12) when pdf $f(r,t)$ satisfies the following Fokker-Planck equation describing the movement of D2D devices in V

$$\frac{\delta f}{\delta t} + div_r\big(u(r,t)f\big) - \frac{\lambda(t)}{2}\Delta f = 0, \tag{14}$$

where the latter can be solved for a given initial condition $f(r,t)|_{t=0} = p(r)$.

The solution is based on the time horizon T. In what follows, $u(r,t)$ is drift velocity and $\lambda(t)$ is non-negative diffusion coefficient. In this study the functions $u(r,t)$, $\alpha(t)$ and $p(r)$ are assumed to be known. In practice, these values need to be estimated using statistical methods.

B. SIR CDF Evolution Equation

Before we proceed addressing the general case, we first consider the special case. Particularly, we construct the evolution equation for $g(S,t)$ when the SIR in (8) at any time t is a monotonic function of a distance r between pare of points. Note that it includes the case of most propagation models that are usually written in the form $L(r) = Ar^{-\gamma}$, where $L(r)$ is the path loss at the separation distance r, A is the coefficient capturing the effect of operating frequency and antenna gains, and γ is the so-called path loss exponent [23].

For the sake of exposition, in what follows, we consider one-dimensional non-stationary pdf of the distances $f(x,t)$. Then pdf of the random variable S takes the following form

$$g(S,t) = f\Big(S^{-1}(x,t),t\Big)\frac{\delta}{\delta x}\Big(S^{-1}(x,t)\Big)\Big|_{x=x(S,t)}. \tag{15}$$

The time dependence of pdf $g(S,t)$ is induced by the non-stationary character of pdf $f(x,t)$ and by self-consistent average field $U(x,t)$ according to (8) as

$$\frac{\delta S(x,t)}{\delta t} = -\frac{S}{U}\frac{\delta U}{\delta t}. \tag{16}$$

From (8) and (14) it follows that

$$\begin{aligned} &\frac{\delta U(x,t)}{\delta t} = \int \phi\big(|x-x'|\big)\frac{\delta f(x',t)}{\delta t}dx' = \\ &\int \phi\big(|x-x'|\big)\Big(-\frac{\delta\big(u(x',t)f(x',t)\big)}{\delta x'} + \frac{\lambda}{2}\frac{\delta^2 f(x,t)}{\delta x'^2}\Big)dx' = \\ &\frac{\delta}{\delta x}\int \phi\big(|x-x'|\big)u(x',t)f(x',t)dx' + \frac{\lambda}{2}\frac{\delta^2 U(x,t)}{\delta x^2}. \end{aligned} \tag{17}$$

From (15) it follows that the partial time difference of pdf $g(S,t)$ can be obtained by differentiating of the complex function $f\big(x(S,t),t\big)$ as

$$\begin{aligned} \frac{\delta g(S,t)}{\delta t} &= -f\big(x(S,t),t\big)\frac{\delta^2 S/\delta x\delta t}{1/(\delta S/\delta x)^2} + \\ &+ \frac{1}{\delta S/\delta x}\Big(\frac{\delta f\big(x(S,t),t\big)}{\delta t} + \frac{1}{\delta S/\delta x}\frac{\delta f\big(x(S,t),t\big)}{\delta x}\frac{\delta S}{\delta t}\Big). \end{aligned} \tag{18}$$

The difference $\delta S(x,t)/\delta x$ is calculated with using (16) and by substituting $x = x(S,t)$ to the resulting expression leading to

$$\frac{\delta g(S,t)}{\delta t} = \frac{\delta\phi(x)/\delta x}{NU(x,t)}\Big|_{x=x(X,t)} - \frac{\phi(x)}{NU^2(x,t)}\frac{\delta U(x,t)}{\delta t}\Big|_{x=x(S,t)}, \tag{19}$$

and also the second derivative is

$$\frac{\delta^2 g(S,t)}{\delta t} = -\frac{\delta\phi(x)/\delta x}{NU^2(x,t)}\frac{\delta U(x,t)}{\delta t}\Big|_{x=x(S,t)} - \\ - \frac{\phi(x)}{NU^2(x,t)}\frac{\delta^2 U(x,t)}{\delta x \delta t}\Big|_{x=x(S,t)} + \\ + \frac{2\phi(x)}{NU^3(x,t)}\frac{\delta U(x,t)}{\delta x}\frac{\delta U(x,t)}{\delta t}\Big|_{x=x(S,t)}.$$

Thus, the final equation for SIR evolution becomes

$$\frac{\delta g(S,t)}{\delta t} = \\ -\frac{1}{\delta S/\delta x}\Big(\frac{\delta\big(u(x,t)f(x,t)\big)}{\delta x} - \frac{\lambda(t)}{2}\frac{\delta^2 f(x,t)}{\delta x^2}\Big)\Big|_{x=x(S,t)} + \\ \frac{1}{(\delta S/\delta x)^2}\frac{\delta f(x,t)}{\delta x}\Big(\frac{\delta\phi(x)/\delta x}{NU(x,t)} - \frac{\phi(x)}{NU^2(x,t)}\frac{\delta U(x,t)}{\delta x}\Big)\Big|_{x=x(S,t)} + \\ \frac{f(x,t)}{1/(\delta S/\delta x)^2}\Big(\frac{\delta\phi(x)/\delta x}{NU^2(x,t)}\frac{\delta U(x,t)}{\delta t} + \frac{\phi(x)}{NU^2(x,t)}\frac{\delta^2 U(x,t)}{\delta x \delta t} - \\ \frac{2\phi(x)}{NU^3(x,t)}\frac{\delta U(x,t)}{\delta x}\frac{\delta U(x,t)}{\delta t}\Big)\Big|_{x=x(S,t)}. \quad (20)$$

Observe that $\delta S(x,t)/\delta t$ is defined using (17) while $\delta U(x,t)/\delta x$ can be obtained from (8). Finally, the $\delta f\big(x(S,t),t\big)/\delta t$ is determined by the kinetic equation (14). Trivially that for the practical case, we have obtained rather complex evolution equation (18) for pdf $g(S,t)$. Although the structure of (18) is more simple compared to the kinetic equation in (14) for pdf $f(x,t)$, we still cannot obtain the solution in the closed-form. Thus, in what follows, we proceed using the mixed analytical-simulation framework.

IV. Non-Stationary SIR Trajectory Modeling

Consider again (14) and treat it as an empirical equation assuming that the drift $u(r,t)$ and diffusion $\lambda(t)$ are obtained from the sample distribution function of concrete time-series of coordinates and their increments. The typical drift for normalized non-stationary random variable is presented in Fig. 1.

In what follows, we consider two special cases for diffusion: (i) $\lambda = 0.001$ (normalized units), and (ii) $\overline{\lambda} = 10\lambda = 0.01$. In Fig. 2, a feasible trajectory of SIR obtained using (20) is shown. It is generated for $N = 10$ devices assuming reflection boundary conditions and $\lambda = 0.001$. The number of time steps is $T = 3000$. For this special case $P\{S \leq s_{cr}\} \equiv P_1 = 0.33$.

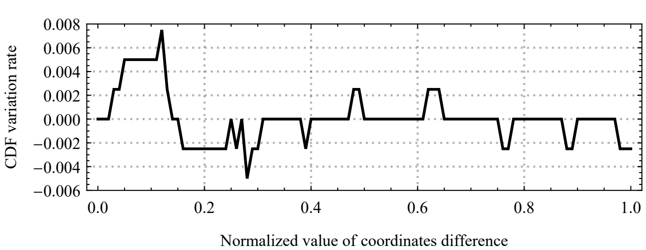


Fig. 1. The typical drift parameter in the empirical kinetic equation.

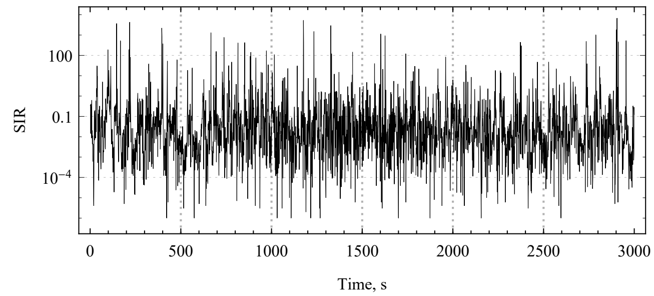
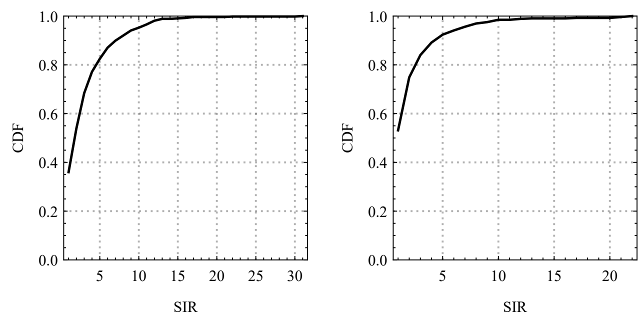


Fig. 2. The SIR trajectory for diffusion parameter α.



(a) $SIR > 0.01$ (b) $SIR \leq 0.01$

Fig. 3. The CDFs of SIR for diffusion $\lambda = 0.001$.

The conditional probability distribution over the total length of trajectory (i.e., for initial time moment $n = 0$) for two considered cases is obtained. In these illustrations, the CDFs in the form $A_m = \sum_{i=1}^{m} a_i$ and $B_m = \sum_{i=1}^{m} b_i$ are presented, respectively. The first one, depicted in Fig. 3(a), corresponds to the probabilities a_i assuming $S > s_{cr}$, and the second one, shows b_i assuming $S \leq s_{cr}$, is provided in Fig. 3(b). Observe that in a stationary case, the distributions A_m and B_m need to be geometric, i.e., $a_{i+1}/a_i = 1 - P_1$ and $b_{i+1}/b_i = P_1$. Figs. 3(a) and 3(b) show, that this is definitely not the case and, thus, the distribution of SIR values is not stationary. Indeed, the probabilities $a + i$ for $S > s_{cr}$ are $a_1 = 0.39$, $a_2 = 0.19$, and $a_3 = 0.1$ and are approximately geometrical with parameter 0.5. However, the hypothesis of geometric distribution (and, thus, stationarity) is neglected as the next three probabilities are $a_4 = a_5 = a_6 = 0.05$ greatly deviating from the geometric distribution with parameter 0.5.

In Fig. 4 the SIR trajectory for $\overline{\lambda} = 0.01$ corresponding to $P\{S \leq s_{cr}\} \equiv P_2 = 0.41$ is shown. Observe that the higher the diffusion coefficient in Fokker-Planck equation (14) leads to the lower probability of steady communication. Similarly to the previous case, the SIR CDF is non-stationary. The condition $A_m \sum_{i=1}^{m} a_i$ for $S > s_{cr}$ and $B_m = \sum_{i=1}^{m} b_i$ for $S \leq s_{cr}$ are shown in Figs. 5(a) and 5(b).

V. Conclusions

In this paper, we have developed a methodology for time-dependent performance analysis of D2D communications us-

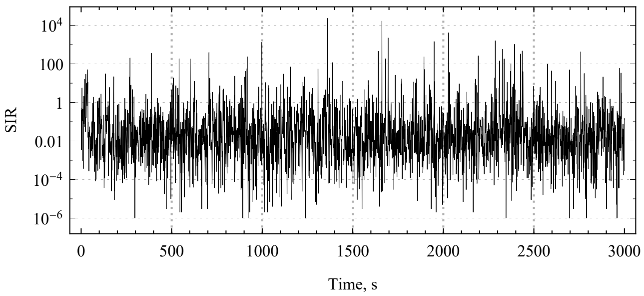
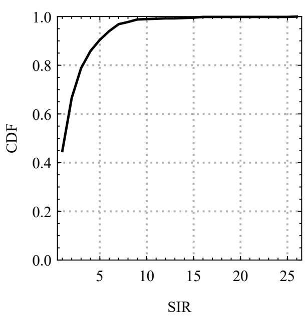
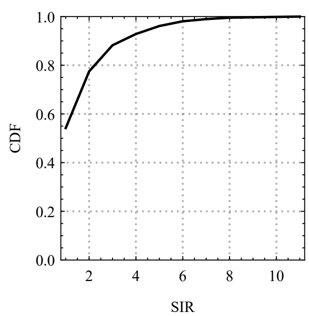


Fig. 4. The SIR trajectory for diffusion 10λ.



(a) $SIR > 0.01$



(b) $SIR \leq 0.01$

Fig. 5. The CDFs of SIR for diffusion $10\lambda = 0.01$.

ing the theory of kinematic equations. We have derived the time-evolution equation for the the SIR CDF representing the stochastic movement of users using the Fokker-Plank equation. Although this equation can be written in explicit form, the analytical solution is available in special cases only.

We have then developed a generic mixed simulation-analytic approach for performance assessment of D2D communications. Compared to the conventional performance analysis techniques, concentrating on ergodic quantities, such as outage probability, the proposed approach allows to get time-dependent metrics including the CDFs of the steady communications periods and outages.

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