General Clipping Modeling and DSP-Based Mitigation for Wideband A/D Interface and RF Front-End of Emerging Radio Receivers

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***Abstract*—Emerging wireless communications concepts, such as cognitive radio, bring various challenges in the implementation of radio receivers. One of the concerns is the dynamic range of the receiver front-end, which might be insufficient in certain situations, e.g., if strong blocker signals are present concurrently with weaker interesting signals. Overdrive of the receiver front-end causes signal clipping and therefore considerable amount of nonlinear distortion is created. This paper derives a general parametric clipping model using time-dependent Fourier series and analyses the model in different clipping scenarios. The paper also proposes a method to exploit the derived model in a radio receiver for digital post-processing in order to mitigate unwanted clipping distortion.**

I. Introduction

Dynamic range is one of the main concerns in emerging wideband radio receivers for wireless communications, the ultimate example being a cognitive radio [1], [2]. Partially the requirement for high dynamic range is due to the modern communication waveforms, which tend to have high peak-to-average power ratio [3]. However, even more important reason is that wideband radio receivers suffers from a blocker problem [4]. It is possible that the receiver front-end is not selective enough to attenuate out-of-band blockers, which may cause inband interference because of receiver nonlinearities. Other scenario is a wideband receiver digitizing several signals at once, which may cause high-power signals to block weaker signals due to the receiver nonlinearities. Strong blocker signals can overdrive the A/D interface and therefore induce signal clipping, which heavily distorts the received waveform causing a vast amount of nonlinear distortion.

This paper proposes that time-dependent Fourier series can be used to model clipping occurring in the A/D interface of a radio receiver. Similar approach is also proposed in [5], but it considers only zero-symmetric clipping occurring equally in I and Q branches of a complex signal. In this paper, the clipping model is expanded to cover also more realistic scenarios, where clipping is non-symmetric w.r.t. zero and unequal in I and Q branches. This is important since real radio receivers suffer from non-idealities such as DC offset and I/Q imbalance [6]. The derived clipping model is useful for understanding better the clipping phenomenon and nonlinear distortion it causes. In addition, this paper proposes a clipping mitigation technique based on the derived parametric model.

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II. A General Clipping Model

A homodyne receiver architecture is a typical structure for modern communications receivers. A complex signal comprising I and Q branches are separately digitized using two A/D converters, which together are called an I/Q ADC. This is depicted in Fig. 1. Due to the differences in A/D converters and other electrical components, the clipping may occur differently in I and Q branches. Hence the following clipping model is physically well-grounded.

The input signal for the I/Q ADC in Fig. 1 is defined here as an analytic bandpass signal

$$v_{IN}(t) = v_{IN,I}(t) + jv_{IN,Q}(t) = A(t)e^{j\theta_c(t)}, \qquad (1)$$

where $A(t) \in [0,1]$ is the normalized signal envelope and $\theta_c(t) = \omega_c t + \varphi(t)$ includes angular frequency ω_c and phase $\varphi(t)$. Signal $v_{IN}(t)$ gets clipped, if it exceeds the full-scale range of the I/Q ADC. Therefore, the clipped output signal of the I/Q ADC is $v_{OUT}(t) = v_{OUT,I}(t) + jv_{OUT,Q}(t)$, where

$$v_{OUT,I}(t) = \begin{cases} v_{IN,I}(t), & V_{L,I} \le v_{IN,I}(t) \le V_{H,I} \\ V_{H,I}, & v_{IN,I}(t) > V_{H,I} \\ V_{L,I}, & v_{IN,I}(t) < V_{L,I}, \end{cases} \qquad (2)$$

$$v_{OUT,Q}(t) = \begin{cases} v_{IN,Q}(t), & V_{L,Q} \le v_{IN,Q}(t) \le V_{H,Q} \\ V_{H,Q}, & v_{IN,Q}(t) > V_{H,Q} \\ V_{L,Q}, & v_{IN,Q}(t) < V_{L,Q}. \end{cases} \qquad (3)$$

As can be seen from (2) and (3), clipping levels are defined separately for I and Q branches. In order to create even more

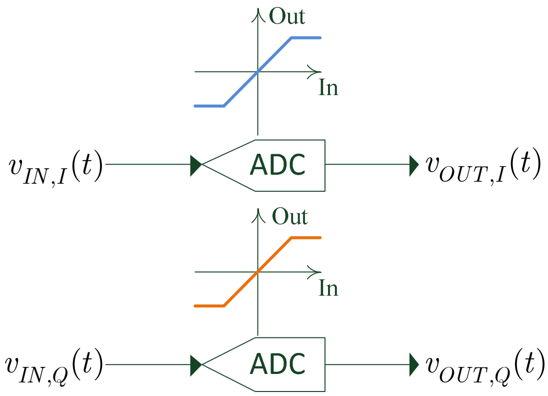

Fig. 1. Communication receivers typically use an I/Q ADC, which consists of two separate analog-to-digital converters. Input-output characteristics are depicted in the block diagram in order to illustrate the clipping phenomenon.

general model, for both branches the higher clipping levels $V_{H,I}$, $V_{H,Q} \in [0,1]$ and the lower clipping levels $V_{L,I}$, $V_{L,Q} \in [-1,0]$ are described independently.

Signal clipping causes nonlinear distortion to the signal and different distortion orders can be modeled separately using time-dependent Fourier series. Hence, the output signal can be written as

$$v_{OUT}(t) = \sum_{m=-\infty}^{\infty} a_m(t) e^{jm\theta_c(t)}, \tag{4}$$

where $a_m(t) = a_{m,I}(t) + ja_{m,Q}(t)$ are the time-dependent Fourier coefficients. The same approach is also used in [5], but this paper expands the model to more general case, where clipping is non-symmetric w.r.t. zero and the clipping levels are unequal in I and Q branches. Fig. 2 illustrates the signal clipping as a function of θ_c over a period of 2π and therefore covers all the possible values θ_c can have at a certain time moment t. Generally, the Fourier coefficients for I branch are calculated as

$$a_{m,I}(t) = \frac{1}{2\pi} \int_{2\pi} v_{OUT,I}(t) e^{-jm\theta_c(t)} \mathrm{d}\theta_c(t). \tag{5}$$

As shown in Fig. 2, the integration period is divided into five parts, i.e., the I branch Fourier coefficients are

$$\begin{aligned} a_{m,I}(t) = \frac{1}{2\pi} \Bigg[& \int_{-\pi/2}^{-r_{H,I}(t)} v_{IN,I}(t) e^{-jm\theta_c(t)} d\theta_c(t) \\ & + \int_{-r_{H,I}(t)}^{r_{H,I}(t)} V_{H,I} e^{-jm\theta_c(t)} d\theta_c(t) + \int_{r_{H,I}(t)}^{\pi - r_{L,I}(t)} v_{IN,I}(t) e^{-jm\theta_c(t)} d\theta_c(t) \\ & + \int_{\pi - r_{L,I}(t)}^{r_{L,I}(t)+\pi} V_{L,I} e^{-jm\theta_c(t)} d\theta_c(t) + \int_{r_{L,I}(t)+\pi}^{3\pi/2} v_{IN,I}(t) e^{-jm\theta_c(t)} d\theta_c(t) \Bigg] \end{aligned} \tag{6}$$

and similarly the Q branch Fourier coefficients are

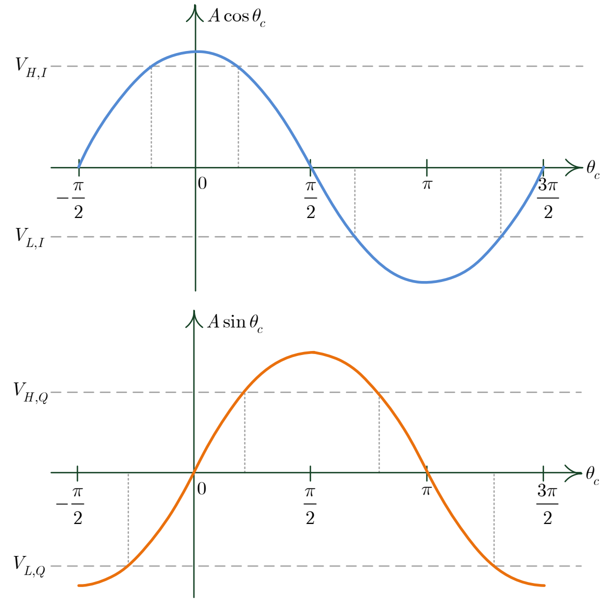

Fig. 2. Illustration of signal clipping for I and Q branches. Signals are presented as a function of the instantaneous angle θ_c over one period of 2π, i.e., the model covers all the possible values of θ_c.

$$\begin{aligned} a_{m,Q}(t) = \frac{1}{2\pi} \Bigg[& \int_{-\pi/2}^{-r_{L,Q}(t)} V_{L,Q} e^{-jm\theta_c(t)} d\theta_c(t) \\ & + \int_{-r_{L,Q}(t)}^{r_{H,Q}(t)} v_{IN,Q}(t) e^{-jm\theta_c(t)} d\theta_c(t) + \int_{r_{H,Q}(t)}^{\pi - r_{H,Q}(t)} V_{H,Q} e^{-jm\theta_c(t)} d\theta_c(t) \\ & + \int_{\pi - r_{H,Q}(t)}^{r_{L,Q}(t)+\pi} v_{IN,Q}(t) e^{-jm\theta_c(t)} d\theta_c(t) + \int_{r_{L,Q}(t)+\pi}^{3\pi/2} V_{L,Q} e^{-jm\theta_c(t)} d\theta_c(t) \Bigg], \end{aligned} \tag{7}$$

where the following auxiliary variables are utilized:

$$r_{H,I}(t) = \arccos \frac{V_{H,I}}{A(t)}, \tag{8}$$

$$r_{L,I}(t) = \arccos \frac{-V_{L,I}}{A(t)}, \tag{9}$$

$$r_{H,Q}(t) = \arcsin \frac{V_{H,Q}}{A(t)}, \tag{10}$$

$$r_{L,Q}(t) = \arcsin \frac{-V_{L,Q}}{A(t)}. \tag{11}$$

In addition, the following auxiliary variables are introduced to make subsequent equations more concise:

$$s_{H,I}(t) = \sin\big(r_{H,I}(t)\big) = \sqrt{1 - \left(\frac{V_{H,I}}{A(t)}\right)^2}, \tag{12}$$

$$s_{L,I}(t) = \sin\big(r_{L,I}(t)\big) = \sqrt{1 - \left(\frac{V_{L,I}}{A(t)}\right)^2}, \tag{13}$$

$$s_{H,Q}(t) = \cos\big(r_{H,Q}(t)\big) = \sqrt{1 - \left(\frac{V_{H,Q}}{A(t)}\right)^2}, \tag{14}$$

$$s_{L,Q}(t) = \cos\left(r_{L,Q}(t)\right) = \sqrt{1 - \left(\frac{V_{L,Q}}{A(t)}\right)^2}. \tag{15}$$

The final form of Fourier coefficients after calculating the integrals are shown on the bottom of the page, where time index t is omitted in order to make the presentation more concise. Additionally, the equations for the Fourier coefficients are represented (on the bottom of the page) in a case, where clipping is non-symmetric w.r.t. zero but equal in I and Q branches. Notation of $\pm$ and $\mp$ is used in such a manner that the upper signs in an equation are valid at the same time and correspondingly the lower signs. An important note about the model is that it is assuming, for all t, that $A(t) \geq V_{H,I}$, $A(t) \geq V_{H,Q}, A(t) \geq -V_{L,I}$ and $A(t) \geq -V_{L,Q}$, i.e., the signal envelope is always clipped or at the clipping level. When this is not the case (the signal envelope is not clipping at the moment), the corresponding clipping levels should be set equal to $\pm A(t)$, depending if it is the higher ($+$) or the lower ($-$) clipping level, for computational purposes. When the envelope is equal to clipping level, it means that the signal is not clipped, but the clipping model is still valid and hence the model can be used also for the unclipped signal parts by tuning the clipping levels as described. In practice, e.g. in clipping mitigation, it is often unnecessary to apply the model for unclipped signal parts since obviously $v_{OUT}(t) = v_{IN}(t)$.

Simplifications of the clipping model can be carried out even further by considering a case, where clipping is zero-symmetric but unequal in I and Q branches, i.e., $V_I = V_{H,I} = -V_{L,I}$, $V_Q = V_{H,Q} = -V_{L,Q}$, $r_I(t) = r_{L,I}(t) = r_{H,I}(t)$, $r_Q(t) = r_{L,Q}(t) = r_{H,Q}(t)$, $s_I(t) = s_{L,I}(t) = s_{H,I}(t)$, $s_Q(t) = s_{L,Q}(t) = s_{H,Q}(t)$. Then the Fourier coefficients can be presented in a form of

$$a_m(t) = \begin{cases} \frac{A(t)}{2} \pm \frac{1}{\pi}\Big[A(t)\big(r_Q(t) \mp r_I(t)\big) \\ +V_Q s_Q(t) \pm V_I s_I(t)\Big], & m = \pm 1 \\ 0, & m = 0, \pm 2, \pm 4, \ldots \\ \frac{2}{\pi m(m^2-1)}\Big[mA\big(s_I(t)\cos(mr_I(t)) \\ +s_Q(t)\sin(mr_Q(t))\big) \\ -V_I \sin(mr_I(t)) \\ -V_Q \cos(mr_Q(t))\Big], & m = \pm 3, \pm 5, \ldots \end{cases} \tag{16}$$

The most simple case (also presented in [5]) is the one, where zero-symmetric clipping occurs equally in I and Q branches, i.e., $V = V_I = V_Q$, $s(t) = s_I(t) = s_Q(t)$. It follows that the Fourier coefficients are then

$$a_m(t) = \begin{cases} A(t) + \frac{2}{\pi}\big[Vs(t) - A(t)r_I(t)\big], & m = +1 \\ 0, & m = 0, \pm 2, \pm 4, \ldots \\ 0, & m = -1, +3, -5, \ldots \\ \frac{4}{\pi m(m^2-1)} \\ \cdot\big[mA(t)s(t)\cos(mr_I(t)) & m = -3, +5, -7, \ldots \\ -V\sin(mr_I(t))\big], \end{cases} \tag{17}$$

Due to the symmetry and equality of the clipping levels, the last case presented in (17) provides the most simple model. This is stemming from the fact that only odd-order nonlinear distortion is caused. To be more specific, there is distortion only on every other odd order, i.e., on +1, –3, +5, –7,... as defined in (17).

For general case (non-symmetric w.r.t. zero and unequal clipping in I and Q branches):

$$a_m = \begin{cases} \frac{1}{\pi}\Big\{A(s_{L,I} - s_{H,I}) + V_{H,I}r_{H,I} + V_{L,I}r_{L,I} + j\Big[A(s_{L,Q} - S_{H,Q}) + V_{H,Q}(\frac{\pi}{2} - r_{H,Q}) + V_{L,Q}(\frac{\pi}{2} - r_{L,Q})\Big]\Big\}, & m = 0 \\ \frac{A}{2} \pm \frac{1}{2\pi}\Big[A(r_{L,Q} \pm r_{L,I} + r_{H,Q} \mp r_{H,I}) + V_{H,Q}s_{H,Q} \pm V_{H,I}s_{H,I} - V_{L,Q}s_{L,Q} \mp V_{L,I}s_{L,I}\Big] & m = \pm 1 \\ \frac{1}{\pi m(m^2-1)}\Big\{mA\big(s_{H,I}\cos(mr_{H,I}) - s_{L,I}\cos(mr_{L,I})\big) - V_{H,I}\sin(mr_{H,I}) - V_{L,I}\sin(mr_{L,I}) \\ -j[mA\big(s_{H,Q}\cos(mr_{H,Q}) - s_{L,Q}\cos(mr_{L,Q})\big) + V_{H,Q}\sin(mr_{H,Q}) - V_{L,Q}\sin(mr_{L,Q})]\Big\}, & m = \pm 2, \pm 4, \ldots \\ \frac{1}{\pi m(m^2-1)}\Big\{mA\big(s_{H,I}\cos(mr_{H,I}) + s_{L,I}\cos(mr_{L,I}) + s_{H,Q}\sin(mr_{H,Q}) + s_{L,Q}\sin(mr_{L,Q})\big) \\ -V_{H,I}\sin(mr_{H,I}) + V_{L,I}\sin(mr_{L,I}) - V_{H,Q}\cos(mr_{H,Q}) + V_{L,Q}\cos(mr_{L,Q})\Big\}, & m = \pm 3, \pm 5, \ldots \end{cases}$$

For non-symmetric case with equal clipping in I and Q branches ($V_L = V_{L,I} = V_{L,Q}$, $V_H = V_{H,I} = V_{H,Q}$, $s_L = s_{L,I} = s_{L,Q}$, $s_H = s_{H,I} = s_{H,Q}$):

$$a_m = \begin{cases} \frac{1+j}{\pi}\Big[A(s_L - s_H) + V_H r_{H,I} + V_L r_{L,I}\Big], & m = 0 \\ A + \frac{1}{\pi}\Big[A(-r_{L,I} - r_{H,I}) + V_H s_H - V_L s_L\Big], & m = +1 \\ \frac{1-j}{\pi m(m^2-1)}\Big[mA\big(s_H\cos(mr_{H,I}) - s_L\cos(mr_{L,I})\big) - V_H\sin(mr_{H,I}) - V_L\sin(mr_{L,I})\Big], & m = \pm 2, \pm 6, \ldots \\ \frac{1+j}{\pi m(m^2-1)}\Big[mA\big(s_H\cos(mr_{H,I}) - s_L\cos(mr_{L,I})\big) - V_H\sin(mr_{H,I}) - V_L\sin(mr_{L,I})\Big], & m = \pm 4, \pm 8, \ldots \\ \frac{2}{\pi m(m^2-1)}\Big[mA\big(s_H\cos(mr_{H,I}) + s_L\cos(mr_{L,I})\big) - V_H\sin(mr_{H,I}) + V_L\sin(mr_{L,I})\Big], & m = -3, +5, -7, \ldots \\ 0, & m = -1, +3, -5, \ldots \end{cases}$$

III. Clipping Mitigation

The derived clipping model can also be used for clipping mitigation purposes, e.g., in a radio receiver. From the received signal $v_{OUT}(t)$, it is possible to extract the estimate of $a_{0,I}(t)$, which is here denoted as $\hat{a}_{0,I}(t)$ and corresponds to the signal content around 0 Hz. The extraction can be done, e.g., using a narrow lowpass filter. If reliable estimates of the angular frequency ω_c and the phase $\varphi(t)$ can be obtained, then the only unknown variable in the equation of the sent signal $v_{IN}(t)$, see (1), is the signal envelope $A(t)$.

In the derivation of the envelope estimate $\hat{A}(t)$, the well-known infinite series representations

$$\sqrt{1-x} = 1 - \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{16} - \ldots \quad (18)$$

$$\arccos x = \frac{\pi}{2} - x - \frac{1}{2}\frac{x^3}{3} - \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} - \ldots \quad (19)$$

are employed. A rough estimate of $A(t)$ is obtained using only the first two terms of (18) and (19). In case of non-symmetric but equal clipping in I and Q, the envelope estimate is

$$\hat{A}(t) = \frac{V_L^2 - V_H^2}{\pi(2\hat{a}_{0,I}(t) - V_H - V_L)}. \quad (20)$$

After calculating $\hat{A}(t)$, the values can be used in (1) to obtain an estimate of unclipped signal. It is worth noticing that (20) is exploiting the fact that $a_{0,I}(t) \neq 0$, when clipping is not symmetric w.r.t. zero. In a real hardware, this can considered to be a rather realistic assumption. The estimator in (20) requires knowledge about the clipping levels and these can be estimated in practice as shown, e.g., in [7].

IV. Simulation Example and Compensation Performance

Fig. 3 shows a simulation example of a QPSK signal, which is clipped non-symmetrically but equally in I and Q branches. The original signal has a center frequency of 10 MHz and oversampling ratio of 256 is used here only for illustrational purposes, i.e., to show the clipping distortion without aliasing. The clipped signal has new frequency content around zero (among others), but not at -10 MHz, 30 MHz, -50 MHz, ... and therefore matches with the clipping model derived in Section II for non-symmetric but equal I and Q clipping.

The bottom part of Fig. 3 illustrates the spectrum after the signal has been reconstructed using the mitigation technique proposed in Section III, when perfect estimates of ω_c, $\varphi(t)$ and clipping levels are assumed. It can be seen that the strongest distortion components are considerably suppressed. For example, the third order distortion is here suppressed by 17 dB. However, due the rough approximations made in the derivation of (20), the clipping mitigation is not perfect. More accurate estimates of $\hat{A}(t)$ could be obtained, but then also an increase in computational complexity is expected.

V. Conclusion

The paper derived a general signal clipping model using time-dependent Fourier series and also proposed a simple clipping mitigation method based on the derived model. The mitigation technique showed promising performance for clipping distortion suppression and has potential for even better performance, if combined with other clipping mitigation techniques or if the derived Fourier model is exploited even more extensively.

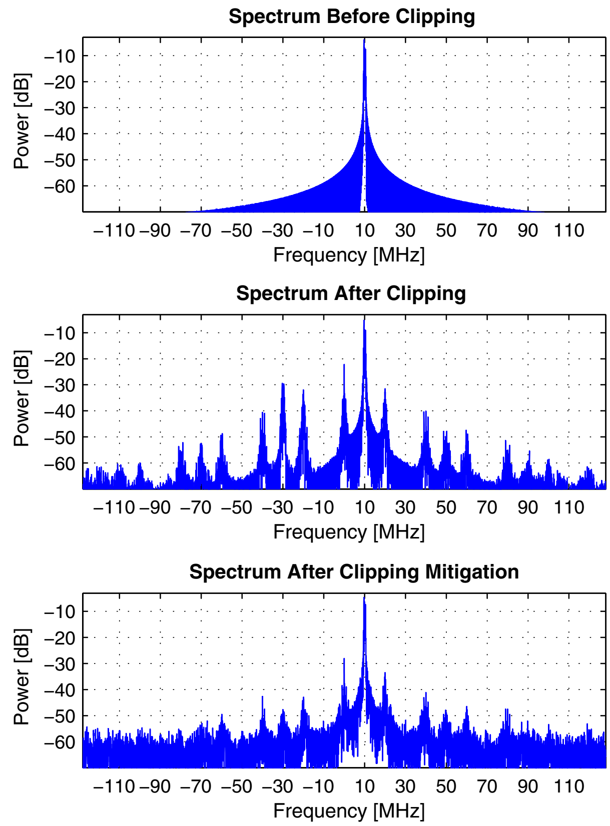

Fig. 3. Simulation example of a QPSK signal, which gets clipping non-symmetrically w.r.t. zero but equally in I and Q branches. The clipping distortion is then mitigated using the proposed method.

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